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REGRESSION MODELS OF SPECIFIC FUEL CONSUMPTION CURVES AND CHARACTERISTICS OF ECONOMIC OPERATION OF INTERNAL COMBUSTION ENGINES

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Abstract. *The present study deals with the procedure of establishment of regression models of closed curves for specific fuel consumption constant values, as well as of curves for the minimum specific fuel consumption in internal combustion engines, which is illustrated on a concrete example. Characteristics and development trends of automatic transmissions which provide the operation of an engine under all exploitation regimes on a minimum fuel consumption curve, i.e. on the characteristic of an economic engine operation, are shown in order to illustrate how actual these issues are nowadays. Regression models are presented in the form of a polynomial in the function of working regime parameters – effective pressure and number of revolutions. A graphical representation of specific fuel consumption constant value curves is given for a concrete example used in this research study, in the form of a relief topographic characteristic.*

Key Words: *Regression Model, Internal Combustion Engine, Specific Fuel Consumption, Economic Operation Characteristic, Automatic Transmission.*

1. INTRODUCTION

Economic performance of an engine, expressed by specific fuel consumption, is considered to be an extremely important indicator of the advanced technological & economical level of automotive and mobile working machines. The decrease in specific fuel consumption has been over a long period of time one of the primary goals of the development and upgrading of driving systems in these engines.

Specific fuel consumption is directly dependant on the engine efficiency coefficient and net fuel heating value:

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$$b_e = \frac{3600}{H_u \cdot \eta_M} \left[\frac{kg}{KWh}\right] \tag{1}$$

where: H_u *[KJ/kg]* – net fuel heating value, η_M – engine efficiency coefficient.

The best insight into understanding of engine efficiency within the entire working range, as a function of the effective pressure *p* (i.e. moment Te) and the number of revolutions *n*, is provided by a universal diagram with closed curves of specific fuel consumption constant values, Fig. 1a.

The lowest specific fuel consumption point, usually referred to as the "economy pole", is presented in Fig. 1a as E_p. This point appears at average number of revolutions and load that is in OTO engines usually ≈90% and in diesel engines ≈75% of the maximum one, [1].

From the point of view of fuel consumption, an ideal engine working regime is at the point E_p. Thus, the aim is to achieve the longest possible operation of an engine at that point, i.e. in the area very close to it, with the maximum permitted consumption of fuel b_{eg}.

The area in which $b_e < b_{eg}$ is considered to be the area of economic engine operation.

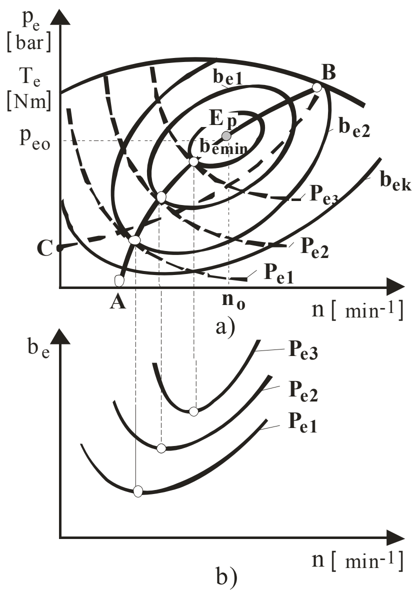

Fig. 1. Impact of Transmission Characteristics on Fuel Consumption in Internal Combustion Engines: a) Universal Diagram of Specific Fuel Consumption, b) Specific Fuel Consumption Curves

However, an engine may also work frequently at partial loading, i.e. at partial characteristics. Each power level P_{ei}, i=1,...,n has a separate fuel consumption curve and each of them has a minimum fuel consumption point, Fig. 1b.

By mapping the minimum specific fuel consumption points at the universal diagram, the AB curve, which represents the characteristic of minimum specific fuel consumption or economic engine operation, is obtained, [2], [3].

How far the operating regimes will be from the economic operation characteristic depends primarily on regulation and control abilities of the driving system as shown in Fig. 1a.

For example, for a classical type of a 4-step or 5-step gearbox, minimum specific fuel consumption is presented by BC curve.

The BC curve in Fig. 1a corresponds to a classic-type, for example, 4-step or 5-step gearbox.

By implementation of automatic transmissions, together with corresponding regulation and control systems, it may be achieved that an engine works under almost all exploitation conditions on the curve AB, i.e. on the minimum specific fuel consumption characteristic [4].

Thus, specific fuel consumption at variable engine load depends to a considerable extent on regulation and control abilities of transmission to enable the engine performance on the minimum specific fuel consumption curve.

2. AUTOMATIC TRANSMISSIONS – CHARACTERISTICS AND DEVELOPMENT TRENDS

Until recently, the world of automatic transmissions has been dominated by automatic hydrodynamic-mechanical gearboxes. Nowadays, though, there are popular alternatives available. These include: AMT (automated manual transmission), CVT (continuously variable transmission) and DCT (dual clutch transmission), [5].

Automated manual transmissions (AMT) are those which offer less operating comfort compared to other transmissions; as a consequence, car producers have not accepted this type of transmission, [5].

Dual clutch transmission (DCT) represents a semi-automated transmission and it seems that its bright future is foreseeable in Europe. Dual shift gearbox (DSG) transmissions are now offered on vehicles as the Golf, Touran and the 3.2 liter Audi T.T. [5] According to some forecasts, the DSG transmission is expected to account for 25% in 2014, [6].

The CVT transmissions have been present for a rather long time, with the highest standards of operating comfort and fuel economy benefits of up to 10 per cent when compared to the automatic 4-speed multi-ratio transmissions, [5]. These transmissions are popular in Japan (Mazda and Toyota), with a rising popularity in North America, while still searching for their place in Europe. Further development of these transmissions continues by application of new materials and electronics. There are, for example, Audi with its Multitronic, Ford with the C-Max and Mercedes-Benz with the latest generation A-Class.

The Multitronic, unlike former CVTs with a rubber trapezoidal belt (V-belt), to be replaced later with a steel belt, uses a link conveyor and also has an incorporated torque sensor. [6] The comparative characteristics of AUDI A6 with varied transmissions are shown in Table 1 [6].

Table 1. Characteristics of Audi A6 with Varied Transmissions, [6]

	0 – 97km/h	Fuel consumption
A6 with 5-speed gearbox (manual)	8.2 seconds	9.9 l /100km
A6 with 5-speed gearbox (automatic–Tiptronic)	9.4 seconds	10.6 l /100km
A6 with Multitronic CVT	8.1 seconds	9.7 l /100km

In automatic hydrodynamic-mechanical transmissions there is a common trend of increasing the number of gears. The increase in the number of gears reduces the fuel consumption. There is hardly any difference, though, in the fuel savings between the CVT transmission and transmissions with 6-step automatic gearbox. Over the last few years 6 and 7-speed automatic transmissions have been gaining a very rapid market penetration. At Mercedes-Benz, the new generation 7-speed automatic transmissions are replacing the 5-speed transmissions. US Ford and General Motors have gone into co-production of automatic 6-speed transmissions, [5].

Automatic transmissions in tractors, i.e. working machines, are based on the application of hydrodynamic-mechanical or hydrostatic-mechanical power transmitters. As described by [2], if in tractors a hydrostatic transmitter with manual drive is replaced with a hydrostatic transmitter with automatic drive, the tractor performance in corn cultivation will be increased by 23% and fuel consumption will decrease by 18.3%. According to the same Reference, when the mechanical transmitter is replaced with a hydrostatic one having automatic drive, the tractor performance will increase by 32.4% and fuel consumption will be reduced by 6.4%.

In the development and upgrading of automatic transmissions, the introduction of a sophisticated electronic drive through changes in the transmission ratio is of a very special importance. Porsche, BMW, Audi and VW use adaptive programs of control through changes in the transmission ratio, which are able to learn and easily adapt themselves to new situations, [7].

In the domain of mobile working machines, solutions of automatic control of hydrostatic transmissions have been developed by application of electronic control systems capable of optimal drive adaptation to the external conditions. For example, in loading shovels manufactured by Libherr, Zettelmeyer etc., Rexroth hydrostatic transmissions are applied with the automatic multi-step Ecomat type gearbox manufactured by Zanradfabrik, [8].

In these transmissions, the hydrostatic transmission and multi-step gearbox control is performed through an electronic control block with a microprocessor. The microprocessor may incorporate two control programs, such as: either at the minimum fuel consumption or maximum performance.

The adaptive control based on the optimality criterion requires a mathematical model in which the dependence among variables is previously defined, [9]. Specifically, for the adaptive control based on the minimum specific fuel consumption, a mathematical model will be needed to define the dependence of the specific fuel consumption on the effective pressure and number of revolutions. The establishment of such a model and of the model of a minimum specific fuel consumption curve represents the basic scope and ultimate goal of the present study.

3. SPECIFIC FUEL CONSUMPTION REGRESSION MODEL

3.1. Methodology of Establishment of Regression Model

Generally, the specific fuel consumption is a nonlinear function of working regime parameters:

$$b_e = f(\mathbf{X}) \tag{2}$$

where:

$\mathbf{X}^T = [x_1, x_2, ..., x_n]$ is a vector of working regime parameters.

The nonlinear function $f(\mathbf{X})$ can be linearized in the vicinity of the working point by expanding it in the Taylor series, and after being limited to a finite number of k terms, the following expression is obtained:

$$b_e = \sum_{j=0}^{k} a_j \cdot f_j(\mathbf{X}) \tag{3}$$

where:

a_j – unknown regression coefficient

$f_j(\mathbf{X})$ – known functions of working regime parameters (incoming variables).

At a limited number of known values of b_e, coefficients a_j cannot be precisely determined; instead an estimate of coefficients $\hat{a}_j$ is given and the regression equation assumes the following form:

$$\hat{b}_e \approx \sum_{j=0}^{k} \hat{a}_j \cdot f_j(\mathbf{X}) \tag{4}$$

For N values of b_e (for example, obtained experimentally) we have evaluation:

$$\hat{b}_e \approx \sum_{j=0}^{k} \hat{a}_j \cdot f_{ij}(\mathbf{X}), \ \ i = 1, 2, ..., N \tag{5}$$

Estimate $\hat{a}_j$ of unknown coefficients a_j can be determined by application of the least square fit method. It is based on the condition that the sum of squares differences of N known values of b_e and values obtained according to the regression equation, in the same regimes $\hat{b}_e$, is a minimum one. i.e.:

$$R = \min \sum_{i=1}^{N} \left(b_{ei} - \hat{b}_{ei} \right) \tag{6}$$

A necessary and sufficing condition for the minimum of function (6) is that partial derivatives of this function by required coefficients are equal zero:

$$\frac{\partial R}{\partial \hat{a}_j} = 0, \ \ j = 0, ..., k \tag{7}$$

A system of ordinary equations is obtained from condition (7). It can be written in vectors matrix form as follows:

$$\mathbf{F}^T \cdot \mathbf{F} \cdot \hat{\mathbf{a}} = \mathbf{F}^T \cdot \mathbf{b}_e \quad (8)$$

where:

$\mathbf{F} = \begin{bmatrix} f_{10} & f_{11} & \cdots & f_{1k} \\ f_{20} & f_{21} & \cdots & f_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ f_{N0} & f_{N1} & \cdots & f_{Nk} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_1^T \\ \mathbf{f}_2^T \\ \vdots \\ \mathbf{f}_N^T \end{bmatrix}$ is a rectangular matrix, having dimensions [N x (k+1)],

defined by the values of function f_{ij} at N of known values b_e, $\mathbf{f}_i^T = [f_{i0} \quad f_{i1} \quad \cdots \quad f_{ik}]$ and $\hat{\mathbf{a}} = [\hat{a}_1 \quad \hat{a}_2 \quad \dots \quad \hat{a}_k]^T$ - vector of estimate of required (unknown) coefficients of the regression equation.

Fischer's information matrix $\mathbf{F}^T \cdot \mathbf{F}$ is a square, positively determined and non-degenerative, when N ≥ k+1, i.e. when values b_e are known at least at k+1 point of incoming variables, [10]. In that case, the Fischer's matrix has a reverse matrix $(\mathbf{F}^T \cdot \mathbf{F})^{-1}$. Therefore, the vector of the coefficient evaluation $\hat{\mathbf{a}}$ is obtained from the solution of the system of ordinary equations:

$$\hat{\mathbf{a}} = (\mathbf{F}^T \cdot \mathbf{F})^{-1} \cdot \mathbf{F}^T \cdot \mathbf{b}_e \quad (9)$$

After determination of numerical values of the $\hat{a}_j$ coefficient estimate, verification of model adequacy shall be carried out by application of the Fischer's test (F-criterion) and checking of accuracy by calculation of an average square deviation.

3.2. Examples of Regression Models Formation

Determination of a regression model of specific fuel consumption is illustrated in this paper on the basis of a diesel engine *IMR S 44/V*, power $P_e = 55$ *KW*, at $n = 2600$ min^{-1}. A universal diagram of specific fuel consumption for this engine is presented in Fig. 2, [1].

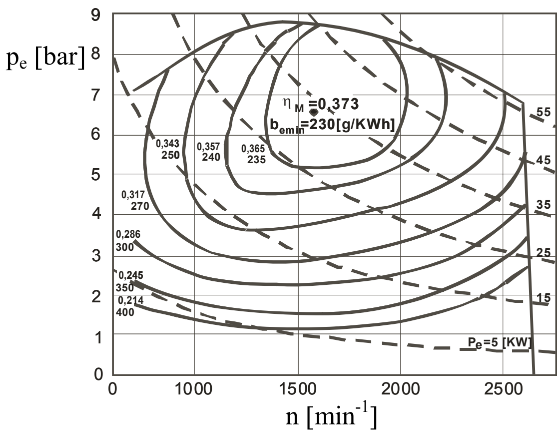

Fig. 2. Universal Diagram of Specific Fuel Consumption for Diesel Engine IMR S44/V.

According to [11], static characteristics of an internal combustion engine can be adequately described by polynomials of a corresponding degree.

In this study, the specific fuel consumption regression model in the function of working regime parameters – effective pressure *p* and number of revolutions *n*, is presented in the form of a third-degree polynomial:

$$\hat{b}_e = \hat{a}_1 + \hat{a}_2 \cdot p_e + \hat{a}_3 \cdot n + \hat{a}_4 \cdot p_e \cdot n + \hat{a}_5 \cdot p_e^2 + \hat{a}_6 \cdot n^2 + \\ + \hat{a}_7 \cdot p_e \cdot n^2 + \hat{a}_8 \cdot p_e^3 + \hat{a}_9 \cdot n^3 + \hat{a}_{10} \cdot p_e^2 \cdot n \quad (10)$$

Values of the estimate of coefficients $\hat{a}_j$, *j*=1, …, 10 are determined on the basis of the values b_e obtained from the diagram, Fig. 2, at 134 points.

With such values of coefficients $\hat{a}_j$, the specific fuel consumption regression model of the engine under consideration is:

$$\hat{b}_e = 486.101444739 - 55.747407844 \cdot p_e - 0.176743156 \cdot n - 0.012269358 \cdot p_e \cdot n + \\ + 9.008414186 \cdot p_e^2 + 0.000105622 \cdot n^2 + 0.000001499 \cdot p_e \cdot n^2 - \\ - 0.387251128 \cdot p_e^3 - 0.000000016 \cdot n^3 + 0.000187109 \cdot p_e^2 \cdot n \quad (11)$$

The F-criterion calculated value is $F_R = 0.981$, while the table value at the significance level of *0.05* is *F=1.18*, [12]. As $F_R < F_T$, the regression model is considered to be adequate.

The average square difference of values b_e obtained from Fig.2, and the values according to the regression equation $\hat{b}_e$, is $\sigma_{be}=1.33$, for 134 given points, what is considered to be acceptable accuracy.

On the basis of expression (11), curves b_e=*const.*, are drawn in Fig. 3. The 3D relief characteristic $\hat{b}_e = f(p_e, n)$ is presented in Fig. 4.

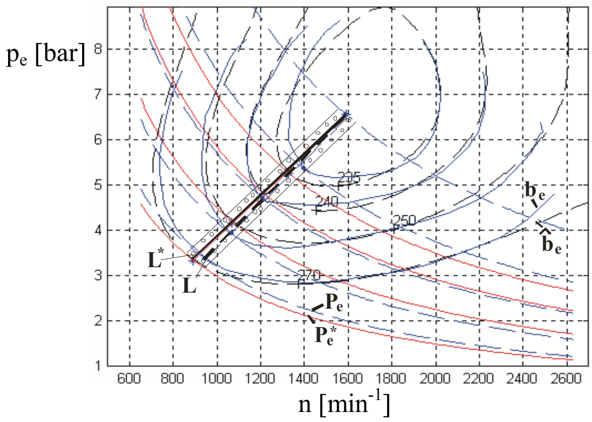

Fig. 3. Constants Values of Specific Fuel Consumption Curves and Straight Line of Minimum Specific Fuel Consumption for Diesel Motor IMR S44/V; — Constants Values of Specific Fuel Consumption Curves That Is Gives Producer (b_e); --- Constants Values of Specific Fuel Consumption Curves Obtained of Regression Models ($\hat{b}_e$); --- Constants Values of Power Curves (P_e); — Constants Values of Power Curves (P^*_e); --- (L) and — (L^*) Straight Line of Minimum Specific Fuel Consumption

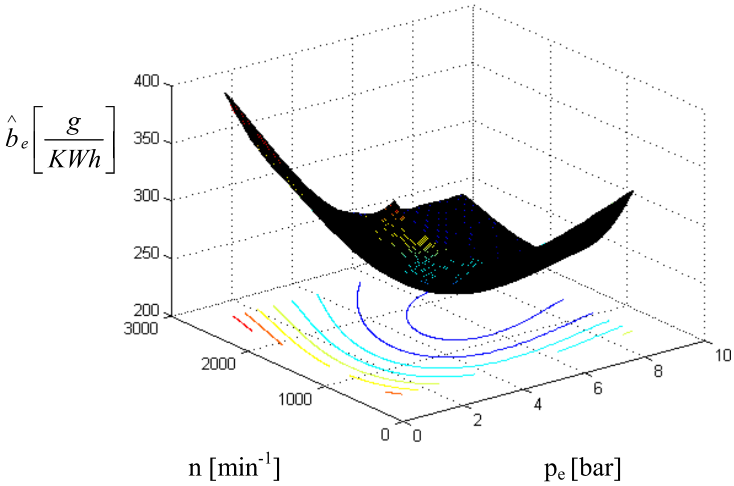

Fig. 4. 3D Relief Characteristic of Specific Fuel Consumption for IMR S44/V.

A 3D relief characteristic of efficiency coefficient $\hat{\eta}_M = f(p_e, n)$, as given in [13], is presented in Fig.5.

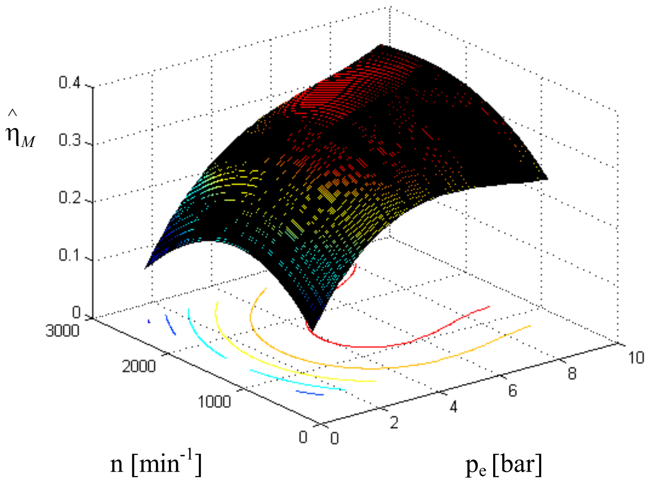

Fig. 5. 3D Characteristic of the Efficiency Coefficient for IMR S44/V

Additional to the constants values of specific fuel consumption curves b_e and $\hat{b}_e$, Fig. 3 also contains the constants values of power curves P_e and P^*_e. Points of contact, the constants values and the constants values of power curves represent points of minimum specific fuel consumption at given power, while coordinates (p_e,n) represent relevant power parameters.

Points of contact of curves b_e and P_e define regression line L:

$$p_e = -1.23558 + 0.004834 \cdot n, \quad 919 \langle n \langle 1590 \frac{1}{\min} \tag{12}$$

while points of contact of curves $\hat{b}_e$ and P_e^* define regression line L*:

$$p_e = -0.73627 + 0.004558 \cdot n, \quad 919 \langle n \langle\ 1590 \frac{1}{\min} \tag{13}$$

Straight lines L and L* represent characteristics of minimum specific fuel consumption, i.e. of economic engine performance. On the basis of Fig. 3 and expressions (12) and (13), it is evident that there is a good compatibility of these characteristics. An analogous procedure may be used to obtain also the characteristics of economic engine operation for the scope of working regime parameters $n>n_0$ and $p_e>p_{eo}$.

The effect of the engine operation at the characteristic of minimum specific fuel consumption could be better perceived through the analysis of a universal characteristic given in Fig. 2.

It is apparent that with the engine operation at the regulation characteristic, when power is $P_e=35$ *KW*, $n_M \approx 2600$ *min*$^{-1}$, specific fuel consumption is $b_e \approx 300$ *g/KWh*. For almost the same power and $n_M \approx 1600$ *min*$^{-1}$, at the "economy pole", specific fuel consumption is $b_e \approx 230$ *g/KWh*, what represents a value lower by 23%.

3.3 Utilization Importance of the Presented Results

The presented systematic procedure and obtained regression models of specific fuel consumption constant value curves and characteristics of economic engine operation represent the primary results of the present research. The utilization importance of these results is viewed in the domain of development of the automatic control system hardware and software for an engine drive at economic regime.

In the synthesizing of this automatic control system, the economic engine operation characteristic will not defined by a line but by a field of a specific width as shown in Fig. 3. The width of this field should be the least possible but at the same time large enough in order to avoid cyclical regime variations which are extremely undesirable.

The automatic control system should comprise a measuring system for determination of values of working regime parameters at a given loading level. It is on the basis of these information and presented regression models of specific fuel consumption that the electronic unit of the system generates control signals for the engine drive at economic regime.

4. CONCLUSION

It is on the basis of the contents presented in this study that we arrive at the following conclusions regarding the specific fuel consumption in internal combustion engines:

- curves of specific fuel consumption constant values can be adequately described by a regression third-degree polynomial in the function of effective pressure and number of revolutions,

- the minimum specific fuel consumption characteristic within the range of minimum values of effective pressure and number of revolutions to the values corresponding to the "semi-economy" can be described by a regression line, and, obtained regression models have utilization importance in the development of automatic control systems for engines drive at economic working regime.

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REGRESIONI MODELI KRIVIH SPECIFIČNE POTROŠNJE GORIVA I KARAKTERISTIKE EKONOMIČNOG RADA MOTORA SUS

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U radu je prikazan, i na konkretnom primjeru ilustrovan, postupak formiranja regresionih modela zatvorenih krivih konstantnih vrijednosti specifične potrošnje goriva i krive minimalne specifične potrošnje goriva. Aktuelnost problematike je ilustrovana prikazom karakteristika i trendova razvoja automatskih transmisija kojima se obezbijeđuje rad motora u svim režimima eksploatacije na krivoj minimalne specifične potrošnje goriva- tj. na karakteristici ekonomičnog rada motora. Regresioni modeli dati su u obliku polinoma, u funkciji parametara režima rada – efektivnog pritiska i broja obrta. Za konkretni primjer data je i grafička interpretacija krivih konstantnih vrijednosti specifične potrošnje goriva u obliku reljefne topografske karakteristike.

Ključne reči: *regresioni model, motor SUS, specifična potrošnja goriva, karakteristika ekonomičnog rada, automatska transmisija.*