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PRESSURE CONTROL BY THE HYDRAULIC NONLINEAR SERVOVALVE

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Abstract. *Electrohydraulic servovalves are very often used as commanding devices in the hydraulic systems. For their control many algorithms are developed. This paper gives a control algorithm based upon digital sliding regime. The properties of the proposed algorithm are compared to those of the linear PD controller by computer simulation.*

Key Words: *Hydraulic Servovalve, Variable Structure Control System, Digital Sliding System*

1. INTRODUCTION

The electrohydraulic servosystems, despite of having a high degree of nonlinearity in their operation, are very often used as commanding devices in the hydraulic systems. The control of these and many other processes implies numerous obstacles even though it is possible to reduce the degree of the servovalve model to the third order model without losing nonlinearity.

A frequent method for using linear controllers is linearization of the system around a given working point. The controller is then designed for optimal conditions around the same working point. In order to increase the range of usage, fast response and accuracy it is necessary to use nonlinear controllers. There is a great number of diverse nonlinear controllers which are used for controlling the piston position in the servovalve but not for pressure control.

This paper shows a nonlinear control algorithm designed by means of the theory of the variable structure control system and it is based upon digital sliding regime. The motive for doing this paper is an attempt to apply the digital sliding regime to the needs of pressure control by means of a servovalve in order to confirm the superiority of the presented algorithm with respect to the classical solutions. The basic advantages of the sliding regime, known to a small number of experts in the field of automatic control, include system invariability and robustness, the operation with the systems whose parameters need

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not to be accurately known (only the interval of their usage) and the system state motion which does not depend on the parameters of the model but on those of control only.

2. MATHEMATICAL MODEL

The system for pressure control by means of the hydraulic servovalve is described in Fig. 1 in the form of a block diagram:

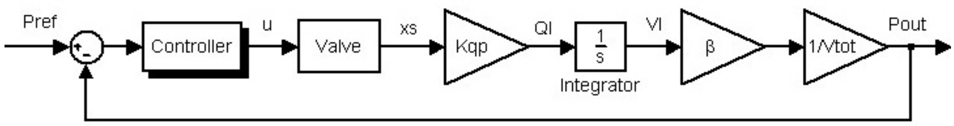

Fig. 1. Block Diagram for Pressure Control

Outlet pressure pout of the system is subtracted from given pref, thereby obtaining input into the controller. On the basis of the output from controller u the position of piston xs is controlled in the electrohydraulic servovalve on the basis of which the oil flow which nonlinearly depends on the pressure drop is regulated.

Oil flow is calculated as:

$$\dot{V}_l = Q_l = k_{qp} x_s \tag{1}$$

Equation (1) is nonlinear since it comprises coefficient kqp which depends on pressure drop Δp that is:

$$\Delta p = p_s - p_{out}\ , \tag{2}$$

$$k_{qp} = k_q \Delta p\ . \tag{3}$$

In equations (2) and (3) ps represents feed pressure while kq combines the constant which comprises valve outlet geometry, discharge coefficient and properties of oil as a working medium.

In order to obtain outlet pressure of oil pout, pressure Q_l, is integralled thereby giving parameter V_l, multiplied by factor β / V_{tot}, where β is elasticity modulus while V_{tot} is total oil volume in the hydraulic system.

Since the equations describing the dynamics of the pressure control system are nonlinear, it is necessary to do linearization by some of the techniques in order to obtain a more adequate model for regulator design. The very process of linearization is explained in [10] by which a linear third-order model is obtained introducing a set of nonlinear transformations as well as a new coordinate of state z:

$$z = -2\sqrt{p_s - p_{out}} \tag{4}$$

Linear model with limited input is given by state equation (5):

$$\begin{bmatrix} \dot{x}_{lin} \\ \dot{x}_s \\ \dot{z} \end{bmatrix} = \begin{bmatrix} -\frac{1}{T} & 0 & 0 \\ \frac{1}{A_s} & 0 & 0 \\ 0 & \frac{\beta}{V_{tot}} k_q & 0 \end{bmatrix} \begin{bmatrix} x_{lin} \\ x_s \\ z \end{bmatrix} + \begin{bmatrix} \frac{G}{T} \\ 0 \\ 0 \end{bmatrix} sat(u) \tag{5}$$

where x_{lin} oil flow in the valve, $T = 0.0015$s time constant of the linear block, $A_s = 0.0002\text{m}^2$ servovalve piston area, $G = 0.0615$ total amplifying of the linear block, $k_q = 0.0745\text{m}^2\text{s}^{-1}\text{bar}^{-0.5}$ conversion factor of the flow, $\beta = 10350$ elasticity modulus, $V_{tot} = 0.0001\text{m}^3$ total oil volume. Feeding pressure is of value $p_s = 138$bars, maximal servovalve piston travel $x_{smax} = 0.003$m and maximal flow in the valve is $Q_{smax} = 1.7143\ 10^{-5}\text{m}^3\text{s}^{-1}$.

With the linear system model in this form it is possible to design many different linear regulators including the control using digital sliding regime.

3. Control Algorithm of the Digital Sliding Regime

Regarding the complexity of the problem of pressure control by means of a hydraulic servovalve, due to the mathematical model nonlinearity, the solution should be looked for in the domain of nonlinear control algorithms. It is well known that there is an infinite number of nonlinear control laws. It is upon the designer to make the right choice for the sake of achieving special characteristics of the designed system. The system is required not only to do good control under normal conditions but also to be robust to the parameters' changes and the effect of the external disturbances. Having all this in view, the interest lies in the variable structure systems with sliding working regimes.

The dominant problem in the practical realization of the variable structure control system with sliding working regime is that of so-called non-modeled dynamics which does not usually represent a serious problem in the systems with linear control laws of the PID type. The non-modeled dynamics which exists in almost every practical application, due to small transport or inertia delays in the object, actuator or information channel which are either missed to be calculated in the model or neglected in the design, causes the phenomenon of chattering of the control signal which leads to the loss of the system motion invariantness. The chattering in the electromechanic system gives rise to unpleasant sound signals (rumbling) and to the fast wear of the mechanical parts which should not be tolerated.

Many papers are dealing with the problem of chattering reduction or elimination. This problem has not been studied enough and fully solved. A number of published control algorithms claim to be chattering-free but under certain conditions. In this paper the control algorithm which also belongs to the group of chattering-free algorithms will be used.

In the general case, for multivariable control system represented by vector state equation:

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x},t) + b(\mathbf{x},t)\mathbf{u}(t)\,, \tag{6}$$

where **x**- n-dimensional while **u** -m-dimensional vectors, m discrete functions represented in form $\mathbf{s}(\mathbf{x})$ are introduced and variable structure control:

$$\mathbf{u}(\mathbf{x},t)=\begin{cases}\mathbf{u}^{+}(\mathbf{x},t) & \text{za} \quad \mathbf{s}(\mathbf{x})>0 \\ \mathbf{u}^{-}(\mathbf{x},t) & \text{za} \quad \mathbf{s}(\mathbf{x})<0\end{cases}; \ \mathbf{u}^{+}(t)\neq u^{-}(t)\,, \tag{7}$$

So that in the reaching regime for finite time, state $\mathbf{s}(\mathbf{x})=0$ should be realized.

In this way, the system design is reduced to two steps:

1. Choice of discrete surface $\mathbf{s}(\mathbf{x})=0$ with desired dynamics of the system motion which is of lower order than the order of the given object dynamics.

2. Design of variable structure control $\mathbf{u}(\mathbf{x},t)$ so that any state $\mathbf{x}$ achieves the discrete plane for finite time and that continuous sliding regime is realized on it.

In this way, the variable structure control system reaches asymptotic stability.

The response of such a system, Fig. 2a, consists of three phases, namely:

- achievement,
- sliding, and,
- stable state.

If the system is now taken in a time discrete form which is needed for the control parameters design,

$$\mathbf{x}(k+1)=\mathbf{A}\mathbf{x}(k)+\mathbf{b}u(k)\,, \tag{8}$$

where $\mathbf{x}$ - n-dimensional vector, u - scalar, $\mathbf{A}$ – matrix and $\mathbf{b}$ – vector of respective dimensions. When the variable structure digital control system is applied, the system response can be divided into three phases. However, two types of the system trajectories in the sliding regime emerge here, Fig. 2b.

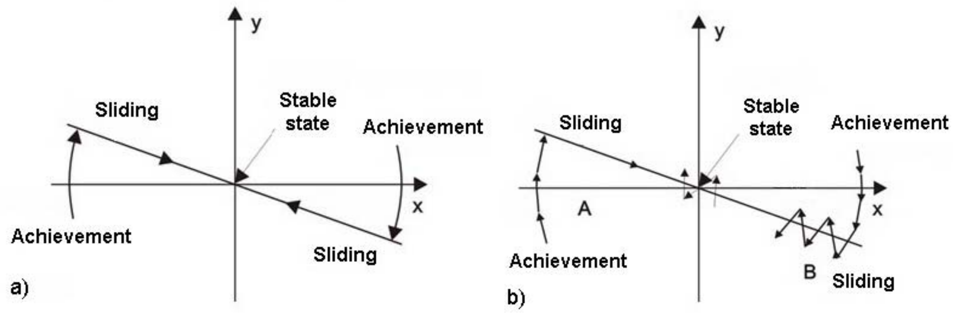

Fig. 2. Variable Structure System Trajectories: a) Analog, b) Digital

Trajectory A is considered as an ideal one. In order to obtain it, the system state should get to the sliding hyperplane exactly at the causation period which rarely happens in practice. Trajectory B represents the system motion for the real case. The system state describes a zigzag motion about the sliding plane [14]. To provide stability and quality guarantee, such motion must fulfill the following requirements:

a) From any initial state, the system state must move towards the sliding straight line and it must be cut for finite time,
b) When the trajectory cuts the sliding hyperplane for the first time, it must cut it again every next period of causation thereby giving resulting zigzag motion, and,

c) Amplitude of the stated motion must not grow in time, that is, it must remain within the range defined in advance.

The motion fulfilling the conditions b) and c) is called quasi-sliding regime. This term was first introduced in paper [15]. If the motion fulfills all the three conditions, the system is stable. Here we cannot speak about global asymptotic stability or absolute stability since oscillations of chaotic type will emerge in the vicinity of the zero balance state. It should be said that, in variable structure analog systems in which there is a delay in the formation of control laws, the quasi-sliding regime and similar motion in the vicinity of the balance state also emerge. Therefore, under the real conditions, due to the presence of non-ideality, asymptotic stability cannot be expected. From this standpoint, it is interesting to study the system's behavior in the vicinity of the balance state in order to determine the motion range.

In this paper the variable structure digital control algorithm published by domestic authors [12] is applied. The characteristic of this algorithm is to reach the sliding hyperplane for finite time with smooth decline. The control law comprises two components:

- nonlinear,
  whose task is to bring the system state into the close vicinity of the sliding plane, and,
- linear
  by which the sliding plane is reached in an interval of choice and it provides for further existence of sliding.

In order to explain the chosen variable structure digital control algorithm and to stress its advantages, it is best to start from the linear system described by equation (6) for which control is given in the form,

$$\begin{aligned} &u(t) = u(kT), \quad kT \le t \le (k+1)T, \\ &k \in N_0 = \{0,1,2,\ldots\}, \quad T > 0. \end{aligned} \tag{9}$$

Equivalent discrete model of system (6) is given in the form:

$$\delta\mathbf{x}(kT) = \mathbf{A}_\delta(T)\mathbf{x}(kT) + \mathbf{b}_\delta(T)u(kT), \tag{10a}$$

where
$$\delta\mathbf{x}(kT) \stackrel{\Delta}{=} \frac{\mathbf{x}((k+1)T) - \mathbf{x}(kT)}{T},$$

$$\mathbf{A}_\delta(T) = \frac{e^{\mathbf{A}T} - I_n}{T}, \mathbf{b}_\delta(T) = \frac{1}{T}\int_0^T e^{\mathbf{A}\tau}\mathbf{b}d\tau \tag{10b}$$

Since pair ($\mathbf{A}$,$\mathbf{b}$) is completely controllable while $\mathbf{A}_\delta(T)$ and $\mathbf{b}_\delta(T)$ are analytical functions of T, then pair ($\mathbf{A}_\delta(T)$, $\mathbf{b}_\delta(T)$) is likewise completely controllable. If the discrete function is given by equation,

$$s = \mathbf{c}_\delta(T)\mathbf{x}, \tag{11}$$

where $c_\delta(T) \in R^{1\times n}$, it is necessary, in order to reach sliding working regime, to provide $s = 0$.

Assumption

$$\mathbf{c}_\delta(T)\mathbf{b}_\delta(T) = 1. \tag{12}$$

This assumption ensures that relative degree of parameter s, if taken as output, with respect to the control signal, should be one. This is the usual condition of variable structure control system.

The next relation defines achievement law (further (kT) is written in short (k)):

$$\delta s(k) = -\Phi(s(k), \mathbf{X}(k)), \tag{13a}$$

where

$$\delta s(k) \overset{\Delta}{=} \frac{s(k+1) - s(k)}{T} = \mathbf{c}_\delta(T)\delta\mathbf{x}(k), \tag{13b}$$

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}(k) \\ \hat{\mathbf{x}}(k) \end{bmatrix} = \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}(k-1) \end{bmatrix}, \tag{13c}$$

Where, by definition $\hat{\mathbf{x}}(0) = \mathbf{x}(0)$.

Substituting (10a) in (13b) and putting their result in (12) and solving it for u(k), the control signal is obtained:

$$u(k) = -\mathbf{c}_\delta(T)\mathbf{A}_\delta(T)\mathbf{x}(k) - \Phi(s(k), \mathbf{X}(k)). \tag{14}$$

Function Φ is chosen in such a way that control law (14) has two regimes, namely, nonlinear and linear. The linear regime works in the vicinity and in the very sliding plane. The sliding plane vicinity is denoted as $\mathbf{S}(T)$ and defined as

$$\mathbf{S}(T) = \{\mathbf{X} \in R^{2n} : s = |\mathbf{c}_\delta(T)\mathbf{x}| < \varepsilon T + T\eta_1\|\mathbf{x}\|_1 + T\eta_2\|\hat{\mathbf{x}}\|_1\},\ \ \varepsilon > 0,\ \eta_1, \eta_2 \geq 0. \tag{15}$$

In equation (15) $\|\mathbf{x}\|_1 = \sum_{i=1}^{n} |\mathbf{x}_i|$.

In order to obtain the needed control parameters, system (10) with control (14) is transformed into regular form through the following coordinates' transformation:

$$\mathbf{x} = \mathbf{P}_1(T)\mathbf{P}_2(T)\begin{bmatrix} \tilde{\mathbf{x}} \\ s \end{bmatrix},\quad \mathbf{x} \in R^{n-1}, \tag{16a}$$

where [18]

$$\mathbf{P}_1(T) = [\mathbf{b}_\delta(T)\quad \dots\quad \mathbf{A}_\delta^{n-1}(T)\mathbf{b}_\delta(T)]\begin{bmatrix} a_1(T) & \dots & a_{n-1}(T) & 1 \\ a_2(T) & \dots & 1 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{bmatrix} \tag{16b}$$

and $a_i(T)$ are coefficients of characteristic polynomial $\det(\delta\mathbf{I}_n - \mathbf{A}_\delta(T))$,

$$\mathbf{P}_2(T) = \left[\begin{array}{c|c} \mathbf{I}_{n-1} & \mathbf{0}_{(n-1)\times 1} \\ \hline -\mathbf{c}_1(T) & 1 \end{array}\right], \tag{16c}$$

$$\mathbf{c}(T) = \mathbf{c}_\delta(T)\mathbf{P}_1(T) = [\mathbf{c}_1(T)\quad 1]. \tag{16d}$$

Assuming that pair ($\mathbf{A}_\delta(T)$,$\mathbf{b}_\delta(T)$) is completely controllable, matrix $\mathbf{P}_1(T)$ $\mathbf{P}_2(T)$ is regular.

By solving equation (16d) matrix $c_\delta(T)$ is obtained:

$$c_\delta(T) = [c_1(T) \mid 1]P_1^{-1}(T), \tag{17}$$

The needed parameters for calculating matrix $c_\delta(T)$ are obtained by means of the following equations:

$$c_i(T) = \frac{1}{(i-1)!} \frac{d^{i-1}\prod_{j=1}^{n-1}(\delta - \delta_j(T))}{d\delta^{i-1}} \Big|_{\delta=0}, \tag{18}$$

$$\begin{gathered} \delta_i(T) = \frac{e^{-\alpha_i T} - 1}{T}, \quad \alpha_i > 0, \\ i \neq j \Rightarrow \alpha_i \neq \alpha_j, \quad i, j = 1, \ldots, n-1 \end{gathered}, \tag{19}$$

Function $\Phi(s,X)$ is calculated as:

$$\Phi(s, X) = \Phi(s) = \min\left(\frac{|s|}{T}, \sigma + q|s|\right)\operatorname{sgn}(s), \tag{20}$$

where ρ and σ are real numbers such that $0 \leq qT < 1$ and $\sigma > 0$.

4. Simulation Results

As has already been said in Introduction, the pressure regulation by means of the hydraulic servovalve represents a big problem which can be solved by using adequate control. The control used here is designed by using the theory of the variable structure control system, on the basis of the digital sliding regime, as briefly explained in Section 3.

Control parameters (14) for discrete model (10), obtained at discretization of T = 0.0015s, calculated by equations (17,18,19 and 20) are:

$$c_\delta(T) = \begin{bmatrix} 0.024296 & 0.0034984 & 2.2528\mathrm{E}-009 \end{bmatrix},$$

$$c_\delta(T)\mathbf{A}_\delta(T) = \begin{bmatrix} 1.2383 & 0.01737 & 0 \end{bmatrix},$$

The results of the control obtained by using digital control are compared with those when the linear PID regulator is used, of the following amplifications $K_P = 0.00005$, $K_I = 0.00005$ and $K_D = 0.0000005$.

Fig. 3 shows the results of the simulations while tracking the bouncing signal; it can be seen that pressure regulation by digital control is better since it provides for fast response while with the use of the PID regulator fast response can also be obtained but it is coupled with over-jumps which are of referential value.

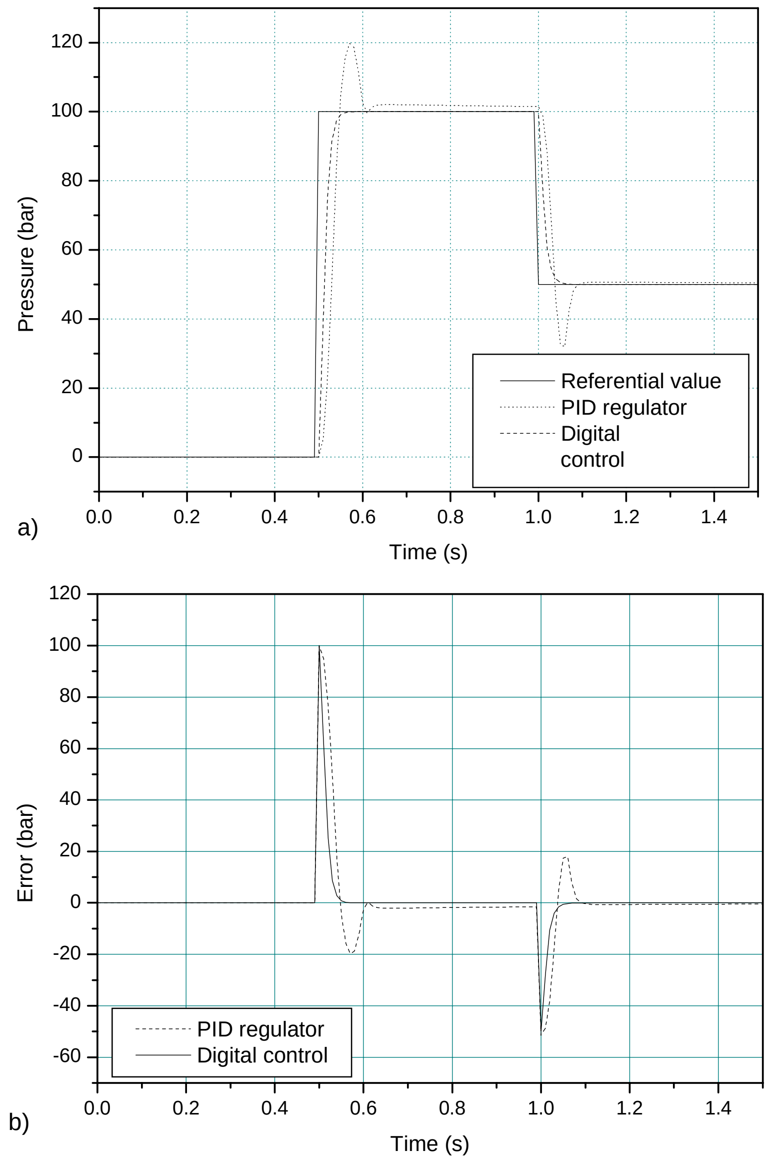

Fig. 3. Simulation Results: a) Tracking Bouncing Signal of Referential Pressure, b) Error

5. Conclusion

This paper presents a control algorithm based on the digital sliding regime designed by means of the theory of the variable structure control system. The characteristics of the proposed algorithm are compared with those of the linear PID controller by computer simulation and the values of the given control are displayed. It is shown that by fast response of the pressure regulation by digital control no over-jumps of referential value emerge while this can be avoided with the PID regulator only by reducing proportional strengthening, that is, by reducing the response velocity.

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UPRAVLJANJE PRITISKOM HIDRAULIČNIM NELINEARNIM SERVOVENTILOM

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Elektrohidraulični servoventili se veoma često koriste kao komandni uređaji u hidrauličnim sistemima. Za njihovo upravljanje razvijeno je dosta algoritama. U ovom radu predstavljen je algoritam upravljanja baziran na digitalnom kliznom režimu. Osobine predloženog algoritma upoređene su sa linearnim PID kontrolerom simulacijom na računaru.

Ključne reči: *Hidraulični servoventil, sistem upravljanja promenljive strukture, digitalni klizni režim*