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INFLUENCE OF ANGULAR BALL BEARING DEFORMATION ON TRUCK-CRANE DYNAMIC STABILITY

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Abstract. *The paper analyses the influence of angular ball bearing deformation at the connecting point of the lower and upper machine assemblies of the truck-crane using the dynamic model with five degrees of free motion. For differential equations derivation Lagrange equations of the second order are applied. Theoretical results obtained are verified by the numerical example for various working operations of the truck-crane.*

Key Words: *Angular Ball Bearing, Deformation, Truck-crane, Dynamic Stability*

1. INTRODUCTION

In the process of object designing it is necessary to notice all values which essentially influence its dynamic behavior. Analyzing available professional literature [1, ..., 7] we have noticed that mechanical-mathematical models established so far give approximate value of the angular ball bearing (Fig. 1) at the connecting point of the upper and lower machine assemblies considering it an absolutely rigid body. Professional literature does not offer any values of angular rigidity of the bearing discussed. At the same time, the practical problem of wearing out of some particular parts of the bearing after years of exploitation, particularly of the rolling bodies, should not be neglected, as they cause the increase of clearances and dynamic load. However, the last mentioned problem could be solved by frequent check ups and repairs of the angular ball bearing. After recognizing the problems, the question arises: how big mistake has been made by approximating the analyzed bearing as a rigid body? In order to get the answer to this question, the bearing at the point of connecting lower and upper machine assemblies will be considered an elastic body. Swiveling angle of the bearing will be taken into account by introducing an appropriate generalized coordinate.

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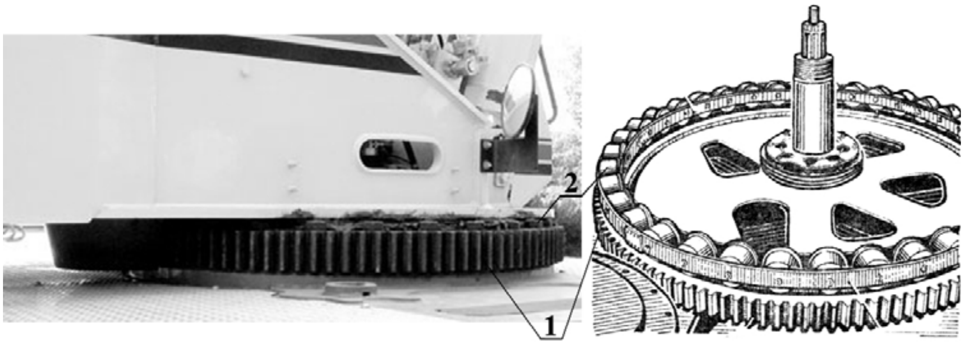

Fig. 1. Angular Ball Bearing at the Point of Connection of Lower and Upper Machine Assemblies (1 – gear, 2 – barrel)

2. CRITERIA OF STABILITY

In the study of professional literature [1, ..., 8] it can be noticed that defining of stability is mostly achieved by means of

- relation between stability momentum and overturn momentum,
- support resistance values, and,
- limit angle.

Stability defining by means of relation between stability momentum and overturn momentum, as well as its defining by means of limit angle can be used in cases where high preciseness is not necessary. In order to get more precise results with respect to stability, it is necessary to define the precise position of the overturn edge. In the case of absolute rigid supports the position of overturn edges is precisely defined. As the truck-crane rests upon elastic supports of the kind of hydraulic outriggers and pneumatic tyres on elastic base, it is necessary to define the actual position of overturn edges which is a very difficult task. Because of that, the approach considered gives only approximate results that can be applied in the first phase of the designing process. Defining of stability by means of the support resistance value offers more reliable results.

In this work the **criterion of ground deflection under the support** [9] will be applied, which is, in its essence, most similar to the criterion of stability defined by means of the support resistance. According to the criterion adopted here, the object with four supports is **completely stable** if the ground deflections under each of the supports are higher than zero. The object can be considered **conditionally stable** if the ground deflections under three supports are higher than zero, i.e. in the case that separation from the ground of only one support occurred. The object is **unstable** if the deflections in the ground under two supports at best are higher than zero, i.e. in the case of separation of at least two supports from the ground. In order to apply previous definitions, the truck-crane model should contain a generalized coordinate which can be connected with the ground deflection under the support. The generalized coordinate observed can be accepted in such a way that its positive value (higher than zero) corresponds to the deflection of the ground. If the generalized coordinate realizes negative values it can be concluded that the loss of contact between the observed support and the ground occurred. The previous definitions of stability can be applied only for real (deformational) characteristics of the ground. It is interesting to analyze a limit case which refers to an absolutely rigid ground. Then the deflection of the ground is always equal to zero. According to the previously established definitions, it follows that the object is unstable. However, in the case of an absolutely rigid ground the loss of stability will appear only when the support separation from the ground occurs. Then the stability can be established by defining the values of forces within supports, i.e. if they are higher or lower than zero.

3. MECHANICAL–MATHEMATICAL MODEL

The mechanical-mathematical model of every object should reflect its behavior as accurately as possible, and, on the other hand, it can be defined only by means of parameters that have significant influence on the problem discussed. Yet, the term "significant" is a term of relative value, which means that we should not be prevented from analyzing the influence of some other parameters which have not been analyzed up to now, and which, in our opinion, might have certain importance in solving the posed problem. Having in mind all the above said, and on the basis of critical analysis of mechanical models presented in the professional literature, we are going to define a mechanical-mathematical model of the truck-crane for the case of analysis of influence of angular ball bearing deformation on dynamic stability of the truck-crane.

Outriggers (item 1 – Fig. 2) in the model accepted here, are considered deformational in the direction of their alongside axes. Their deformations in the direction of the other two axes of the Descartes coordinate system are neglected. Explanation can be found in the fact that the outrigger is affected by the load with the dominant component having the direction of alongside axis. One should also have in mind that the outrigger length is short in comparison with its cross-section length, which considerably adds to its greater rigidity in transversal directions than in the axial one. Outriggers also have the muffling characteristic. While manipulating the load, the truck-crane

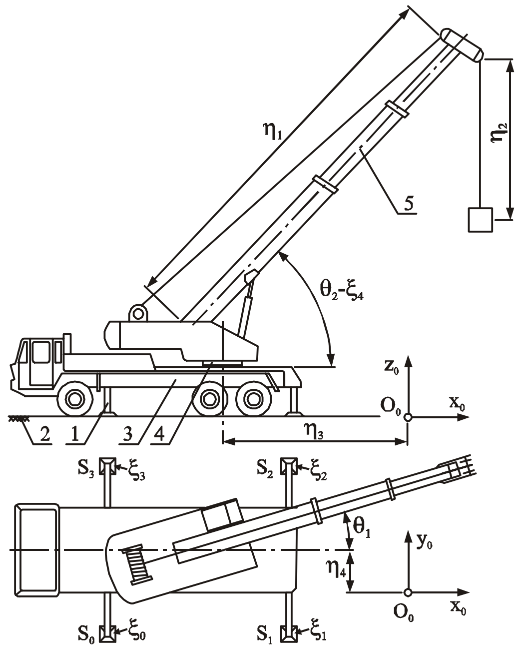

Fig. 2. Mechanical Model of the Truck-crane (1-outrigger, 2-ground, 3-chassis, 4-angular ball bearing at the point of connection of lower and upper machine assemblies, 5-telescope boom)

is most frequently supported by outriggers. Depending on the type of the truck-crane, but also on the working conditions and load mass that is being lifted, resting of the truck-crane can be performed either on pneumatic tyres or using a combination of pneumatic tyres and outriggers. The case when the truck-crane carries certain load, and, at the same time, moves with the speed of lower intensity can be also analyzed. Pneumatic tyres will be considered deformational in the direction of axis perpendicular onto the ground, and they will also be considered to have the quality to muffle oscillation. The ground (item 2 – Fig. 2) is also characterized by rigidity and ability to muffle oscillation. On the basis of the preliminary discussion the mechanical-mathematical model is being established with the following generalized coordinates:

- ξ_0, ξ_1, ξ_2, ξ_3 – distances between the lower outrigger surface and non-deformed ground surface under the points S_0, S_1, S_2, S_3, respectively,
- ξ_4 – swiveling angle of angular ball bearing at the point of connection of lower and upper machine assemblies.

The truck-crane performs the working operations which are defined by the following values (Fig. 2)

- θ_1 – the angle of rotation (turning) of the upper machine assembly,
- θ_2 – rotation angle of the boom in relation to the stand – raising and lowering of the boom,
- η_1 – telescopic boom length,
- η_2 – distance from the boom top to the centre of the load,
- η_3 – the covered path which describes the truck-crane motion by means of pneumatic tyres in the direction of coordinate axis $''x''$,
- η_4 – the covered path which describes the truck-crane motion by means of pneumatic tyres in the direction of the coordinate axis $''y''$.

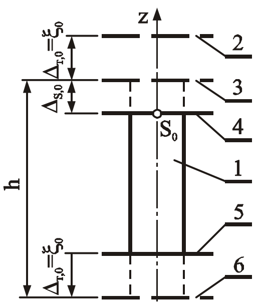

Fig. 3. Modeling of the Outriggers and Ground Connection

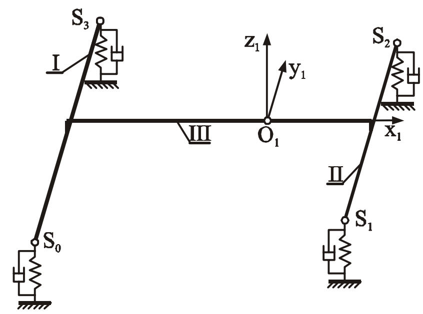

Fig. 4. Modeling of the Truck-crane Chassis

By analyzing the movement of the point S_0 (the place of outrigger and chassis connection) all the parameters which refer to moving of the points S_1, S_2 and S_3 will be defined. In the case of a contact between the outrigger and the ground the distance between the outrigger lower surface and non-deformed ground surface under the point S_0 (generalized coordinate ξ_0) corresponds to the deflection of the ground when measured beneath the point observed. The following units and characteristic planes that are needed for further analysis are indicated in Fig. 3: 1 – outrigger, 2 – position of the point S_0 with non-deformed outrigger and non-deformed ground, 3 – position of the point S_0 with non-deformed outrigger and deformed ground, 4 – position of the point S_0 with deformed outrigger and deformed ground, 5 – non-deformed ground, 6 – deformed ground. In mechanical sense it can be considered that the chassis in the point S_0 is rested across two in a series of the connected springs of different rigidity. Fig. 3 also includes the following signs: h – height of outriggers, $\Delta_{T,0}$ – deformation of ground, $\Delta_{S,0}$ – deformation of outriggers.

Truck-crane chassis (item 3 – Fig. 2) is composed of the group of beam carriers. The biggest load and greatest geometrical characteristics have two transversal beam carriers (items I and II – Fig. 4) which connect outriggers, as well as the two alongside beam carriers. The distance between the two beam carriers is short in comparison to their length, and as such they can be substituted by one carrier (item III – Fig. 4), geometrical characteristics of which correspond to those of both the alongside carriers. The carriers by which chassis is modeled are absolutely rigid.

The truck-crane boom (item 5 – Fig. 2) is approximated as an absolutely rigid beam carrier with overhang, where one support represents the connection of the boom and the lower machine assembly (point A), and the other represents the connection of the boom and hydro-cylinder (3) (point B) – Fig. 5. The boom, in case of absolute rigid bearing at the connection point of lower and upper assemblies, forms angle θ_2. Due to introduced deformational character of the bearing, the real angle has lower value which is $\theta_2 - \xi_4$.

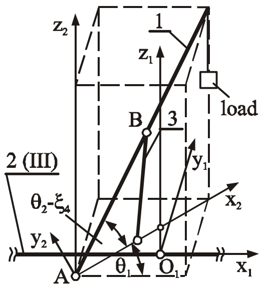

Fig. 5. Modelling of the telescope boom (1) and chassis (2) connection

The carriers by which the chassis and the boom are modeled are discussed as carriers with continually distributed mass.

For calculation of differential equations of motion, Lagrange equations of the other kind will be used [1, 8, 9]

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\xi}_i}\right) - \frac{\partial E_k}{\partial \xi_i} + \frac{\partial \Phi}{\partial \dot{\xi}_i} + \frac{\partial E_p}{\partial \xi_i} = Q_i^n, \qquad i = 0,1,2,3,4, \tag{1}$$

where E_k, E_p, Φ and Q_i^n – are kinetic energy, potential energy, function of the dissipation (dissipative force), and corresponding generalized non-conservative force, respectively.

Kinetic energy of the mechanical system is defined as

$$E_k = \sum_{i=1}^{6} E_{k,i} = E_{k1} + E_{k2} + E_{k3} + E_{k4} + E_{k5} + E_{k6}, \tag{2}$$

where:

- E_{k1} – kinetic energy of chassis,
- E_{k2} – kinetic energy of driver's cab and driving-unit for motion on wheel,
- E_{k3} – kinetic energy operator's cab and driving-unit for the truck-crane during the working operations,
- E_{k4} – kinetic energy of counterweight,
- E_{k5} – kinetic energy of telescope boom,
- E_{k6} – kinetic energy of load.

Potential energy of the mechanical system observed is defined as follows

$$E_p = \sum_{i=1}^{8} E_{p,i} = E_{p1} + E_{p2} + E_{p3} + E_{p4} + E_{p5} + E_{p6} + E_{p7} + E_{p8} , \quad (3)$$

where:

- E_{p1} – potential energy of outriggers and ground,
- E_{p2} – potential energy of chassis,
- E_{p3} – potential energy of driver's cab and driving-unit for motion on wheel,
- E_{p4} – potential energy of angular ball bearing at the point of connection of lower and upper machine assemblies,
- E_{p5} – potential energy operator's cab and driving-unit for the truck-crane during the working operations,
- E_{p6} – potential energy of counterweight,
- E_{p7} – potential energy of boom,
- E_{p8} – potential energy of load.

Function of dissipation (dissipative force) is defined by applying the following expression:

$$\Phi = \Phi_1 + \Phi_2 , \quad (4)$$

where:

- Φ_1 – function of dissipation of outriggers and ground,
- Φ_2 – function of dissipation of cylinder for boom elevation.

Generalized force is of the form

$$Q_i^n = 0, \ i = 0,1,2,3,4 . \quad (5)$$

At the initial moment of time the generalized coordinates have the values which are the same as their static deformations.

With more complex spatial systems the biggest problem is how to define the speeds and coordinates of the characteristic points of the system analyzed. Derivation of their expressions in the developed form is an exceptionally complicated task. Also, their later differentiating gives us final forms of differential equations of motion, which are in their scope exceptionally complex. As such, they also produce difficulties in their numerical solutions. These difficulties are evident in the impossibility of their solving or in considerably prolonged time needed for their calculation. The problem observed is solved by defining coordinates of any characteristic point of the truck-crane in any moment of time in relation to static coordinate system $O_0x_0y_0z_0$ (Fig 2). Within the frame of the model a number of mobile coordinate systems are defined $O_kx_ky_kz_k$ ($k \geq 1$), whose motion is also defined in relation to static coordinate system. In that way we obtain expressions by which necessary coordinates, speeds and acceleration are defined in the form of matrix [1]. Such an example enables simple defining of speed of very complex mechanical systems and considerably facilitates the process of derivation of the final forms of differential equations of motion, and shortens the time needed for their solution.

When into Lagrange equations of the other type (1) expressions for kinetic (2) and potential energies (3), function of dissipation (4) and generalized force (5), five differential equations of the second order are obtained in the following form

$$[\ddot{q}] = [A], \tag{6}$$

where:

- $[\ddot{q}] = [\ddot{\xi}_0 \quad \ddot{\xi}_1 \quad \ddot{\xi}_2 \quad \ddot{\xi}_3 \quad \ddot{\xi}_4]^T$,
- $[A] = f(\xi_i, \dot{\xi}_i, \theta_j, \dot{\theta}_j, \eta_k, \dot{\eta}_k, L_i, c_i, \delta_i ...)$ – matrix of the 5×5 order, which is the function of the generalized coordinate ξ_i (i = 0,1,2,3,4), of the values θ_j (i = 1, 2) and η_k (i = 1...4), their first derivates per time, geometrical values of the truck-crane (L_i), rigidity of the ground, outriggers, and of angular ball bearing (c_i), coefficient of ground damping, outriggers and of cylinder for boom elevation damping...

Differential equations (6) can be solved numerically, by means of a computer, introducing the changes:

$$X_0 = \xi_0,\ X_1 = \dot{\xi}_0,\ X_2 = \xi_1,\ X_3 = \dot{\xi}_1,\ ...,\ X_8 = \xi_4,\ X_9 = \dot{\xi}_4. \tag{7}$$

In this way, ten differential equations of the first order are obtained, which are convenient for numerical solution of some of the programs such as Matlab and Mathcad. In order to solve all this, a program is made in this paper applying the program package "*MathCad 2001i Professional*" [9]. The program has been written for already known functions of changes of the following values: θ_1, θ_2, η_1, η_2, η_3, η_4 (Fig. 2) [10], which correspond to real exploitation conditions. The written program could be modified so as to consider values θ_1, θ_2, η_1, η_2, η_3, η_4 as generalized coordinates, so appropriate differential equations would be also derived for them [9].

4. NUMERICAL EXAMPLE. ANALYSIS OF THE RESULTS

Theoretical results obtained are discussed on a numerical example of the truck-crane AD-16 (maximum lifting capacities m_q= 16 t) designed at Ivo Lola Ribar – Železnik (Belgrade) [11].

Dynamic stability analysis for the case of rotating upper machinery assembly of the truck-crane is realized for the load mass value of 3 t and angles θ_1=0^0 and θ_2=45^0 (Figs. 6 and 7). Maximum value of the angle speed of the upper machinery assembly rotation is 0,0675 s^{-1}. By the analysis of the swiveling angle change of the angular ball bearing for various values of angular rigidity of the angular ball bearing – c_L (Fig. 6) great deviations are noticed. So, in case of lower rigidity there is appearance of the swiveling angle values of

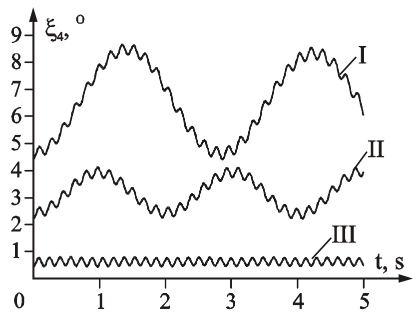

Fig. 6. Changes of the Swivelling Angle of Angular Ball Bearing (ξ_4=ξ_4(t)) for Various Angular Rigidity Values of the Angular Ball Bearing for the Case of the Rotation of the Upper Machine Assembly (I–c_L=0,5·10^{7}Nm/rad, II–c_L=10^{7}Nm/rad, III–c_L=5·10^{7}Nm/rad)

the angular ball bearing that cannot be neglected, as well as the appearance of oscillation amplitudes (curves I and II). Angular moving observed causes the change of the boom centre position and what is even more important, the change of the load centre position. As a result of low values of the bearing rigidity increased values of load swaying angle may appear. For higher rigidity values ($c_L > 5\cdot10^7$ Nm/rad) the size of swiveling angle of angular ball bearing as well as oscillation amplitude have considerably lower values ($\xi_4 < 0{,}8^0$).

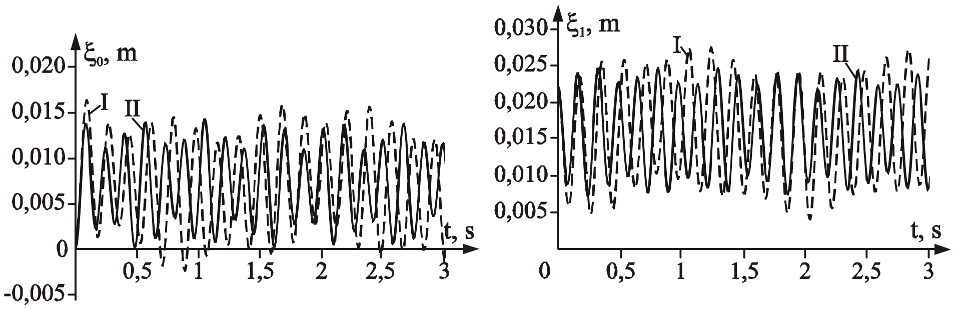

Fig. 7. Changes of the Distance Between the Lower Outrigger Surface and Non-defined Ground Surface under Supports S_0 and S_1 for Various Angular Rigidity Values of the Angular Ball Bearing (I – $c_L = 10^7$ Nm/rad, II – $c_L = 5\cdot10^7$ Nm/rad)

Considerable influence of angular rigidity of the angular ball bearing onto truck-crane stability is noticed by analyzing corresponding graphs in Fig. 7. In case of rigidity $c_L = 10^7$ Nm/rad (curve I) for nearly 36% larger amplitudes of oscillation of generalized coordinate ξ_0 appear than in the case when the bearing rigidity has the value of $c_L = 5\cdot10^7$ Nm/rad (curve II). From the aspect of stability it is also necessary to underline the fact that the use of angular ball bearing with lower angular rigidity values is more unfavorable. The proof is the result of this numerical analysis, according to which for $c_L = 10^7$ Nm/rad the value is $\xi_{0,\min} = -0{,}00279$ m, while for $c_L = 5\cdot10^7$ Nm/rad minimum value of the generalized coordinate observed is $\xi_{0,\min} = 0{,}00012$ m.

Dynamic stability analysis for the case of raising the truck-crane boom is achieved for the load mass value of 3 t and angle $\eta_1 = 0^0$. At the initial moment of time the angle between the boom and the stand has got the value of 45^0. Angular rigidity of the angular ball bearing is $c_L = 5\cdot10^7$ Nm/rad. With this working operation of the truck-crane, and in real exploiting conditions analyzed, higher values of the swiveling angle of the angular ball bearing are noticed in respect to the case of upper machinery assembly rotation (Fig. 8). The difference is approximately 67%.

One of the frequent working operations which occur with telescopic booms is the change of its length. The operation observed is known in practice as boom telescoping. During exploitation the operators most often carry out the process of telescoping when the boom is without load, i.e. the load is not suspended from the hook. Explanation can be found in the fact that the increase of the boom length can cause stability loss of the truck-crane, i.e. an overturn could occur. Telescoping process is most frequently carried out under the action of hydro-cylinders. In some cases construction solution of the whole hydro-system is not intended for carrying out the telescoping process, and that is why the manufacturers in that case do not recommend mentioned working operation with the suspended load. Telescopic boom length, in the case discussed here, at the initial moment of time was 12,342 m (only one segment of the boom is extended). In the analysis the case of extending the other segment of the boom is discussed (Fig. 9). With the increase of the telescopic boom length the momentum of bending at the position of the bearing increases, and its value directly affects the increase of bearing deformation. Maximum value of the angle of bearing deformation is 1^0.

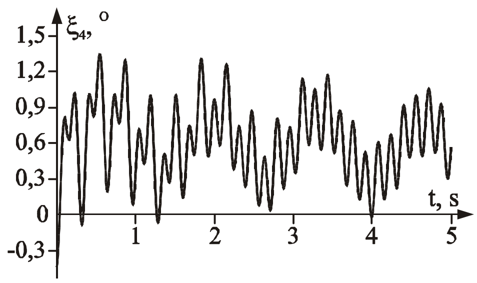

Fig. 8. Function of Dependence $\xi_4 = \xi_4(t)$ for the Case of the Boom Raising

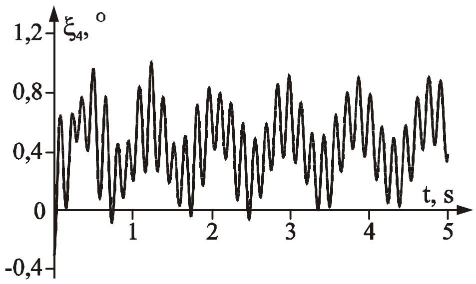

Fig. 9. Function of Dependence $\xi_4 = \xi_4(t)$ for the Case of the Telescoping of the Telescopic Boom

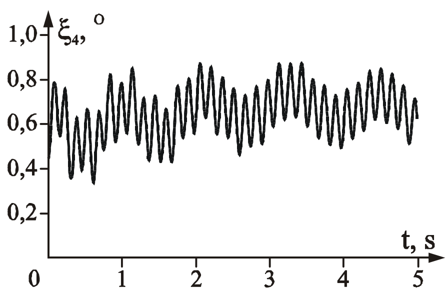

Fig. 10. Function of Dependence $\xi_4 = \xi_4(t)$ for the Case of Load Lifting

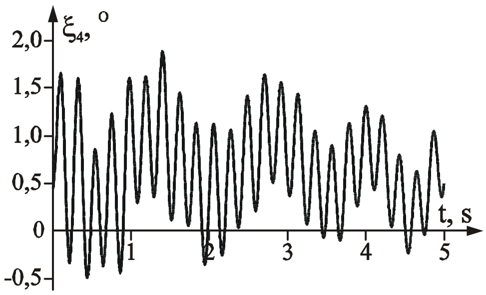

Fig. 11. Function of Dependence $\xi_4 = \xi_4(t)$ for the Case of the Truck-crane Motion with Pneumatic Tyres across the Ground

Lifting the load from the ground, i.e. base, depending on the force of the load lifting rope, can be studied in the following characteristic cases:

- the rope is tight, and the force in the rope before the beginning of the process of load lifting is equal to the load weight, i.e. the case of static equilibrium occurs,
- the rope is tight and the force in the rope before the beginning of the load lifting procedure is weaker than the load weight, but higher than zero,
- the rope is not tight (it is loose), and the force in the rope before the beginning of the load lifting procedure is equal to zero.

In case when the rope is tight, and the force in the rope is equal to the load weight, the initial conditions are defined for the real weight of the load hung on the hook. Numerical analysis of the load lifting procedure for this case is shown in Fig. 10. Maximum value of the swiveling angle of the angular ball bearing at the point of connection between the upper and lower machine assemblies in this case is $0{,}87^0$.

The truck-crane can move on pneumatic tyres with the load on the hook, or it might be in traffic without load. Manufacturers' recommendation is not to include in traffic the truck-crane with pneumatic tyres with the load. Even so, this case will be also discussed because in this way many useful pieces of information on the truck-crane capacity during the given motion in real conditions of exploitation could be obtained. Hypothesis has been introduced that the ground across which the truck-crane is moving is absolutely flat, i.e. that there are not any rough, uneven spots. Such hypothesis is introduced, as it is impossible to define precisely the shape of the ground surface. In case of bigger rough spots an additional dynamic load occurs, as well as the possibility of uneven distribution of the load on the supports, i.e. pneumatic tyres. For the case of truck-crane motion in the direction of the coordinate axis x, the maximum speed value of the motion is 1 km/h. The boom is set at the rear part of the truck-crane, and its position is more precisely defined by angle values $\theta_1 = 0^0$ and $\theta_2 = 45^0$. The swiveling angle of the angular ball bearing has higher values than in the course of other working operations. In this case its maximum value is $1{,}89^0$ (Fig. 11).

5. Conclusion

The task of this paper was to analyze the influence of angular ball bearing deformation on the truck-crane dynamic stability. The problem is solved by application of Lagrange equations of the second order. Through analysis of the obtained numerical results a considerable influence of deformational characteristics of the angular ball bearing on the amplitude of the distance between the lower outrigger surface, and non-deformed ground surface under the support can be noticed. The swiveling angle value of the truck-crane angular ball bearing is relatively small. With introduction of some modifications, the model established could be applied to other similar crane-transport machines [9].

References

1. Šelmić R., "Prilog istraživanju dinamičkih opterećenja noseće konstrukcije auto-dizalice sa specijalnom (ojačanom) šasijom, hidropogonom i teleskopskom strelom", doktorska disertacija, Mašinski fakultet, Beograd, 1979.
2. Волков Д. П., Манасян В. Г., Шелмич Р. Р., "О динамической устойчивости самоходных строительных кранов", *ICMFMDI-2000*, Машински факултет, Београд, стр. ИП10-14
3. Вайнсон А. А., "Строительные краны," *Машиностроение*, Москва, 1969.
4. Коган И. Я., "Строительные башенные краны," *Машиностроение*, Москва, 1964.
5. Babin N., Vladić J., Stokić D., "Prilog određivanju stepena sigurnosti protiv preturanja dizalice, IV seminar o transportnim procesima u industriji", *SMEITS*, Mašinski fakultet, Beograd, 1980., p.p. 105-115,
6. Alamoreanu M., "Jib Cranes Stability Criteria to Overturning," *XVI ICMFMDI 2000*, Mašinski fakultet, Belgrade 2000., p.p. IP6-9,
7. Никитинский М. П., "Исследование распределения давлений на опоры пневмоколесных кранов", канд. диссертация, Московский Государственый Строительный Университет, Москва, 1973.
8. Dubowsky S., Gu P. Y., Deck J. F., "The Dynamic Analysis of Flexibility in Mobile Robotic Manipulator Systems", *Proc. VIII World Congress on the Theory of Machines and Mechanisms*, Prague, Czechoslovakia, July 1991.
9. Mijailović R., "Investigation of Parameters Relevant for Dynamic Stability and Preventive Safety of the

Truck-crane in Traffic and Transport," ph. dissertation manuscript, Faculty of Transport and Traffic Engineering, University of Belgrade, Serbia and Montenegro, 2004.
10. Kilicislan S., Balkan T., Ider S. K., »Tipping Loads of Mobile Cranes with Flexible Booms,« *Journal of Sound and Vibration*, 223/1999.
11. Šelmić R., "Ispitivanje i proračun vitalnih delova auto-dizalice AD-16, sa predlogom mera za poboljšanje konstrukcije," studija Instituta Saobraćajnog fakulteta, Beograd, 1986.
12. Волков Д.П., Манасян В.Г., Шелмич Р.Р., "Влияние колебаний на устойчивость самоходных стреловых кранов," *Автомобильный транспорт*, серия "Совершенствование машин для земляных и дорожных работ", Харков, выпуск 5/2000, стр. 9-13.
13. Mijailović R., Šelmić R., «Mechanical and Mathematical Spatial Modeling of Truck Crane Telescope Boom,' *Naučnotehnički PREGLED*, Beograd, 2003, str. 51-57.
14. Шелмич Р., Мияилович Р., "Анализ механическо-математической модели телескопической стрелы автокрана, Международная научно-техническая конференция "ИНТЕРСТРОЙМЕХ-2002", Могилевский государственный технический университет, Могилев, 2002, *Сборник трудов*, стр. 447-451.
15. Шелмич Р., Мияилович Р., "Анализ динамической устойчивости телескопической стрелы автокрана," *XVII International Conference on "Material Flow, Machines and Devices in Industry"*, Faculty of Mechanical Engineering University of Belgrade, Belgrade 2002, Proceedings, pp. 1.107-1.111.

UTICAJ DEFORMACIJE RADIJALNO-AKSIJALNOG LEŽAJA NA DINAMIČKU STABILNOST AUTO-DIZALICE

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U radu se analizira uticaj deformacije radijalno-aksijalnog ležaja na mestu veze donjeg i gornjeg stroja auto-dizalice na dinamičku stabilnost auto-dizalice, korišćenjem dinamičkog modela sa pet stepeni slobode kretanja. Za izvođenje diferencijalnih jednačina kretanja primenjene su Lagranžove jednačine druge vrste. Dobijeni teorijski rezultati su verifikovani numeričkim primerom za različite radne operacije auto-dizalice.

Ključne reči: *radijalno-aksijalni ležaj, deformacija, auto-dizalica, dinamička stabilnost.*