

## FUNCTIONAL OPTIMIZATION OF THE CAM MECHANISMS THROUGH USING HELICAL SPRINGS WITH VARIABLE GEOMETRY

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**Abstract.** *By using springs that have a progressive (non-linear) elastic characteristic, the contact between cam and cam follower can be ensured on the whole cycle. The values of the resulting force acting on the cam follower can be reduced, as well as the values of the resistant torque reduced to the cam's shaft, respectively to the actuating motor's power.*

**Key words:** *Cam, Cam Follower, Spring, Variable, Geometry.*

### 1. INTRODUCTION

Mechanisms using superior kinematical pairs need technical solutions in order to ensure permanently the contact between the elements' profiles. Cam mechanisms, referred to in the paper, use superior kinematic pairs as the contact between the cam and the cam follower. The contact, in the case of cam mechanisms, can be ensured in two ways: by shape or by force.

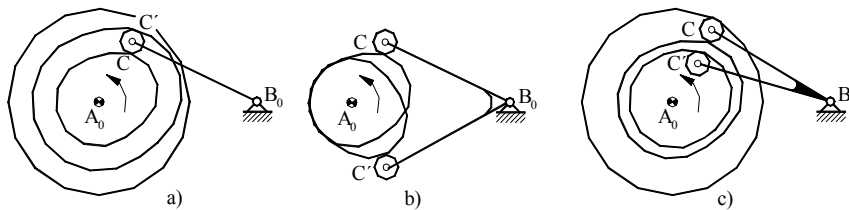


Fig. 1.1 Solutions for ensuring the contact between cam and cam follower by shape

In the first case, some solutions can be used: cam with channel (groove) (Fig. 1.1.a), cam and counter-cam (Fig. 1.1.b) or cam with slot/notch (Fig. 1.1.c). These cams ensure

the permanent contact between the cam and the cam follower with the help of the two profiles, but they also lead to design complication and implicitly to manufacturing costs' increase, because of the high execution precision implied [2], [4].

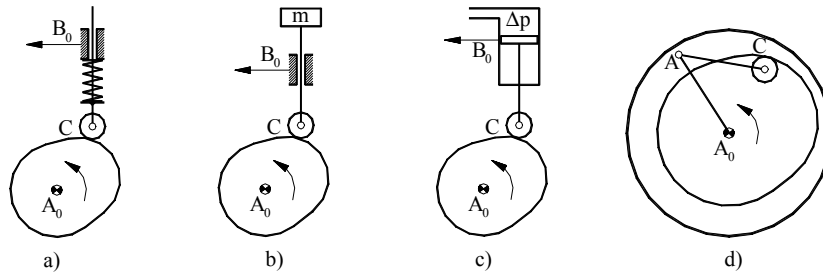


Fig. 1.2 Solutions for ensuring the contact between the cam and the cam follower by force

In the second case, in order to ensure the contact between the cam and the cam follower, forces or exterior moments can be used, generated with help of spiral or helical springs (Fig. 1.2.a), by gravitation or through additional masses acting over the cam follower (Fig. 1.2.b), through hydro-pneumatic forces (Fig. 1.2.c) or through centrifugal forces at mechanisms running at high angular velocity (Fig. 1.2.d) [2], [4].

2. CAM MECHANISMS' KINETOSTATIC CALCULUS

For the cam mechanism in order to transmit accurately the movement law for which it was designed, the permanent contact ensurance between the cam and the cam follower is vital. The paper studies the cam mechanisms using springs in order to ensure the contact between the cam and the cam follower.

The force system acting on the cam follower is composed of [1],[4]:

$F_u$  – the useful force needed in the considered technological process,

$F_i$  – the inertial force, given by the relation

$$F_i = -m \cdot a \tag{2.1}$$

where:

$m$  – is the cam follower's total mass (including the pieces joined to the cam follower),

$a$  – is the cam follower's acceleration,

$F_e$  – the spring's elastic force.

For the forces acting on the cam follower, the positive direction was considered as orientated from the cam follower to the cam.

Useful forces and inertial forces respectively can lead the cam follower's roll to detach, if their direction is reverse as in Fig. 2.1. Especially the inertial forces acting on the cam follower cause the detachment, because the acceleration presents a variation during

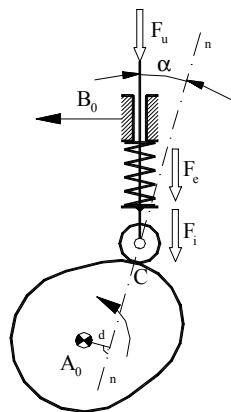


Fig. 2.1. The force system acting in a cam mechanism

the movement that implies shifting direction (acceleration respectively deceleration). The most frequent danger of detachment appears in the descent phase within the interval of the cam follower's movement acceleration.

The spring used in order to ensure contact is calculated so that it should be able to overcome, during the whole kinematical cycle, the forces attempting to detach the cam follower's roll. During the whole kinematical cycle, the following relation must be satisfied [1], [4]:

$$F(\varphi) = F_u(\varphi) + F_i(\varphi) + F_e(\varphi) > 0 \quad . \quad (2.2)$$

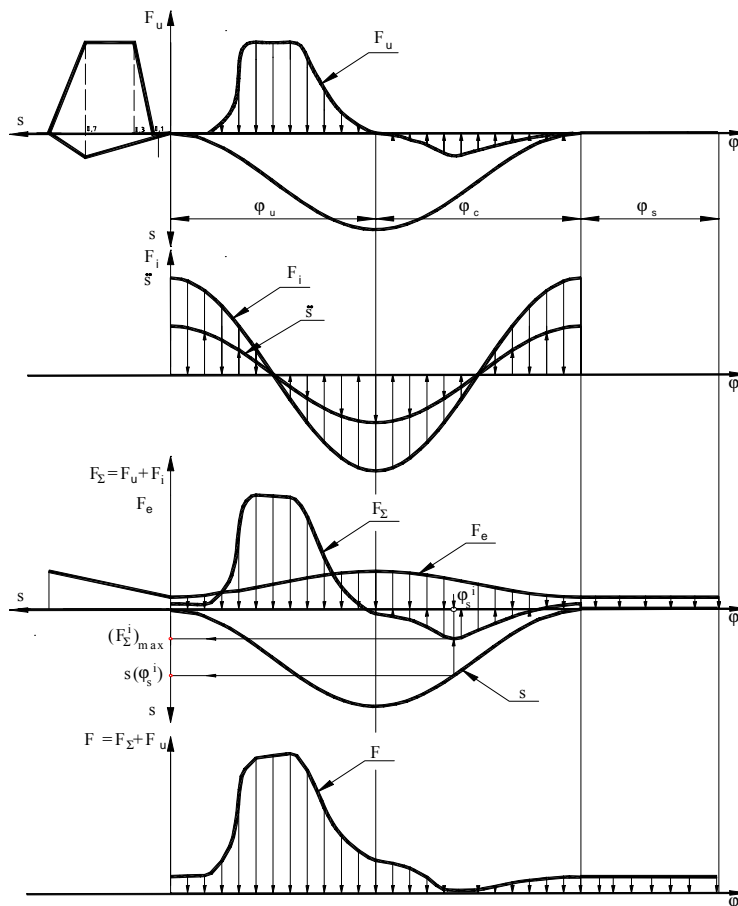


Fig. 2.2. Variation diagrams for the force system acting on the cam follower (example)

In order to establish the characteristic of the spring that satisfies the condition (2.2), the variation in the useful force  $F_u(\varphi)$  and in the inertial force  $F_i(\varphi)$ , respectively and their sum  $F_\Sigma(\varphi)$  (see Fig. 2.2) must be known. The angular positions  $\varphi_s^i$  are then identified, where the maximum detachment danger is met, where the maximum sum force  $(F_\Sigma^i)_{max}$  appears. Elastic forces achieved by the spring must be superior as absolute value to these

maximum values  $(F_{\Sigma}^i)_{\max}$ . It is known that in the kinetostatic calculus for dimensioning the cam mechanisms' spring, the spring must ensure a torque or force surpassing for up to 30% the sum of the useful torque or forces and the inertial ones, when functioning at the maximum speed allowed by its design. Thus, the safety forces will be determined in the shape of:

$$F_S = (1.1 \dots 1.3) \cdot (F_{\Sigma}^i)_{\max} \quad \varphi = \varphi_s^i \rightarrow s = s(\varphi_s^i), \quad (2.3)$$

and the spring loads itself with a force:

$$F_{e0} = (0.3 \dots 0.6) \cdot (F_{\Sigma}^i)_{\max} \quad \varphi = 0 \rightarrow s = 0. \quad (2.4)$$

When using springs with linear characteristic, the spring's characteristic has the expression:

$$F_e = F_{e0} + k \cdot s(\varphi), \quad (2.5)$$

where:  $k$  is the spring's elastic constant.

When using springs with non-linear characteristic, these are mathematically defined on domains:

$$F_e = \begin{cases} F_{e0} + k_1 \cdot s(\varphi) & 0 < \varphi \leq \varphi_1 \\ F_{e1} + k_{22}(s(\varphi) - s(\varphi_1)) + k_{23}(s(\varphi) - s(\varphi_1))^2 & \varphi_1 < \varphi \leq \varphi_2 \end{cases}, \quad (2.6)$$

where, depending on the wanted characteristic, the pairs of values  $(\varphi_1, F_{e1})$ ,  $(\varphi_2, F_{e2})$ , respectively the elastic constants  $k_1$ ,  $k_{22}$ ,  $k_{23}$  are indicated.

Through using springs that have a progressive (non-linear) elastic characteristic, the contact between the cam and the cam follower can be ensured on the whole duration of a cycle, the values of the resulting force  $F(\varphi)$  acting on the cam follower can be reduced, as well as the values of the resistant torque reduced to the cam's shaft and the actuating motor's power respectively.

### 3. HELICAL SPRINGS WITH PROGRESSIVE ELASTIC CHARACTERISTIC

The progressive (non-linear) load-deflection characteristic can be achieved in various manners:

- By using mixed suspensions, combining a cylindrical helical spring with an additional elastic element (cave rubber spring, elastic membrane); the subassembly's characteristic becomes progressive once the auxiliary spring is acted
- By using a cylindrical helical spring with variable pitch or a profiled helical spring, both made of constant-diameter wire (main disadvantage: growth in spring height and superior quality steel consumption).
- By using *helical springs with variable geometry*, namely springs made of profiled wire, wrapped on a cylindrical or profiled director surface with a constant or variable pitch.

The main variable-geometry helical spring types are presented in figure 3.1. The wire (bar) of which such a spring is manufactured can be of variable section at one or both extremities, with/without constant-diameter central portion (see figure 3.2).

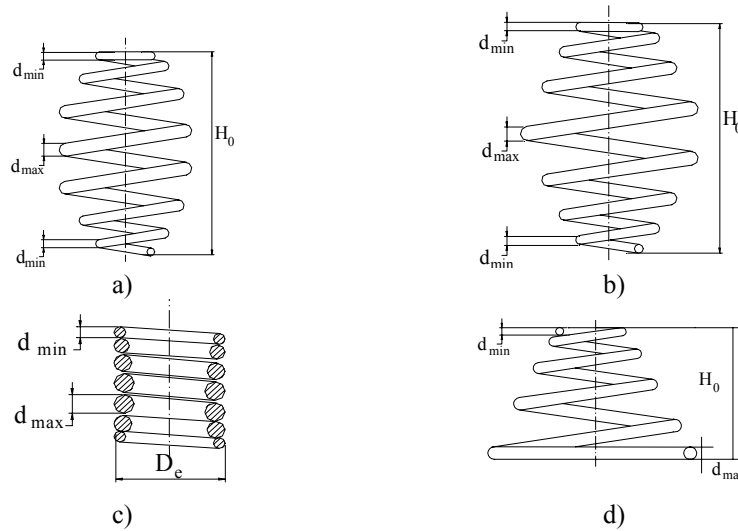


Fig. 3.1 Helical springs with variable geometry

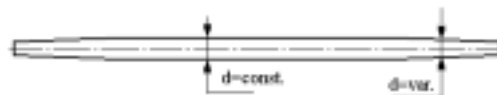


Fig. 3.2 Variable-section bar

The functioning of such a spring presents the following phases (see figure 3.3):

- At reduced intensity centrally axial loads, all active coils participate and the characteristic is pronouncedly linear (1<sup>st</sup> domain);
- Once the load increases the coils with variable-section bar settle successively and the characteristic becomes non-linear and progressive (2<sup>nd</sup> domain);
- After the settlement of all variable-section coils the spring's characteristic becomes again linear (3<sup>rd</sup> domain).

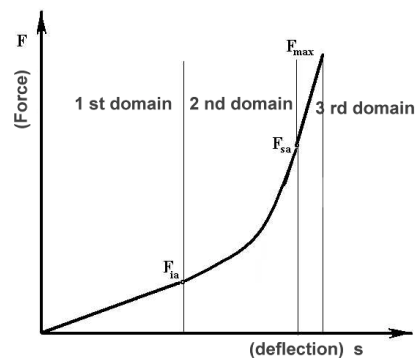


Fig. 3.3 Load-deflection characteristic

On the characteristics' progressive portion, the spring's rigidity is no longer constant, it varies linearly.

The spring can be constructed so that, at an instantaneous load which blocks the respective coils, it won't exceed the admissible tension of the material of which the spring is made.

The main advantages of the new spring category are the following (besides achieving load-deflection dependencies closer to necessities imposed by its functional role): an efficient material use, low weight, large useful deflection ( $f$ ) in respect to the spring's free-state height ( $H_0$ ) and the avoiding of the oscillate system's functioning in resonance regime.

The disadvantage cannot be omitted, either: the relatively high cost to the variable section bar execution technology.

The new spring class is successfully used where a limited room is available at a relatively long needed run: auto-vehicle suspensions and damping apparatus/systems.

In order to determine the number of progressive coils, the diameters characteristic to the variable-section bar and the cylindrical helical spring's middle diameters, the spring characteristic's progressive domain is to be divided into  $\Delta F$  force and  $\Delta f$  deflection steps, accepting thus a first approximation, more precisely that the characteristic's domain' variable rigidity is transformed into a rigidity that is constant on each sections. It is considered that each force step  $\Delta F$  determines a  $\Delta n$  coil fraction's settling. For the coil section  $\Delta n_j$  blocked by the  $F_j$  force, the  $d_j$  bar diameter, the medium diameter corresponding to  $D_{mj}$  and the  $\Delta l$  bar length are calculated [3]. The sum of all  $\Delta n_j$  values represents the total progressive coils' number.

Profiled helical springs made of variable-section bar can be simple-profiled (meaning conical with constant slope angle, conical with constant pitch or paraboloid), double-profiled, or composed of cylindrical portions continued by simple-profiled springs, at one or both extremities.

The number of progressive coils in a simple profiled spring is determined in function of load restrictions, of the helix' slope angle and of the chosen spring variant: conical or paraboloid.

The calculus can be particularized for any possible spring configuration [3].

The load-deflection characteristic can be traced through two methods:

a) the first method [3] proposes an iterative track in calculating the deflection and force corresponding to each of the three zones of the characteristic. This analysis allows a theoretical definition of the elastic characteristic throughout the whole existence range and finally, its graphic representation.

b) the second method, according to [5], is based on the deflection's expression:

$$f = \frac{8}{G} \frac{F}{\pi} \int_{l_0}^{l_1} \frac{D_m^2}{d^4} dl \quad (3.1)$$

The integral has variable limits, because with an increase in the compression force, the progressive coils start to settle and the bar length corresponding to the still active, unsettled coils decreases.

The analytical expression of the load-deflection characteristic's progressive domain (2nd domain, figure 3.3) was established, with a confidence coefficient higher than or at least equal to 0,99:

$$y = a_2 + b_2x + c_2x^2 \quad (3.2)$$

or

$$y = a'_2 + b'_2x + c'_2x^3 \quad (3.3)$$

The modeling of helical springs with variable geometry and the simulating of their behavior under load have been done in the programming medium MATLAB. The soft modules have been conceived so that an adequate program to any possible helical spring configuration can be rapidly created. The particularized programs for the most widely used types of springs have been verified on the studied models.

Since the maximum deviations of the characteristic are situated under the admissible limit of 10%, it may be considered that the theory and the own-elaborated soft are valid.

4. EXAMPLE OF KINETOSTATIC CALCULUS FOR THE CAM MECHANISM

For a rotating cam mechanism with its cam follower in translation and equipped with a roll, the kinetostatic calculus of the resulting force when using a spring with linear characteristic, respectively when using a spring with non-linear characteristic is to be presented in comparison.

In the considered example problem, the following geometric parameters were imposed:

Table 4.1

Cam follower shaft	$s_{max} = 40 \text{ mm}$	Polynomial movement law 3-4-5
Base radius	$R_b = 140 \text{ mm}$	
Cam RPM	$n = 100 \text{ rot/min}$	
Radius of roll	$r = 16 \text{ mm}$	
Climbing run	$\varphi_u = \varphi_1 = 90^\circ$	
Superior stationing	$\varphi_{ss} = \varphi_2 = 90^\circ$	
Descent run	$\varphi_c = \varphi_3 = 90^\circ$	
Inferior stationing	$\varphi_{si} = \varphi_4 = 90^\circ$	
Cam follower mass	$m = 2 \text{ kg}$	

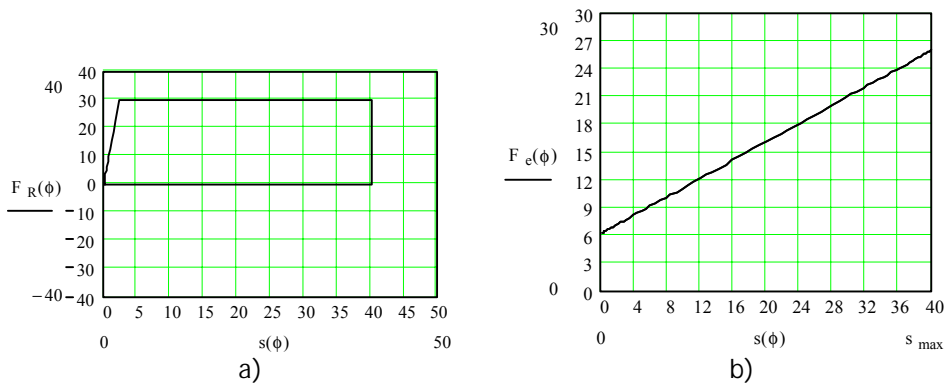


Fig. 4.1 Variation law for the useful force and for the elastic force

The variation law of the useful force is presented in Fig. 4.1a. The dependency between the elastic force and the deflection of the helical spring with linear characteristics is presented in Fig. 4.1b. The spring's characteristic is given by the expression (2.5), where the spring's elastic constant is determined from the relation:

$$k = \frac{F_s - F_{e0}}{s(\varphi_s^i) - s(0)} = 0.495 \text{ N/mm} \quad (4.1)$$

where:  $F_{e0} = 0.3 \cdot F(\varphi_s^i)$ ,  $F_s = 1.14 \cdot F(\varphi_s^i)$ .

The variation diagrams of the force system acting on the cam follower when using a helical spring with linear characteristic are presented in Fig. 4.2.

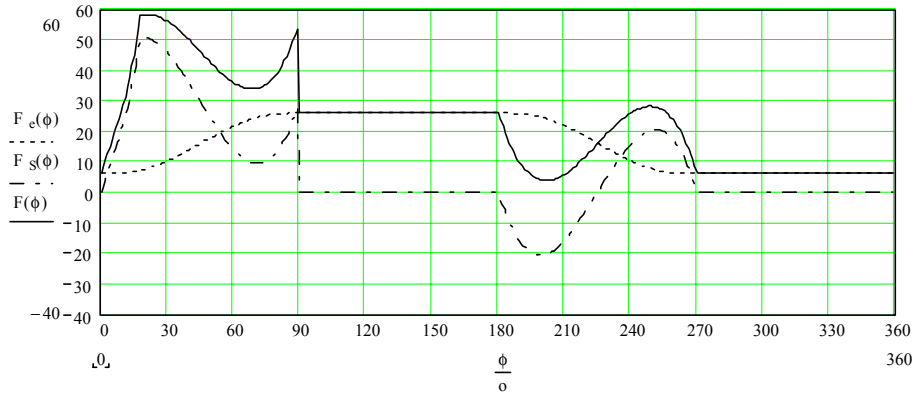


Fig. 4.2. Variation diagrams of the force system acting on the cam follower in case of using helical springs with linear characteristic

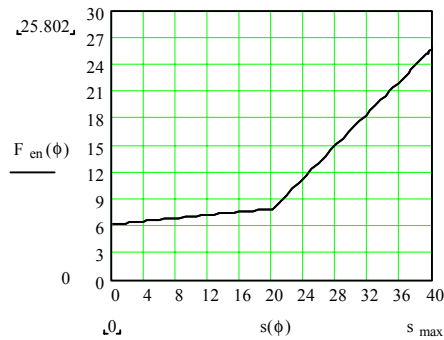


Fig. 4.3 The elastic force's variation law

In Fig. 4.3 the dependency between the elastic force and the deflection of the helical spring is presented. The spring's characteristic is given through a set of points by the expression (2.6), where:

$$F_{e0} = 0.3 \cdot F(\varphi_s^i),$$

$$F_{e1} = F_{e0} + 10\% \cdot (F(\varphi_s^i) - F_{e0}),$$

$$F_{e2} = 1.14 \cdot F(\varphi_s^i)$$

In Fig. 4.4 the variation diagrams of the force system acting on the cam follower when using a helical spring with non-linear characteristic are presented.

Through comparing the two variation diagrams of the force system, the surplus of resulting force in case of the spring with linear characteristic in respect with the spring with non-linear characteristic can be determined. In figure 4.5 this surplus is presented.

The force surplus that is eliminated by using helical springs with non-linear load-deflection characteristic reduces the reaction forces in the cam superior kinematical pair - the cam follower's roll, and implicitly reduces the wear in it.



For the considered cam mechanisms' kinetostatic calculus example, in view of optimizing the cam mechanisms' functioning by using variable-geometry helical springs, it is necessary to design a spring that has a load-deflection characteristic quasi-identical to the one in figure 4.3. The dimensions of the spring (Fig. 3.1 d) designed with help of a dedicated software, out of dimension patterns in the cam mechanism taken into account (as example), will need to have a free-state height  $H_0 = 120$  mm, a maximum diameter  $D_{max} = 16$  mm, a minimum diameter  $D_{min} = 8$  mm,  $d_{max} = 1.2$  mm and  $d_{min} = 0.6$  mm.

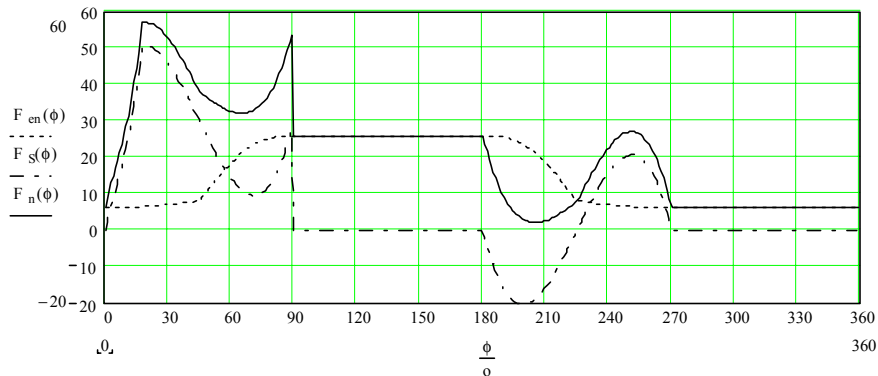


Fig. 4.4. Variation diagrams of the force system acting on the cam follower in case of using helical springs with non-linear characteristic

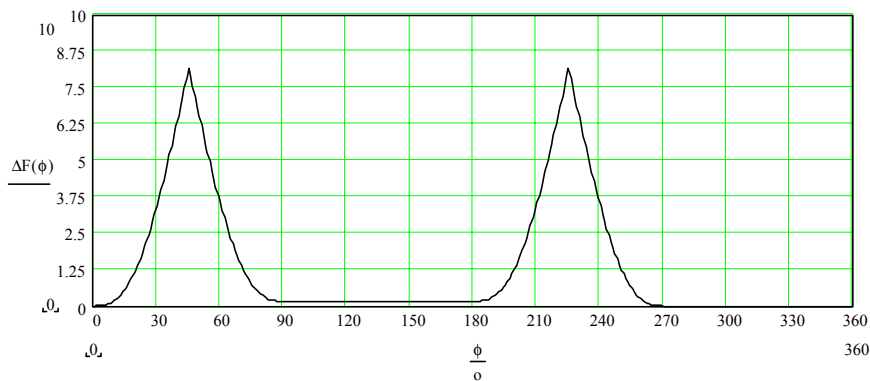


Fig. 4.5. Force surplus of a cam mechanism using a spring with linear characteristic compared to one using a spring with non-linear characteristic

The load-deflection characteristic, compared to the imposed one (Fig. 4.3), can be seen in figure 4.6, and compared to the linear characteristic (Fig. 4.1b), can be seen in figure 4.7.

Through linking the variable-geometry helical springs' domain to the cam mechanisms' domain, an optimization is achievable in the energy consumption of the automations (that use cam mechanisms set in action by a central motor/actuator), of cam mechanisms used in automobile industry, and of other applications of such mechanisms.

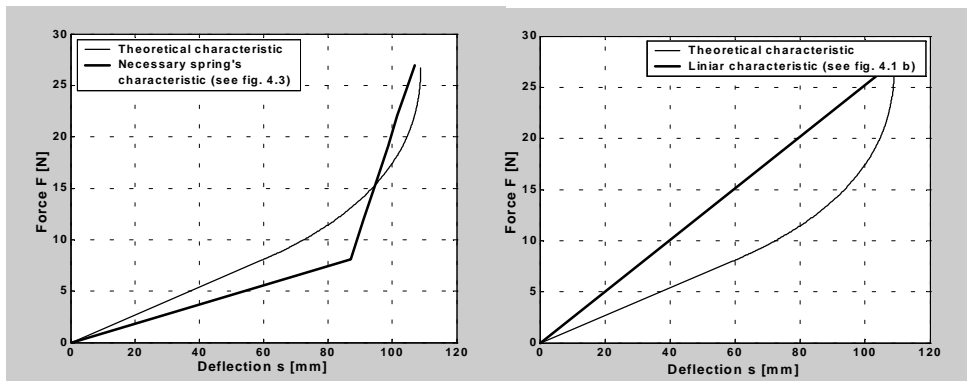


Fig. 4.6 The progressive characteristics' comparison

Fig. 4.7 The linear characteristics' comparison

## 5. CONCLUSIONS

We reserve the right of sustaining that the variable-geometry helical springs' usage in the domain is an absolute novelty. This study could open new perspectives on the horizon of the cam mechanisms' kinetostatic and dynamic optimization.

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## OPTIMIZACIJA RADA BREGASTIH MEHANIZAMA PRI UPOTREBI HELIKOIDALNIH OPRUGA SA PROMENLJIVOM GEOMETRIJOM

Rodica Militaru, Erwin-Christian Lovasz

*Pri upotrebi opruga sa progresivnom (nelinearnom) elastičnom karakteristikom, kontakt između brega i sledbenika može biti osiguran preko celog ciklusa. Veličina rezultirajuće sile koja deluje na sledbenik može biti smanjena, a takođe i vrednost opirućeg momenta reduciranog na breg, odnosno vrednost snage "pogonskog" motora.*