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APPLICATION OF DIGITAL SLIDING MODES TO SYNCHRONIZATION OF THE WORK OF TWO PNEUMATIC CYLINDERS

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Abstract. *The paper considers the problem of ensuring synchronized movement of two pneumatic cylinders. The control system for synchronization was designed by applying the theory of control system with variable structure. The algorithm of control is based on digital sliding regime. Measuring coordinates of state (positions and speeds) directly on the cylinders is supposed to be possible. It is shown that such a system ensures quick synchronization of cylinders under different initial conditions. The applied algorithm was compared with conventional algorithms of control. The quality of work of the considered system was illustrated by computer simulation.*

Key words: *pneumatic actuator, variable structure system, digital sliding regimes.*

1. INTRODUCTION

Pneumatic actuators (cylinders) are widely applied in flexible automation of machines and manufacturing processes. As with other types of actuators (e.g. electric motors), it is frequently demanded that two or more pneumatic cylinders move synchronously. Since these cylinders can have different initial conditions and different loads, it is necessary, as soon as possible, to achieve synchronization of further movement to a given position value. This problem was examined in detail by Unbehauen and Vakilzadeh [1-4] for various types of actuators, with the application of various algorithms of control of conventional type (P, PI, PID). The basic idea is represented in Figure 1. The referent signal is supplied to the system's input. Control signals of cylinders C_1 and C_2, respectively, are obtained as the sum and subtract of the referent input and the signal of the feedback, generated by the control mechanism (synchronizer). The feedback's signal is generated as the function of the difference of vectors of state (of position and speed) for cylinders C_1 and C_2. It is also possible to view the system as a control system with an observer, where, apart from the effect of the signal of the observation error on the observer's input, the effect on the object's input is also achieved. In the given case, C_2 is

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the 'observer', while C_1 is the 'object'. The basic difference from the system with the observer lies in the fact that here both the 'observer' and the 'object' are dynamic elements with the same dynamic properties and approximately the same parameters.

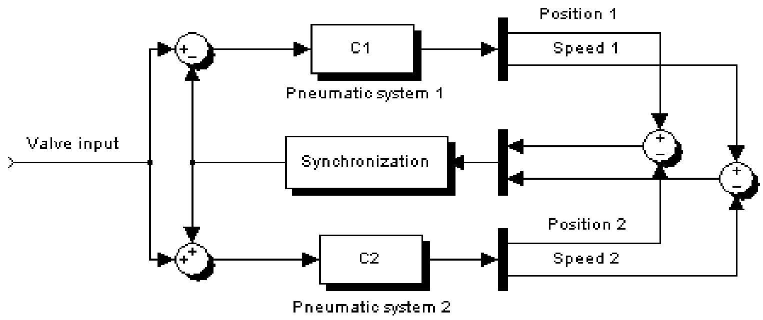


Fig. 1. Schematic representation of synchronization system.

The main goal of this paper is exploration of the possibility of applying the algorithm of control of the variable structure with sliding working regime to the problem of synchronization. The motives for this paper are to be found in numerous publications dedicated to the application of sliding regimes to various control tasks [9], which confirm the superiority of such systems over classic solutions. The basic features of sliding regimes, known to a small circle of experts in the field of automatic control, are:

- theoretical invariance to the external load and internal perturbations (parameters uncertainity) if maching conditions are satisfied [7] and practical robustnes;
- the character of the system's movement is known in advance;
- the movement does not depend on the object's parameters and control, but only on control parameters;
- what is necessary is not exact knowledge of the object's parameters, but only of the range of their possible change;
- it is easier to ensure the system's stability by decomposing the problem of stability into two simpler sub-problems;
- lowering the order of the differential equation which describes movement.

Apart from the above listed features, systems with a variable structure with sliding working regimes have shortcomings. There are two basic of these:

- the necessity of measurability of the full state of the controlled object;
- the occurrence of vibration in the control signal, which may cause excitation of non-modeled dynamics of the object and undesired movement in the area of the predicted trajectory.

The first shortcoming can successfully be solved by the application of the observer, which partly alleviates the second, too. However, algorithms which solve one or both these problems have also been developed.

This paper makes use of a digital algorithm of control of the variable structure, developed at Department of Automatic Control, Faculty of Electronic Engineering – Nis, which was applied for the first time to the problem of controlling the generator of waveforms [5] and detailed explained in [6].

The paper is organized in the following way: the second part explains the method of obtaining mathematical models of control objects. The third part presents an outline of the control algorithm on the basis of [6]. The fourth part presents the effects of the application of the above-mentioned algorithm of control in comparison with conventional algorithms [1-4] by means of computer simulation.

2. MATHEMATICAL MODEL OF THE SYSTEM

In this paper, control objects are pneumatic cylinders with bilateral effect, with an electromagnet-activated distributor (proportional 5/3), position indicator, and pressure sensor. Schematic representation of this system [10], together with all required values, is given in Figure 2.

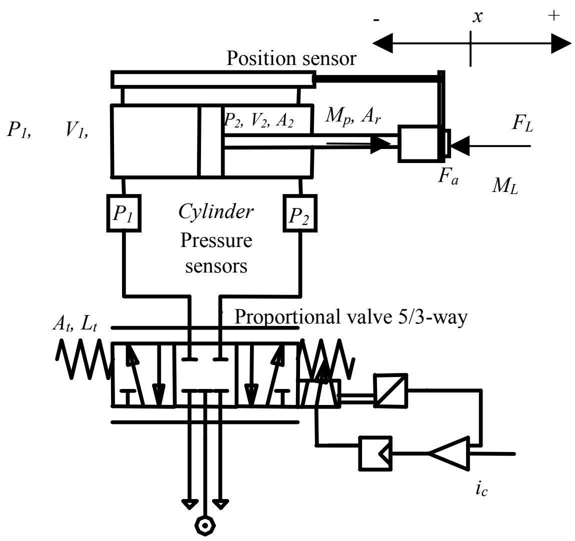


Fig 2. Schematic representation of the pneumatic cylinder-valve system.

The equation which describes the dynamics of one of the cylinders' process [8] is given as:

$$(M_L + M_p)x + \beta x + F_f + F_L = P_1 A_1 - P_2 A_2 - P_a A_r, \tag{1}$$

where: M_L is external mass, M_p is mass of the piston and the piston rod, x is position, β coefficient of damping, P_1 and P_2 absolute pressures in piston and piston rod chambers of the cylinder, P_a atmospheric pressure, A_1 and A_2 piston surfaces on the side of piston and piston rod chambers, and A_r surface of the transverse section of the piston rod.

The equation which describes the change of pressures in piston and piston rod chamber is given as [11]:

$$P = \frac{RT}{V_{0i} + A_i(0.5L \pm x)}(\alpha_{in} m_{in} - \alpha_{out} m_{out}) - \alpha \frac{P_i A_i}{V_{0i} + A_i(0.5L \pm x)} x \quad (2)$$
,

where, $\dot{m}_{in}$ and $\dot{m}_{out}$ are mass flows of air under pressure into and out of the cylinder, L is the cylinder's piston rod range, R is ideal gas constant, T temperature, V_{01} and V_{02} are non-active volumes of cylinders, α, α_{in} and α_{out} are coefficients which depend on the type of the thermal process, and which may have the values from 1 to k, depending on whether the process is isothermal or adiabatic

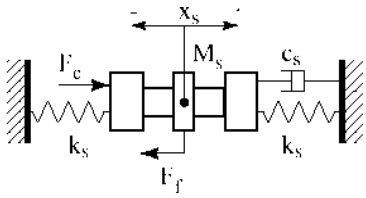


Fig. 2. A simplified proportional distributor for feeding 5/3 with electromagnetic activation

The dynamic model of the valve is represented by equation (3), obtained from figure 2, which represents a simplified proportional distributor for feeding 5/3 with electromagnetic activation.

$$M_S \ddot{x}_S + c_S \dot{x}_S + 2k_S x_S = K_{fc} i_c \quad (3)$$
,

where x_s is the movement of the proportional valve's piston, M_s is the mass of the valve's piston, c_s coefficient of damping, k_s coefficient of the valve's spring, K_{fc} coefficient which refers to electric current, and i_c current intensity.

The standard equation for the flow of mass through surface A_v, which is required in equation (2), is given as:

$$\dot{m}_v = \begin{cases} C_f A_v C_1 \frac{P_u}{\sqrt{T}} & if \ \frac{P_d}{P_u} \le P_{cr} \\ C_f A_v C_2 \frac{P_u}{\sqrt{T}} (\frac{P_d}{P_u})^{1/k} \sqrt{1 - (\frac{P_d}{P_u})^{(k-1)/k}} & if \ \frac{P_d}{P_u} > P_{cr} \end{cases} \quad (4)$$

where $\dot{m}_v$ is mass flow through the related surface, C_f dimensionless value, P_u lower pressure, P_d upper pressure, and C_1 and C_2 are the constants given in equation (5). The constants for air (k=1.4) are C_1=0.040418, C_2=0.156174, P_{cr}=0.528.

$$C_1 = \sqrt{\frac{k}{R}(\frac{2}{k+1})^{\frac{k+1}{k-1}}}; \qquad C_2 = \sqrt{\frac{2k}{R(k-1)}}; \qquad P_{cr} = (\frac{2}{k+1})^{\frac{k}{k-1}} \quad (5)$$
.

The valve's passage surfaces, which appear in equation (4), are calculated in the way shown in equations (6) and (7),

$$A_{vin} = \begin{cases} 0 & if\ x_s \le p_w - R_h \\ n_h\left[2R_h^{\ 2}\arctan\left(\sqrt{\dfrac{R_h - p_w + x_s}{R_h + p_w - x_s}}\right) - (p_w - x_s)\sqrt{R_h^{\ 2} - (p_w - x_s)^2}\right] & if\ p_w - R_h < x_s < p_w + R_h \\ \pi n_h R_h^{\ 2} & if\ x_s \ge p_w + R_h \end{cases} \quad (6)$$

$$A_{vex} = \begin{cases} \pi n_h R_h^{\ 2} & if\ x_s \le -p_w - R_h \\ n_h\left[2R_h^{\ 2}\arctan\left(\sqrt{\dfrac{R_h - p_w + |x_s|}{R_h + p_w - |x_s|}}\right) - (p_w - |x_s|)\sqrt{R_h^{\ 2} - (p_w - |x_s|)^2}\right] & if\ -p_w - R_h < x_s < R_h - p_w \\ 0 & if\ x_s \ge R_h - p_w \end{cases} \quad (7)$$

The value ($2p_w$) is somewhat larger than the radius R_h, and n_h is the number of openings, [8].

The damping force, from equation (1), is calculated according to equation (8)

$$F_f = \begin{cases} F_{sf} & if\ \dot{x} = 0 \\ F_{df}\ \mathrm{sgn}(\dot{x}) & if\ \dot{x} \ne 0 \end{cases} \quad (8)$$

where F_{sf} and F_{df} are static and dynamic damping forces, respectively.

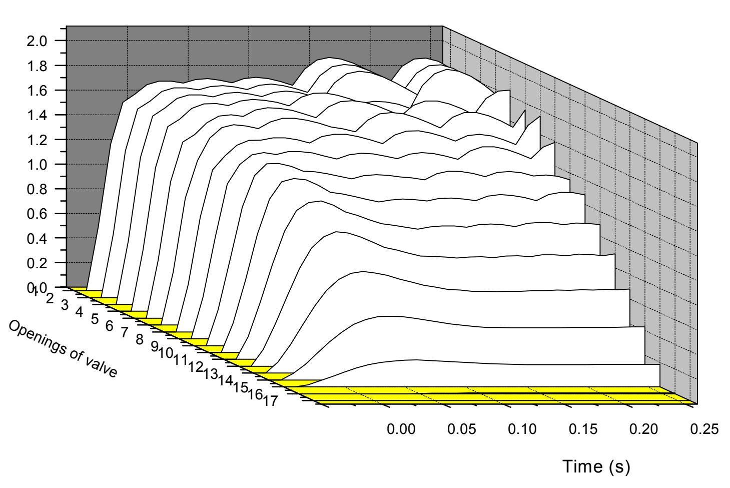
By using the equations given above, it is possible to mathematically describe a pneumatic system for positioning, which consists of:

– a model of the proportional valve;

– a model of pressure change in the piston chamber and the piston rod chamber; and

– a dynamic model of the cylinder with bilateral effect.

The response of the piston rod speeds of the mathematical model, while using feeding pressure 2 bars, and at different openings of the proportional distributor for feeding 5/3, is represented in Figure 3.



Valve	x_s(m)
1	0.003
2	0.0029
3	0.0027
4	0.0025
5	0.0023
6	0.0021
7	0.0019
8	0.0017
9	0.0015
10	0.0013
11	0.0011
12	0.0009
13	0.0007
14	0.0005
15	0.0003
16	0.0001
17	0.00008

Fig. 3. The response of the piston rod speeds.

Since synchronization ends at very small openings of the valve of the distributor for feeding, it was here that identification of the linear model was performed, i.e. when the valve's opening is x_s=0.0001m. In this case, we have a transfer function:

$$\frac{\dot{x}_k}{u}(s) = \frac{66}{1+0.071s} \quad \Rightarrow \quad \frac{x_k}{u}(s) = \frac{66}{s(1+0.071s)} \tag{9}$$

where $\dot{x}_k$ the speed of the cylinder's piston, x_k cylinder's piston position and $u=x_s$ is the valve's opening;
i.e. the model in the state space form is given by the relation (10),

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{bu}$$
$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -14.0845 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 929.577 \end{bmatrix}, \tag{10}$$

where $\mathbf{x} = [\dot{x}_k \ \ x_k]^{\mathrm{T}}$.

3. Algorithm of Digital Control with Sliding Mode

The algorithm of control, whose application to the problem of synchronizing pneumatic actuators is examined in this paper, belongs to the group of digital algorithms of control of variable structure. The goal of synthesis of control is to achieve movement of the system in the space of state on a pre-given hyper-surface, in systems of the higher order, i.e. on a line (most frequently a straight line), in systems of the second order. To do so, what must be ensured is the transfer of the system's state from any initial state to the given hyper-surface and its subsequent movement on it in the sliding regime. This means that the system's phase trajectories all go into the given hyper-surface. Since it is selected so that it passes through the outcome of the space of state, which represents the state of equilibrium, asymptotic stability of the system is also ensured. In this way, the system is brought into equilibrium according to a pre-given trajectory, which may also have attributes of optimality. To summarize, the movement of these system has three phases: (I) the phase of reaching the hyper-surface; (II) the phase of the sliding regime; (III) the phase of steady state.

If the sliding hyper-surface is marked as s(**x**), the conditions are met by satisfying the inequality

$$s(\mathbf{x})\dot{s}(\mathbf{x}) < 0 \tag{11}$$

This condition can be satisfied by applying various algorithms of control. However, they must contain a relay component of type, which may give rise to parasitic movements (chattering) in the area of the hyper-surface s(**x**)=0 even in the steady state. Such movements are especially intrusive in electromechanical systems

$$U_0 \operatorname{sgn}\{s(\mathbf{x})\}, U_0 > 0 \tag{12}$$

The algorithm applied in this paper eliminates or minimizes the problem of vibration to a tolerable level. The control is formed so that it has two components: a relay component, which ensures safe and quick transfer of the system's state near the sliding hyper-surface without intersecting it, and a linear component, which brings the system's state into s(**x**)=0 in the following step (during one discretization period).

In shortest, the applied algorithm , which is described in [6] in detailed, can be represented in the following way.

For a given controllable and observable dynamic system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u; \quad \mathbf{x} \in R^n, u \in R^1, \mathbf{A}_{nxn}, \mathbf{b}_{nx1} \tag{13}$$

With slowly changing parameters within limits known in advance, whose digital equivalent

$$\delta\mathbf{x}(kT) = \mathbf{A}_\delta(T)\mathbf{x}(kT) + \mathbf{b}_\delta(T)u(kT) \tag{14}$$

is obtained by applying relation

$$\mathbf{A}_\delta(T) = \frac{e^{\mathbf{A}T} - \mathbf{I}_n}{T}, \quad \mathbf{b}_\delta(T) = \frac{1}{T}\int_0^T e^{\mathbf{A}\tau}\mathbf{b}d\tau, \tag{15}$$

The sliding regime is organized on hyper-surface s(**x**), whose equation in the state space is

$$s(\mathbf{x}(k)) = s(k) = \mathbf{c}_\delta^{\mathrm{T}}(T)\mathbf{x}(k) = 0; \mathbf{c}_\delta^{T}(T)\mathbf{b}_\delta(T) = 1 \tag{16}$$

The control is formed so as to realize the desired process of reaching the sliding surface, which is defined by the following dynamic relation

$$\delta s(k) = \frac{s(k+1) - s(k)}{T} = -\Phi(s(k), \mathbf{X}(k)) = \mathbf{c}_\delta^{\mathrm{T}}\delta\mathbf{x}(k) \tag{17}$$

where $\mathbf{\Phi}$ is the function to be selected, and

$$\mathbf{X}(k) = \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}(k-1) \end{bmatrix}, \mathbf{x}(k-1)|_{k=0} = \mathbf{x}(0) \tag{18}$$

determines the area near the hyper-surface where linear control has effect.

By substituting (14) in (17) and solving the obtained equation according to *u(k)*, we obtain,

$$u(k) = -\mathbf{c}_\delta^{\mathrm{T}}(T)\mathbf{A}_\delta(T)\varepsilon(k) - \Phi(s(k), \mathbf{X}(k)). \tag{19}$$

By translating system (14) into the regular form, by means of linear transformation,

$$\mathbf{x} = \mathbf{P}_1(T)\mathbf{P}_2(T)\begin{bmatrix} \tilde{\mathbf{x}} \\ s \end{bmatrix}; \tilde{\mathbf{x}} \in \Re^{n-1}, \tag{20}$$

where the sliding hyper-surface becomes a coordinate of state, and where matrices of transformation are

$$\mathbf{P}_1(T) = [\mathbf{b}_\delta(T) \quad \dots \quad \mathbf{A}_\delta{}^{n-1}(T)\mathbf{b}_\delta(T)]\begin{bmatrix} a_1(T) & \dots & a_{n-1}(T) & 1 \\ a_2(T) & \dots & 1 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{bmatrix} \tag{21}$$

$$\mathbf{P}_2(T) = \begin{bmatrix} \mathbf{I}_{n-2} & \mathbf{0}_{(n-1)x1} \\ -\mathbf{c}_1^T(T) & 1 \end{bmatrix}; \mathbf{c}^{\mathrm{T}}(T) = \mathbf{c}_\delta^{\mathrm{T}}(T)\mathbf{P}_1(T) = [\mathbf{c}_1^{\mathrm{T}}(T) \quad 1] \tag{22}$$

we obtain model of the system in the form

$$\begin{aligned} \delta\tilde{\mathbf{x}}(k) &= \mathbf{A}_{11}(T)\tilde{\mathbf{x}}(k) + a_{12}s(k), \\ \delta s(k) &= -\Phi(s(k), \mathbf{X}(k)) \end{aligned}, \tag{23}$$

where,

$$\mathbf{A}_{11}(T) = \begin{bmatrix} \mathbf{I}_{n-2} \quad \mathbf{0}_{(n-2)x1} \\ \hline -\mathbf{c}_1^{\mathrm{T}}(T) \end{bmatrix}. \tag{24}$$

It is proved that selection of function Φ in the form

$$\Phi(s(k), \mathbf{X}(k)) = \min\left(\frac{|s|}{T}, \sigma + \rho\frac{|s|}{T}\right)\mathrm{sgn}(s); \; 0 < qT < 1, \sigma > 0 \tag{25}$$

ensures the transfer of the system's state from any initial state into area X, defined in the following way:

$$S(T) = \left\{\mathbf{X} \in \Re^{2n} : |\, \mathbf{c}_\delta^{\mathrm{T}}(T)\mathbf{x}\,| < \frac{\sigma T}{1 - qT}\right\} \tag{26}$$

within a finite period of time, and ensures that, within a discretization period T, the system's state satisfies condition (16), i.e. $s(k)=0$. Then the first relation (23) becomes (27)

$$\delta x\tilde{\mathbf{x}}(k) = \mathbf{A}_{11}(T)\tilde{\mathbf{x}}(k) \tag{27}$$

so that stability of movement is defined by own values of matrix A_{11}. Since (24), own values of matrix A_{11} are functions of parameters of the sliding hyper-surface only, i.e. of elements of vector $\mathbf{c}_\delta$. Supposing that own values of the given matrix are

$$\delta_i(T) = \frac{e^{-\alpha_i T} - 1}{T}, \quad \alpha_i > 0, \quad i \neq j \Rightarrow \alpha_i \neq \alpha_j, \quad i, j = 1, \dots, n-1 \tag{28}$$

then elements $\mathbf{c_1}$(T) are determined by means of relation

$$c_i(T) = \frac{1}{(i-1)!}\frac{d^{i-1}\prod_{j=1}^{n-1}(\delta - \delta_j(T))}{d\delta^{i-1}} \Big|_{\delta=0}, \tag{29}$$

and vector $\mathbf{c}_\delta^{\mathrm{T}}$(T) is determined by relation

$$\mathbf{c}_{\delta}^{\mathrm{T}}(T) = [\mathbf{c}_1^{\mathrm{T}}(T) \mid 1]\mathbf{P}_1^{-1}(T) \ . \tag{30}$$

4. SYNCHRONIZATION OF THE SYSTEM WITH TWO PNEUMATIC CYLINDERS

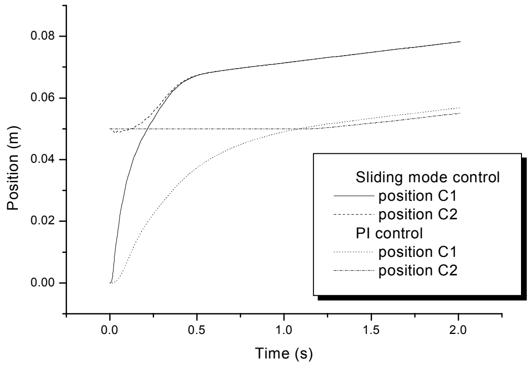
As was said in the introduction, synchronization of the work of two or more cylinders is a sizeable problem, which can be solved by using adequate control. Sections 2 and 3 briefly explained the achievement of a mathematical model of the pneumatic system which contains one cylinder with bilateral effect, as well as the formation of the control signal which enables synchronization.

The pneumatic system to be considered in this section consists of two pneumatic cylinders with bilateral effect whose work should be synchronized, since they have different initial positions. The system's model, given in state space form by equation (10), is discretized by means of transformation (15), with discretization period 0.1s. The sliding hyper-surface coefficient is calculated by means of equation (28), for α=20s^{-1}. Parameters q and σ are taken to be 0 and 10, respectively. Vector $\mathbf{c}_{\delta}$(T)=[0.016333521 0.00100545] and $\mathbf{c}_{\delta}$(T)$\mathbf{A}_{\delta}$(T)= [0 0.0021764]. Different initial positions are reflected in the fact that the piston rod of the first cylinder is already somewhat drawn out, whereas the piston rod of the other cylinder is completely pulled in.

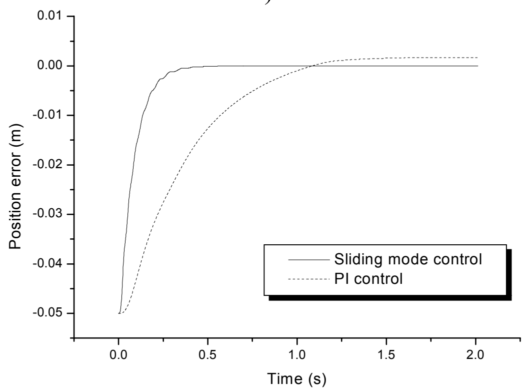
The problem of synchronization is to bring, within a short period of time, the positions of both cylinders (x_1 and x_2) into equal positions, with no differences in further work. In order to successfully solve this problem of synchronization in the manner described in section 3, the algorithm of digital control, given in equation (19), is used, which ensures that errors in positions and speeds of the piston rods are eradicated within a short period of time.

To confirm that the algorithm of control, which enables quick synchronization, was adequately selected, Figure 4 shows simulated results of the positions of standard pneumatic cylinders, type DNU-100-50-PPV-A, manufactured by Festo company, maximal range 0.5m. As has been mentioned, initial positions of the cylinders differ; in this case, the piston rod of the first cylinder C_1 is at initial position x_{10}=0.05m, whereas the piston rod of the other cylinder C_2 is completely pulled in, i.e. x_{20}=0m. Figure 4a shows the work of the system when synchronization is performed, while Figure 4b shows the result of the position error, where can be seen that position error becomes zero within a short period of time. Figure 5 shows the effect of the force of disturbance of value 25N, starting from moment 1.5s, whereby the used algorithm of digital control shows satisfactory results.

The occurrence of chattering is the consequence of the application of the algorithm of control to the real model.

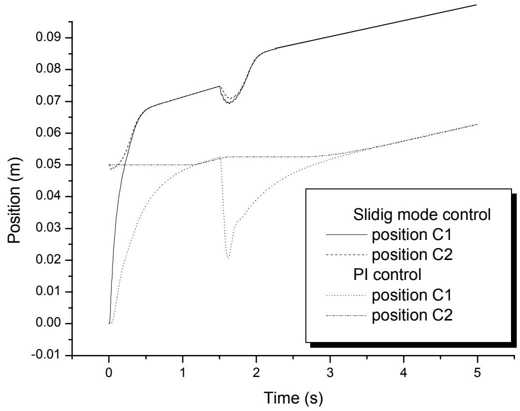


a)



b)

Fig. 4. Simulated results of the positions of standard pneumatic cylinders: algorithm of digital control with sliding mode and PI regulator, and b) position error.



c)

Fig. 5. Simulated results of the positions of standard pneumatic cylinders with the effect of the force of disturbance of value 25N, starting from moment 1.5s.

5. CONCLUSION

The paper outlines the problem of synchronizing the work of two cylinders, and the algorithm of digital control which solves this problem is given. The algorithm of control serves its purpose, since it ensures synchronization of the work of the cylinders within a short period of time, which was demonstrated and explained in section 4. This way of synchronizing cylinders with bilateral effect simplifies and reduces the price of realization to a significant extent

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PRIMENA DIGITALNIH KLIZNIH REŽIMA U SINHRONIZACIJI RADA DVA PNEUMATSKA CILINDRA

Vladislav Blagojević, Čedomir Milosavljević

Razmatra se zadatak obezbeđenja sinhronog kretanja dva pneumatska cilindra. Upravljački sistem za sinhronizaciju projektovan je primenom teorije sistema upravljanja promenljive strukture. Algoritam upravljanja se bazira na digitalnom kliznom režimu. Pretpostavlja se da se na cilindrima mogu neposredno meriti koordinate stanja: pozicije i brzine. Pokazano je da takav sistem obezbeđuje brzu sinhronizaciju cilindara pri različitim početnim uslovima. Izvršeno je upoređenje primenjenog algoritma sa konvencionalnim algoritmima upravljanja. Simulacijom na računaru ilustrovan je kvalitet rada razmatranog sistema.