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DESIGN AND SIMULATION OF MESHING OF INTERNAL INVOLUTE SPUR GEARS WITH PINION CUTTERS

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Abstract. *This paper presents the kinematics and computer aided procedure for the design of internal involute spur gears. Geometric and operating constraints for internal gears are defined. For the defined kinematics model, a computer program is developed. The developed kinematics model is most helpful in designing, contact and stress analyzing, manufacturing, measuring and optimizing internal gear sets. The numerical results are tested by computerized simulation of mesh of internal involute spur gears with pinion cutters to demonstrate the developed model.*

Key words: *internal gear, pinion, interference, design*.

1. Introduction

Internal gears have initially found application in design of planetary gear trains and mechanisms for their high gear ratio and small space requirement. In comparison with an external spur gear, the center distance of an internal spur gear is much shorter. Planetary gear trains have a number advantages as compared to the transmission with fixed shafts. Under similar operating conditions the planetary transmissions serve longer and produce less noise compared to the fixed shaft transmission.

In recent years, many researchers have successfully investigated the surface geometry of spatial conjugate gear pairs [1,2]. The kinematics and geometric relation between the gear and the cutting tool during the profile generation process is the same relation as that developed when a gear meshes with a rack [3]. These analyses have greatly extended understanding of surface geometry and contact kinematics. In this paper, a kinematics model for the internal gear set is developed based on the cutting design parameters of the pinion cutter form used in generating internal spur gears.

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In order to ensure the mounting as well as the correct meshing of the gears, it is necessary to fulfil the requirements regarding their alignment and the clearance between the gears. It is necessary to express the above requirements by the corresponding functional constraints, and based upon them, to identify all relevant values together with the areas of their practical applications.

Computer graphics of the gear set are presented to demonstrate the developed kinematics model.

2. Tip Interference

Interference may occur between internal and external tooth tips. Figure 1. Illustrates the condition when tip interference not exists. The critical point is when the pinion tip intersects the gear tip circle.

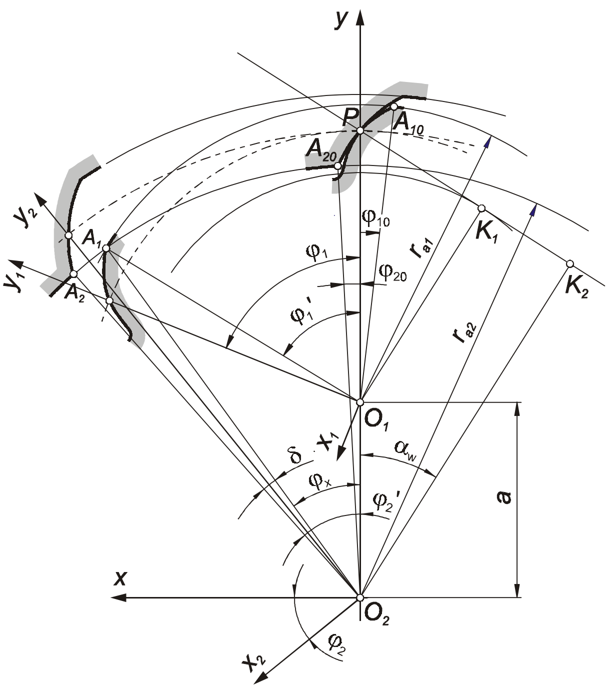

Fig. 1. Checking for tip interference.

Condition for the actual clearance angles between the profiles of a tooth pair measured on the addendum circle of the tooth pair, must be satisfied:

$$\delta = \varphi_2' - \varphi_x > 0 \tag{1}$$

From Fig. 2. the following equation can be derived:

$$r_{a1}^2 = r_{a2}^2 + a^2 - 2 \cdot a \cdot r_{a2} \cdot \cos\varphi_x \tag{2}$$

where r_{a1}, r_{a2} – the tip radii of the pinion and gear respectively,

The gear ratio gives the relation between the twist angles where the corresponding rotation of the internal gear is given by:

$$\varphi_2 = \frac{z_1}{z_2} \cdot \varphi_1 \tag{3}$$

The angles φ_1 and φ_2 are then calculated as follows:

$$\varphi_1^{`} = \arccos \frac{r_{a2}^2 - r_{a1}^2 - a^2}{2 \cdot r_{a1} \cdot a} \tag{4}$$

$$\varphi_2^{`} = \varphi_2 - \varphi_{20} \tag{5}$$

where $\varphi_{10} = inv\alpha_{A1} - inv\alpha_w$

$\varphi_{20} = inv\alpha_w - inv\alpha_{A2}$

α_{A1} and α_{A2} - the pressure angles at pinion and gear tooth tips respectively.

In connection with that, Fig. 2 represents the change of the functional constraints in the function of the rotation of angle φ_1, with a different tooth number difference Δz.

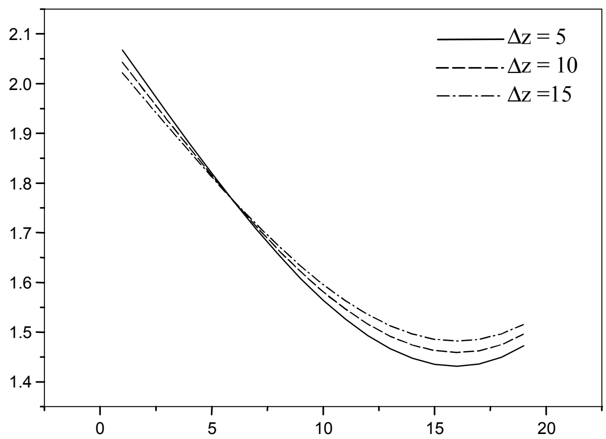

Fig. 2. The effect of the rotation of angle φ_1 upon the space requirements.

Based upon the graphic representation of the results obtained, it follows that the given functional constraint is exceptionally sensitive to the change rotation angle of the external gear – pinion cutter. The tooth number difference is the major influential factor on the clearance angle.

2. SIMULATION OF MESHING

The kinematics and geometric relation between the gear and the cutting tool during the profile generation process is the same relation as that developed when a gear meshes with a rack. Based on the basic law of conjugate action, the common normal of the surfaces at the contact point must pass through the instantaneous contact points on the surface of the cutting tool.

In order to simulate the conditions of meshing, coordinate system $O_1x_1y_1$ and $O_2x_2y_2$ that are rigidly connected to pinion 1 and gear 2, respectively as shown in Figure 3. The meshing of the gear tooth surfaces is considered in the fixed coordinate system Cxy that is rigidly connected to the housing.

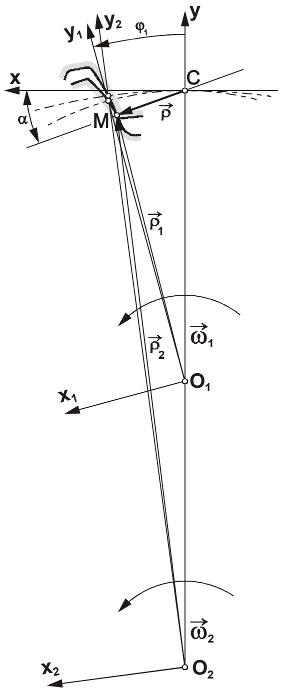

Fig. 3. Coordinates systems for generation of pinion tooth space.

Transforming coordinate system $O_1x_1y_1$ which is attached to the pinion cutter to fixed coordinate system Cxy, we can describe by:

$$\begin{Bmatrix} \vec{i}_1 \\ \vec{j}_1 \end{Bmatrix} = \begin{bmatrix} \cos\varphi_1 & \sin\varphi_1 \\ \sin\varphi_1 & \cos\varphi_1 \end{bmatrix} \begin{Bmatrix} \vec{i} \\ \vec{j} \end{Bmatrix} \tag{6}$$

At the point of contact, due to the tangency of two contacting gear tooth surfaces, the position vectors and their unit normals of both gear tooth surfaces should be the same. Therefore, the following equations must be observed [5]:

$$\vec{\rho}_f^{(p)} = \vec{\rho}_f^{(g)} \tag{7}$$

$$\vec{n}_f^{(p)} = \vec{n}_f^{(g)} \tag{8}$$

The shape of the pinion tooth is represented in coordinate system Cxy by the vector equation. Thus

$$\vec{\rho} = \overrightarrow{CO_1} + \vec{\rho}_1 = -r_{w1}\vec{j} + x_1\vec{i}_1 + y_1\vec{j}_1 \tag{9}$$

where

r_{w1} – cinematic circle of pinion cutter.

We transfer the pinion tooth shape to fixed coordinate system Cxy, using the matrix equation:

$$\vec{\rho} = (x\cos\varphi_1 + y_1 \cdot \sin\varphi_1)\cdot\vec{i} + (-x_1 \cdot \sin\varphi_1 + y \cdot \cos\varphi_1 \cdot r_{w1})\cdot\vec{j} \tag{10}$$

where

$$\begin{Bmatrix} x_1 \\ y_1 \\ t_1 \end{Bmatrix} = \begin{Bmatrix} r\cdot\varphi_1\cdot\cos\varphi_1\cdot\cos^2\alpha - r\cdot\sin\varphi\cdot(1-0{,}5\cdot\varphi_1\cdot\sin 2\alpha) \\ r\cdot\varphi_1\cdot\sin\varphi\cdot\cos^2\alpha + r\cdot\cos\varphi_1\cdot(1-0{,}5\cdot\varphi_1\cdot\sin 2\alpha) \\ 1 \end{Bmatrix}$$

The position vector from O_2 to the cutting point, expressed in fixed coordinate system is given by:

$$\vec{\rho}_2 = \overrightarrow{O_2C} + \vec{\rho} = r_{w2}\vec{j} + x\vec{i} + y\vec{j} \tag{11}$$

By applying the coordinate transformations, the equation of gear tooth surface can be represented in coordinate system $O_2x_2y_2$ as follows:

$$\vec{\rho}_2 = (x\cos\varphi_2 - y\sin\varphi_2 - r_{w2}\sin\varphi_2)\,\vec{i}_2 + (x\sin\varphi_2 + y\cos\varphi_2 + r_{w2}\cos\varphi_2)\,\vec{j}_2 \tag{12}$$

where

$$\begin{Bmatrix} \vec{i} \\ \vec{j} \end{Bmatrix} = \begin{bmatrix} \cos\varphi_2 & \sin\varphi_2 \\ \sin\varphi_2 & \cos\varphi_2 \end{bmatrix} \begin{Bmatrix} \vec{i}_2 \\ \vec{j}_2 \end{Bmatrix}$$

It is practically impossible to present in this paper all the relation between the gear and the cutting tool during the profile generation process.

5. THE RESULTS

Computer graphs of pinion and internal gear can be plotted and represented in fixed coordinate system as shown in Fig 4.

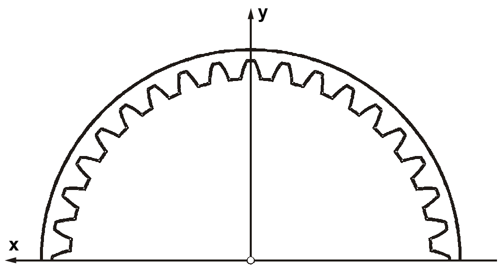

Fig. 4. Computer graph of internal involute spur gear.

6. CONCLUSION

In this paper method for computer generation of involute internal spur gears has been developed. The developed kinematics model is most helpful in designing, contact and stress analyzing, manufacturing, measuring and optimizing internal gear sets. The numerical results are tested by computerized simulation of mesh of internal involute spur gears with pinion cutters to demonstrate the developed model.

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KONSTRUKCIJA I SIMULACIJA SPREZANJA ZUPČANIKA SA UNUTRAŠNJIM OZUBLJENJEM SA ALATOM U OBLIKU ZUPČANIKA

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U radu je prikazan kinematski postupak kontruisanja zupčanika sa unutrašnjim ozubljenjem podržan odgovarajućim kompjuterskim programom. Geometrijska i radna ograničenja za zupčanik sa unutrašnjim ozubljenjem su definisana. Za definisani kinematski model razvijen je kompjuterski program. Razvijeni kinematski model je od izuzetne pomoći u kontruisanju, analizi kontaktnih napona, proizvodnji, kontroli i optimizaciji zupčastih parova sa unutrašnjim ozubljenjem. Numerički rezultati su testirani, kompjuterskom simulacijom sprezanja zupčanika sa unutrašnjim ozubljenjem sa alatom u obliku zupčanika, da potvrde razvijeni model.