FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol.1, N° 8, 2001, pp. 1107 - 1115

MULTICRITERION OPTIMIZATION OF MULTISTAGE GEAR TRAIN TRANSMISSION

UDC 62-233.3.001.26

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Abstract. *This paper presents an analytical and computer aided procedure for the multicriteria design optimization of gear train transmission. By applying the optimization methods in the field of gear transmission design it is possible to define the optimal parameters of the complete gear train transmission and of each transmission stage separately. For the defined multicriterion optimization model of the gear train transmission, computer program based in interactive dialogue is developed. The Monte Carlo method is implemented in the program. The result of program are presented in appropriate tables with Pareto–optimal solutions, target functions and criterions.*

Key words: *gear train transmission, multicriterion optimization, design*

1. INTRODUCTION

Concept from multicriterion optimization and decision theory can play an important role in all stages the design process. The optimizing design theory and methodology will be illustrated through a gear train transmission. Gear train transmissions present a very important group of machine members, which are utilized in a great number of engineering fields and which must satisfy very rigorous technical requirements regarding reliability, efficiency, precise manufacturing of gears, bearing, etc. Commencing with the demands imposed regarding the performance, the most modern experimental testing methods, e.g., the numerical computation methods of machine members, have been introduced. In addition to that, the latest achievements in the fields of technology and testing of the preciseness of manufacturing gears, bearings, etc. have been applied to the manufacturing process.

The development of the computer technology, together with the corresponding computer programs (FEM, AutoCAD, Quick BASIC, etc.), have very quickly found their

Received March 12, 2002

place in the development of the expert system for gearbox design (1) at a high technical level. Thus, it can freely be said nowadays that the gearbox design is no longer a "routine job", which in most cases based upon the designer's experience and knowledge.

This paper demonstrates the application of a nonlinear multicriteria optimization method, with the purpose to build such a powerful method as a module into the gear train design expert system. The introduction of a larger number of criteria considering the desirable performances, even the conflicting ones (axial distance efficiency), represents a significant step towards the reality of a gear train model solved by multicriteria optimization methods.

2. THE FORMULATION OF THE PROBLEM

Gear train transmissions represent complex mechanical systems that can be decomposed into the corresponding number of gears with corresponding interaction. This means that the procedure for multistage gear train optimization can also be carried out through the corresponding number of stages. During the first optimization stage, characterized by comparatively small number variables, the distribution of transmission ratio per gear train stages is defined from the conditions of the minimal volume of the gear sets. During the second stage, introducing a greater number of criteria, which represent the essential gear train performances, solves the multicriteria optimization problem.

The target function for multistage gear train representing the volume of the gear sets can be written in the form the following relation:

$$f(x) = \frac{\pi}{4} d_1^3 \varphi_I \left[(1+u_I^2) + \frac{\varphi_{II}}{\varphi_I} \frac{d_3^2}{d_1^2} (1+u_{II}^2) + \ldots + \frac{\varphi_N}{\varphi_I} \frac{d_{2N-1}^2}{d_1^2} (1+u_N^2) \right] \tag{1}$$

where $u_I, u_{ii}, \ldots, u_N$ – the transmission ratio for particular stages,

$d_1, d_3, \ldots, d_{2N-1}$ – diameters of kinematics circles of the driver gears,

$\varphi = b/d_1$ – ratio of width and diameter of the driver gear kinematics circle.

For the target function stated, it is also necessary to define the functional constraints from the standpoint of the surface strength for the first stage of gearing, which can be written in the following form:

$$g_1(x) = Z \cdot \frac{2 \cdot K \cdot T_1}{d_1^3} \cdot \frac{u_I + 1}{u_I} \leq \frac{[\sigma_H]_I}{S_H} \tag{2}$$

and, from the standpoint of the volume strength:

$$g_2(x) = K \cdot Y \cdot \frac{2 \cdot T_1}{\varphi_I \cdot d_1^2 \cdot m_I} \leq \frac{[\sigma_F]_I}{S_F} \tag{3}$$

where: $K = K_A K_{H\alpha} K_{H\beta} K_{HV}$ – load factor,

T_1 – pinion torque,

S – factor of safety.

In the exactly analogous way, the functional constraints from the standpoint of the surface and volume strength for other transmission stages of gear trains are determined. These strength constraints can be converted to expressions for the module as functions of the number of teeth on the pinion.

For the bending constraints, where Y_{Fa} is the tooth form factor for the highest point of the tooth contact for the given ratio and the number of teeth on the pinion, and lower bound on the module can be determined from bending fatigue:

$$m \geq \left(\frac{2 \cdot K \cdot T_1 \cdot Y_F \cdot S_F}{\varphi_I \cdot z_1^2 [\sigma_F]_I} \right)^{1/3} \tag{4}$$

Based on the assumption that Hertz stress is a measure of the tendency to pit, the module limit based on the contact stress at the lowest point of the single tooth contact can be expressed in the form of the following inequality:

$$m \geq \left(\frac{2 \cdot K \cdot T_1 (u_I + 1) \cdot Z^2 \cdot S_H^2}{\varphi_I \cdot z_1^3 \cdot [\sigma_H]_I^2 \cdot u_I} \right)^{1/3} \tag{5}$$

The efficiency of the Penalty function method is greatly dictated by the choice of the starting point within the feasible domain. As shown in Fig. 1 the feasible domain is the upper right hand corner of the plot.

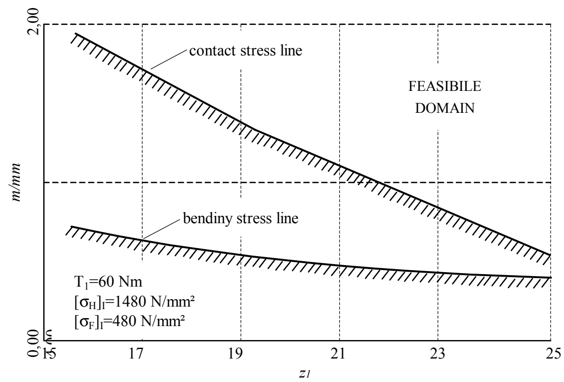

Fig. 1. Feasible domain for optimization procedure.

Commencing from the technical requirement concerning the transmission ratio of a gear train, it is also necessary to determine the functional constraint in the form of the equation:

$$h_I(x) = u - u_I \cdot u_{II} \cdot \ldots \cdot u_N = 0 \tag{6}$$

Input parameters for the optimization will be power to be transmitted, pinion input speed, normal pressure angle and material parameters. Basing upon the determined target function and the constraints, it can be noticed that this problem belongs to the field of nonlinear optimization with the constraints in the form of inequalities. For the solution of this problem, the computer program SUMT, based on the mixed penalty functions, has been applied. Fig. 2 shows a graphic representation of the results of the computer program SUMT. Basing upon the section of the corresponding functions, the domains of the optimum transmission ratios for the multistage gear trains are defined in the following way:

$u < 4.2$ – for single – stage gearboxes,

$u \in [4.2, 12.2]$ – for two stage gearboxes.

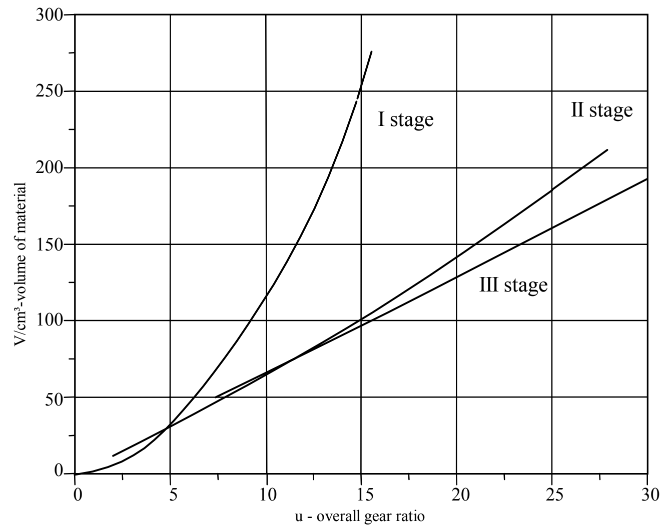

Fig. 2. The relation between the volume of gear train and overall gear ratio.

3. Formulation of the Optimization Model

The criteria regarding the desired performances are expressed by the criteria functions, which, for the best gearbox design, should reach the extreme:

$$\underset{x \in D}{extr}\, f(x)$$

Table 1 presents the objective function for the first stage of gearing.

Table 1. Objective vector function

$f_1(x) = \dfrac{m_n(z_1 + z_2)}{2 \cdot \cos\beta} \dfrac{\cos\alpha_t}{\cos\alpha_{tw}}$ - centre distance	$f_2(x) = \eta$ - efficiency
$f_3(x) = \varepsilon_\alpha$ - contact ratio	$f_4(x) = \alpha_{tw}$ - pressure angle
$f_5(x) = S_{H1} = [\sigma_H]_I / \sigma_{HI}$ - safety factor	$f_6(x) = V$ - volume of material

In addition to that, it is also necessary to include the functional constraints in the form of the inequalities. The Table 2 presents functional constrains.

Table 2. Functional constrains

$g_1(x) = \frac{[\sigma_F]_I}{\sigma_{FI}} - S_F > 0$ - bending stress	$g_2(x) = \frac{[\sigma_H]_I}{\sigma_{HI}} - S_H > 0$ - contact stress
$g_3(x) = z_1 - 2\frac{h - x_1}{\sin^2 \alpha_t} > 0$ - undercuting	$g_4(x) = f_{sa1} - 0{,}25 \geq 0$ - width of creast
$g_5(x) = \alpha_{A2} - \alpha_{E2} > 0$ - interference	$g_4(x) = f_{sa2} - 0{,}25 \geq 0$ - width of creast

Based upon the objective functions given and upon the functional constraints, all the relevant values of the gearbox have also been identified, so that the vector of the variable values can be written in the form of the following relations:

$$x = x(m_n, z_1, x_1, x_2, \varphi) \tag{7}$$

where x_1, x_2 – addendum modification of pinion gear and driven gear respectively.

4. Optimization Procedure

The optimization model is characterized by the existence of mutually conflicting criteria and thus the optimum solution in relation to one object function is not at the same time optimum in relation to another object function. This means that the acceptable solution can be only that permissible solution which is not dominated by another permissible solution. Finding the set of nondominant solutions $\boldsymbol{X^p}$, of the Pareto set is the first step in solving the multicriteria optimization problem.

In solving multicriteria optimization problems it is almost impossible to define explicitly in advance the preference regarding the selection of "the best" design. In such circumstances, the most suitable procedure is applying a method with an interactive approach, in which the decision-maker (the designer) gradually lays his preferences regarding the proposed set of Pareto solutions.

The Pareto solution set gets narrower and narrower because in each iteration the designer gives his own preferences regarding the desired performances, i.e., he places the upper confining values of the objective function vector. In the process, the calculation phase (determination of the Pareto solution set) is alternately replaced by the decision phase, in which the designer analyses the Pareto solution set, i.e., gives suggestions for reducing the observed set. Fig. 3 shows a conceptual flowchart for the interactive design optimization process.

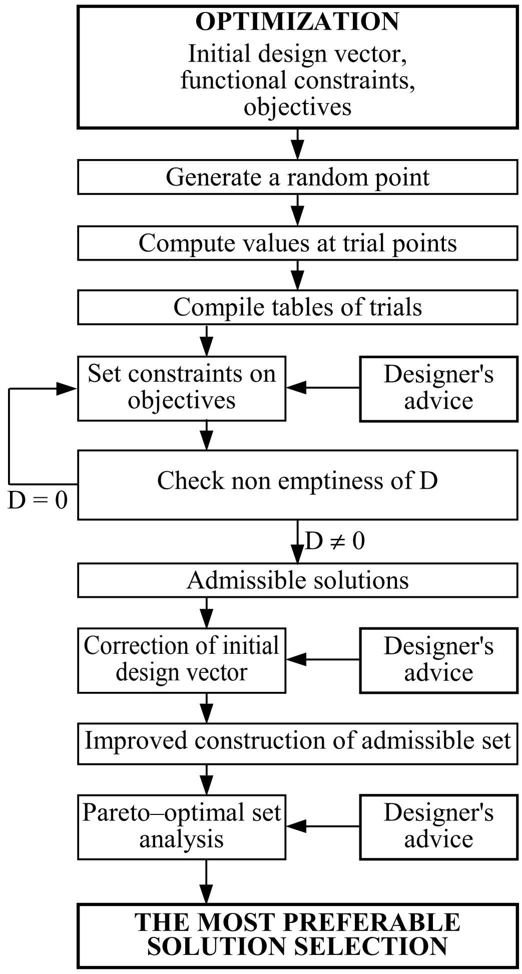

Fig. 3. Flow – chart for optimum design procedure.

5. THE RESULTS

For the proposed model a computer program with an interactive approach has been devised. The approach is based upon generation random numbers according to Monte Carlo approach. In the Monte Carlo approach a certain number of points are picked at random over the estimated range of all of the variables. This may be done formally by obtaining the randomly selected values for x_i from the following equation:

$$x_i = x_i^l + \delta(x_i^u - x_i^l) \quad i = 1,\dots,N \tag{8}$$

where x_i^l – estimated or given lower limit, x_i^u – estimated or given upper limit,

δ – a random number between zero and one.

The results of computer program can be shown in the form of diagram – the criterion space. Figures 4 and 5 present the criterion space for the axial distance – efficiency and criterion space for the axial distance – the volume of material used for gears.

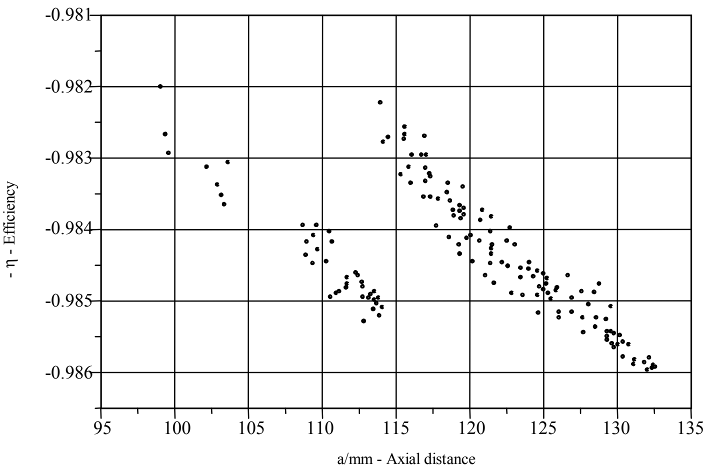

Fig. 4. The criterion space for axial distance – efficiency.

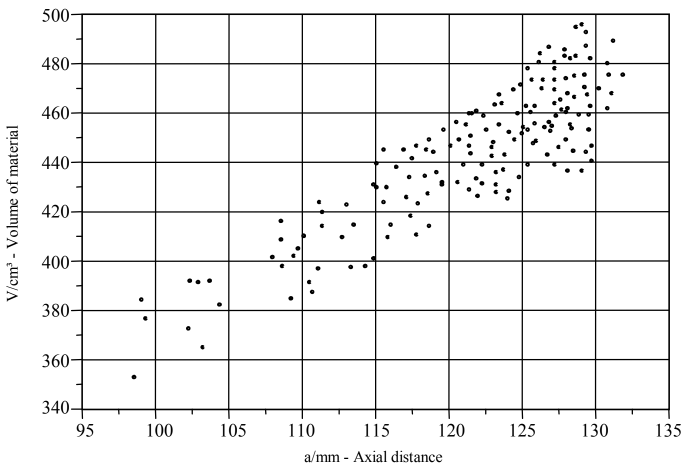

Fig. 5. The criterion space for axial distance – the volume of material used for gears.

Based upon a geometrical interpretation of the results in the criterion space, the following conclusions may be drawn:

- the criteria axial distance–efficiency are mutually conflicting;
- there exists very strong correlation between the criteria axial distance–the volume of material used for gears.

In addition, in Table 3 the Pareto solutions for the given task are shown. In Table 4, the ordered sets of the objective function values for the first stage of gearing.

Table 3. Pareto–optimal solutions

Model	Design variables				
	$x_1=m_n/mm$	$x_2=z_1$	$x_3=x_1$	$x_4=x_2$	$x_5=\varphi$
264	3.5	27	0.086	0.27	0.72
822	3.5	26	0.094	0.29	0.81
1010	3.5	27	0.072	0.25	0.72

Table 4. Objective vector function

Model	Objective functions					
	$f_1=a/mm$	$f_2=\eta$	$f_3=\varepsilon_\alpha$	$f_4=\alpha_w$	$f_5=S_H$	$f_5=V/mm^3$
264	112.88	0.984	1.69	0.363	1.54	4.5 E6
822	119.4	0.983	1.68	0.364	1.54	4.07 E6
1010	109.82	0.983	1.69	0.362	1.53	3.85 E6

Finally in order to materialize practically the design of a gear train transmission, it is necessary to adopt "the best" solution from the suggested set of Pareto solutions. By analyzing the models obtained, it has been decided to adopt the model under number 1010 as "the best" design of the gear train transmission.

6. Conclusion

The paper represents a brief illustration of a wider study undertaken with the aim of building the powerful multicriteria optimization methods into the expert system for gear train transmission design. It points out the necessity of decomposition multistage gear train transmission as complex mechanical systems. In the way, the gear train optimization procedure is also carried out through the corresponding number of stages. In the first optimization stage, the domains of the practical application of gear train transmissions are defined, whereas, during the second stage, the multicriteria optimization problem is solved.

All relevant parameters that effect the sensitivity of the constraints are identified. The procedure developed in this paper is advantageous because the results are practical and no further analysis of the gear train is required. This means that, as early as in conceptual design of gear trains, by utilization of the corresponding optimization model, the best solution of gear train design with a higher reliability and efficiency can be reached.

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VIŠEKRITERIJUMSKA OPTIMIZACIJA VIŠESTEPENIH ZUPČASTIH PRENOSNIKA

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U radu je izložen analitički i numerički postupak višekriterijumske optimizacije višestepenih zupčastih prenosnika. Primenom metoda optimizacije u oblasti zupčastih prenosnika mogu se definisati optimalne parametre za zupčasti prenosnik kao celinu i za svaki stepen prenosa posebno. Za postavljeni višekriterijumski optimizacioni model razvijen je kompjuterski program sa interaktivnim pristupom baziran na generisanju slučajnih brojeva metodom Monte Karlo. Rezultati programa prikazani su u obliku Pareto–optimalnih rešenja, kao i u obliku kriterijumskih prostora.