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THE APPLICATION OF SOME APPROXIMATE SOLUTIONS OF STRESS AND STRAIN CONCENTRATION FOR LIFE ESTIMATION IN THE LOW CYCLE FATIGUE REGION

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Abstract. *The problem of solution of stress and strain distributions around the notches is too complex especially if the local stress and strain are over the elastic limit such as in the low cycle fatigue region exist. In the paper are presented some approximate solutions of the notch effect on the stress and strain concentration. In that way, some practical approximations by the estimation of the stress and strain concentration factors are made which have shown a satisfactory experimental verification. These solutions make possible to determine the fatigue life for a notched parts on the basis of corresponding fatigue life data for unnotched part or smooth specimen.*

Key words: *low cycle fatigue, fatigue life, stress-strain concetration, cyclic stress-strain-curve.*

1. INTRODUCTION

Influence of the notch on the fatigue characteristics is very significant because of the stress and strain concentration which reduces the fatigue strength and fatigue life. This appearance is mainly investigated in the elastic region where the number of cycles to the fatigue crack initiation or to the finally fracture is high one. It is the region of the High Cycle Fatigue, abbreviated — HCF. Therefore, the problem of notch effect is more complex in the cases of elasto-plastic deformation at the notch root where the number of cycle to the crack or to the finally fracture is relatively low, so called Low Cycle Fatigue, abbreviated — LCF. Here it will be considered only this case. Experimental investigations in LCF region have indicated that the fatigue tests were better related to the constant total strain amplitude than to the constant elastic or the plastic one, [1,2]. For the life estimation purpose, a relation between the cyclic stress-strain behavior of smooth and of notched specimens or machine parts subjected to the cyclical elasto-plastic strain history is necessary. This relation is given through the factors of stress and strain concentration.

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2. APPROXIMATE ESTIMATION OF STRESS AND STRAIN CONCENTRATION FACTORS IN ELASTO-PLASTIC REGION

During the fatigue process the stress-strain hysteresis loops corresponding to each cycles have changeable extreme magnitude both of stress and of plastic strain, [9]. For the fatigue life to initial crack it may be assumed only one so-called stabilized hysteresis loop as a represent one. Analogous to the static stress-strain curve corresponds a cyclic stress-strain curve (Fig. 1a). It represents the loci of the tips of the stabilized hysteresis loops from fatigue processes under various total strain amplitude, [1,6].

As known, the theoretical or geometric factor α_k of stress concentration in fully elastic region is defined as

$$\alpha_k = \sigma_{\max} / \sigma_{nom} \ . \tag{1}$$

where σ_{max} and σ_{nom} are the maximum and nominal stress, respectively, [3].

In the elasto-plastic region doesn't exist the proportionality between the stress and total strain and because of that, there is a qualitative difference in the stress and strain concentration. Such stress-strain state around the notch root is mathematically difficult to be described and to be resolved in a general form, [4].

In the elasto-plastic region the elasto-plastic stress concentration factor is defined:

$$\alpha_{k\sigma} = \sigma_{\max} / \sigma_{nom} \tag{1'}$$

and the elasto-plastic strain concentration factor

$$\alpha_{k\varepsilon} = \varepsilon_{\max} / \varepsilon_{nom} \tag{2}$$

where ε_{max}, ε_{nom} are the maximum and the nominal strain at notch root, respectively.

Because of the mentioned difficulties, some approximate proceedings are suggested. A simplest one is Stowel´s formula for elasto-plastic stress concentration factor for an infinite sheet with a circular hole, subjected to an uniform tensile stress σ_{nom} at a remote distance from the hole:

$$\alpha_{k\sigma} = 1 + 2\,E_s / E \tag{3}$$

where E is modulus of elasticity at points remote from the hole where the elastic strains with nominal stress σ_{nom} dominate, E_s represents the secant modulus of elasticity corresponding to the maximum concentrated stress σ_{max} at the points of hole in the smallest net cross-section, where both elastic and plastic strains exist (Fig. 1a), [1,2,4]. The influence of local plasticity is taken into account by the ratio E_s / E.

For fully elastic stress-strain state the maximum stress is less than elastic or proportional limit, then $E_s = E$ and $\alpha_k = 3$ what is equal to the value according to consequent theoretical solution, [3]. This shows that such concept is possible to apply on stress and strain concentration also in the plastic region.

Hardrath and Ohman have been suggested more general formula for plastic stress concentration factor for other types of geometrical discontinuities which are more often in practical use. For the cases where the nominal stress is determined on the net cross-section σ_{nom} and if its value is greater than the proportional or elastic limit (Fig. 1a), this formula is

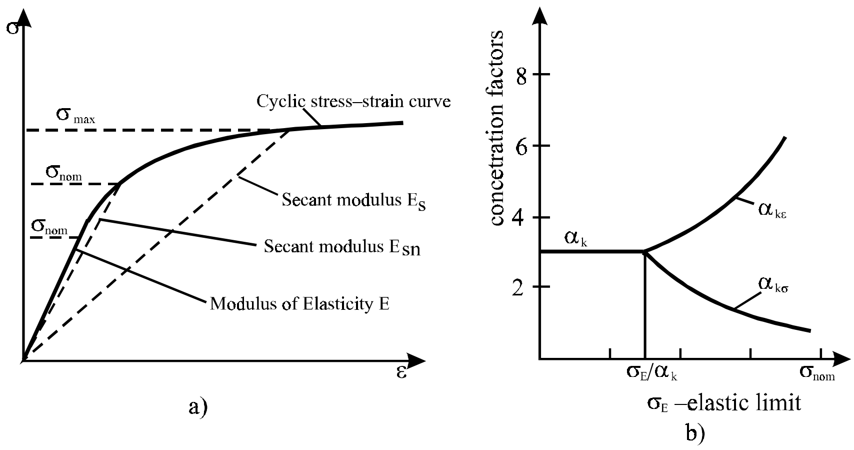

Fig. 1: a) Cyclic stress-strain curve and modules of elasticity;
b) Concentration factors versus nominal stress.

$$\alpha_{k\sigma} = 1 + (\alpha_k - 1) \cdot E_s / E_{sn}, \tag{4}$$

where E_{sn} represents the secant modulus of elasticity corresponding to the nominal stress, [1,2,4]. The maximum strain in the notch root is then

$$\varepsilon_{\max} = \sigma_{\max} / E_s \tag{5}$$

and the nominal strain corresponding to the nominal stress

$$\varepsilon_{nom} = \sigma_{nom} / E_{sn} \tag{6}$$

From Eq. (5) and (6), according to Eq. (2), it follows

$$\alpha_{k\varepsilon} = \alpha_{k\sigma} \cdot (E_{sn} / E_s) \geq \alpha_{k\sigma} \tag{7}$$

because the ratio (E_{sn} / E_s) ≥ 1.

The estimation of the elasto-plastic stress concentration factor requires the values for corresponding secant modules of elasticity E_s and E_{sn} what is only possible by successive approximations. These secant modules are to be based on the static stress-strain curve for parts subjected under static loading but based on cyclic stress-strain curve if the cyclic loading acts (Fig. 1a). Further works showed that a good agreement between the concentration factor values obtained both by analytical way, using these approximate formulas, and by experimental way exists only for the strain value of about 1-2% what is usual case in engineering constructions, [2]. Fig. 1b shows the concentration factor versus the nominal stress. It is seen that in elastic region exists only one, t.i. the theoretical factor but in the elasto-plastic region there are two separate concentration factors, one for stress and other for strain, [1,4].

H. Neuber has also investigated the behavior of stress and strain concentration in the elasto-plastic region for two-dimensional shear-strained prismatical bodies with the sharply curved notch, [5]. By using an nonlinear stress-strain law for the range of greater deformations and by neglecting the notch angle influence, he has obtained a relation between the real stress at notch root and the nominal stress. This leads to a relation between the stress and strain concentrations for fully ideal elastic material and for any real elasto-plastic material the stress-strain curve of which shows a stable monotonic growth (Fig. 1a, Fig. 2), in the form:

$$\sigma_{\max,id} \cdot \varepsilon_{\max,id} = \sigma_{\max} \cdot \varepsilon_{\max} \tag{8}$$

or

$$\alpha_k^2 \cdot \frac{\sigma_{\max}^2}{E} = \sigma_{max}\varepsilon_{max} = const\ . \tag{9}$$

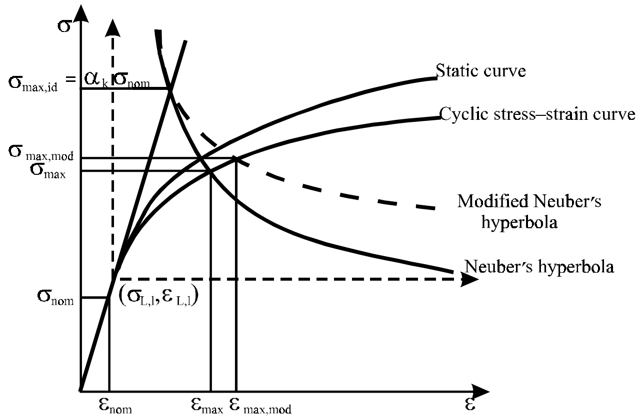

Fig. 2. The estimation of stress and strain concentration at notch root using Neuber´s hyperbola and cyclic stress-strain curve.

In the stress-strain coordinate system it represents a relation called after inventor as Neuber´s hyperbola (Fig. 2, full line). By separate introducing in this equation the stress and the strain concentration factors according to Eq. (1) and (2), it follows:

$$\alpha_k = \sqrt{\alpha_{k\sigma} \cdot \alpha_{k\varepsilon}} \tag{10}$$

t.i. the theoretical concentration factor is the geometric mean of the concentration factors of stress and of strain

This relation could be generalized for the cases of two or three-dimensional stress state. Because of constant value of α_k for a given geometric discontinuity it follows that the product ($\alpha_{k\sigma} \cdot \alpha_{k\varepsilon}$) should be permanently constant. However, according to other consideration, that product is only approximately constant because the decreasing of $\alpha_{k\sigma}$ is not equal to reciprocal value of increasing of $\alpha_{k\varepsilon}$. In the cases when the nominal values increase over the elastic or proportional limit, then $\alpha_{k\varepsilon}$ by increasing reaches a maximum and after that begins to fall, [4].

A further investigation is related on applying of Neuber's hyperbola not only for sharp notches, but for mild ones, as well. According to Neuber the asymptotes of hyperbola are the coordinate axes, what means that the lower limit for stress decreasing at root of sharp notch is equal to zero. For mild notches the lower limit for stress decreasing is not equal to zero but a characteristic amount $\sigma_{L,l}$ with corresponding strain in elastic region, $\varepsilon_{L,l} = \sigma_{L,l}/E$. Mertens and Dittmann have suggested a modification of Neuber's hyperbola which asymptotes passing through point ($\sigma_{L,l}$,$\varepsilon_{L,l}$) are parallel to coordinate axes, [7]. Then the equation of modified hyperbola is

$$(\sigma_{\max,\,id} - \sigma_{L,l}) \cdot (\varepsilon_{\max,\,id} - \varepsilon_{L,l}) = (\sigma_{\max,\,\mathrm{mod}} - \sigma_{L,l}) \cdot (\varepsilon_{\max,\,\mathrm{mod}} - \varepsilon_{L,l}) \quad (11)$$

The modified hyperbola is showed in Fig. 2, dashed line, and it intersects the cyclic stress-strain curve at point with some greater stress and strain values at notch root than the Neuber's hyperbola, full line.

Based on the known, both cyclic stress-strain curve, and theoretical stress concentration factor, the corresponding representative amplitudes of total strain ε_a and of stress σ_a at the notch root are obtained as intersection point of cyclic stress-strain curve with Neuber's hyperbola according to Eq. (8), that passes through the point with stress $\sigma_{\max,\,id} = \alpha_k \cdot \sigma_{nom}$ laying on the ideal elastic Hooke's straight line (Fig. 2).

3. THE CONDITION OF EQUIVALENCE OF FATIGUE PROCESSES IN THE SMOOTH AND IN THE NOTCHED SPECIMEN OR PART

Here will be studied only stress and strain concentration effect and other effects such as the absolute size of part and surface finish will not be considered. The absolute size of part is more significant for estimation the number of cycles for crack growth what is studied in Fracture mechanics.

In order to produce a fatigue process in a smooth specimen the same, as at notch root, it is possible only if both stress and strain histories are also the same during the fatigue processes. Since only the total strain at notch root is possible to be directly registered, it is necessary the measured output of total strain history at notch root to give as input strain history for smooth specimen (Fig. 3). Such an experimental method is firstly suggested by Crews and Hardrath, [2], with the help of which is possible to realize a simultaneous companion fatigue process.

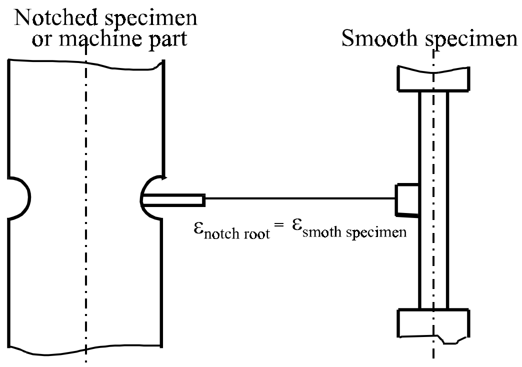

Fig. 3. Simultaneous companion fatigue tests of notched specimen or machine part and of smooth specimen

Under the same strain history, the corresponding stress histories both at the points in notch root and at all points in cross-section of smooth specimen would be also the same because the material is the same. The condition of equality of these fatigue processes may be also described in the stabilized stress-strain state when the stabilized stress amplitudes both in the smooth specimen and at the notch root are mutually equal. Under these conditions, it was firstly supposed that fatigue fracture lives are also mutually the same. However, the experimental results have not fully confirmed this last assumption. Namely, the all fatigue results have been earlier expressed in the number of cycles to total fracture. With that a satisfactory agreement was achieved at higher nominal stresses but at the lower one, the discrepancy was greater and greater, [2]. Afterwards, the further investigations are realised by separation of the life in two phases: the first one is the pure fatigue until the fatigue crack initiation, and the second one is a mixture consisting of fatigue in the instantaneous actual cross section with the simultaneous growth of fatigue crack to the total fracture. Taking into consideration only the first phase, with corresponding number of cycles to fatigue crack initiation, a better agreement in fatigue life for notched and smooth specimen was obtained, [1, 8]. The small discrepancy may be also caused by different stress-strain state in the smooth specimen and at notch root of part. For the case where the stress state at notch root is uniaxial, t.i. the maximum normal stress is at the same time the principal normal stress (for example the edge notched sheet specimen, [2, 3, 9]) this discrepancy is smaller.

4. TOTAL STRAIN AMPLITUDE VERSUS NUMBER OF CYCLES TO FATIGUE CRACK INITIATION

Above mentioned base condition of equivalence of the fatigue processes can be practically used for the purpose of fatigue life estimation of notched specimen or part based on the experimental fatigue life data for smooth specimen. The experimental procedure for smooth specimen is considerable standardized and a great number of available results exists. These results are obtained in a family of fatigue tests under the constant total strain amplitude versus numbers of cycles to crack initiation. By separation the total strain amplitude in its elastic and plastic amplitude, two functions are obtained, (Fig. 4):

$$\varepsilon_{el,a} = \frac{\Delta\varepsilon_{el}}{2} = B \cdot N_A^b \tag{12}$$

for elastic strain amplitude, (Fig. 4, line 1), and

$$\varepsilon_{pl,a} = \frac{\Delta\varepsilon_{pl}}{2} = C \cdot N_A^c \tag{13}$$

for plastic strain amplitude, (Fig. 4, line 2). Here B and C as well as the exponents b and c are the material constants, the values of which are to be estimated based on corresponding fatigue data, [8,10,11,12]. Some investigators have been suggested the constants B and C to estimate assuming that the static tensile test should be considered as a fatigue test by setting $N_A = 1/4;\ 1/2;\ 1$, [1], but the obtained values differ from true values obtained from fatigue test data, [8,12]. Here it is interesting to use only the fatigue life expressed through the number of cycles to crack initiation because it represents a pure boundary between two phases of fatigue process. Because of true values of the constants, an appropriate definition of the initial fatigue crack during the cyclic straining is necessary, what is given in ref. 10,12. By summing of Eq. (12) and (13), the criterion for fatigue crack as function of total strain amplitude versus fatigue life is obtained, (Fig.4, line 3),

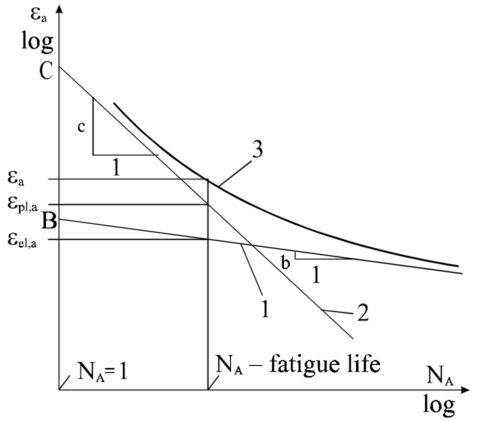

Fig. 4. Elastic, plastic and total strain amplitude versus number of cycles to fatigue crack

$$\varepsilon_a = \varepsilon_{el,a} + \varepsilon_{pl,a} = B \cdot N_A^b + C \cdot N_A^c \quad (14)$$

from which it is possible to obtain the fatigue life N_A by setting the maximum strain value obtained after Eq. (2),(5),(8) or (11) in place of the total strain amplitude, (also shown in Fig. 4).

The same fatigue life is possible to estimate explicitly according to Eq. (12) or (13) if the total strain amplitude is previously separated in elastic and plastic strain amplitude by using the cyclic stress-strain curve and one of approximate methods for estimation of stress and strain concentration factors after Stowel, Hardrath-Ohman or Neuber (Fig. 4). For the practical purposes the Eq. (13) is more often used for such explicit estimation of the fatigue life because the values of constants C and c can be estimated more accurate than of constants B and b, [10,12].

5. Conclusion

In spite of the approximate estimation of stress and strain concentration factors, the experimental results of fatigue life have shown a good agreement with those obtained by application of the methods described. Such confirmation is the best evidence why these approximate solutions of stress and strain concentration have found a wide application for the constructions which work in the low cycle fatigue region.

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PRIMENA NEKIH PRIBLIŽNIH REŠENJA KONCENTRACIJE NAPONA I DEFORMACIJA ZA ODREĐIVANJE VEKA U OBLASTI MALOCIKLIČNOG ZAMORA

Miodrag Janković

Prolem rešavamja raspodele napona i deformacija oko zareza je suviše složen naročito ako su lokalni naponi i deformacije iznad granice elastičnosti kao što je u oblasti malo-cikličnog zamora. U radu su prikazana neka približna rešenja o uticaju zareza na koncentraciju napona i deformacija. Na taj način učinjena su neka praktična uprošćenja pri određivanju faktora koncentracije napona i deformacije koja su pokazala zadovoljavajuću eksperimentalnu proveru. Ova rešenja omogućavaju određivanje zamornog veka za delove sa zarezom na osnovu odgovarajućeg zamornog veka nezarezanih delova ili glatkih epruveta.

Ključne reči: *malociklični zamor, zamorni vek, koncentracija napona i deformacija, ciklična kriva napon-deformacija.*