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NON-LINEAR ELASTIC BONDS IN THE MODELS OF CRANE MECHANISMS

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Saša Marković, Zoran Marinković

Mechanical Engineering Faculty University of Niš
Str. Beogradska 14, 18000 Niš, Yugoslavia

Abstract. *The modelling of crane driving mechanisms i.e. non-linear model characteristics is shown in the paper. Crane mechanisms can be described as elasto-kinetic models with a final number of concentrated masses and elastic bonds between them. Gap as the main reason of non-linearity is being linearized and described as partial-linear characteristic. The other model parameters, the elastic bond rigidity, damping, reduced mass models inertia moments, the shape of external disturbance moment are non-linear but it is replaced with a greater number of linear functions that provide analytical solving of the set differential movement equations. The results received this way are very similar to the experimental recordings, which justifies the linearization of the mentioned functions.*

1. THE INTRODUCTION

Cranes contain different driving mechanisms that perform certain operations and movements with an aim of replacing the load in space. These mechanisms can be sorted out into four groups: for load lifting, for moving, for circular rotating and for the reach change. To be able to do analysis we bring down mechanisms to certain dynamic models. Nowadays, elasto-kinetic models with a final freedom degree movements and the elastic bonds between masses are being used. The adequate results are received from two (Fig. 1) or three reduced mass models with gap influence [1,3]. The external loads influence model masses.

The moment of electro-motor starting in acceleration period or working brake moment at the period of braking influence the first mass. Stationary movement resistance or safety brake conceivably, load the driven mass. In the transient working regimes, i.e. acceleration and braking periods, masses perform forced damped oscillatory movement. Mathematic model is so the system of non-homogenous second order equations.

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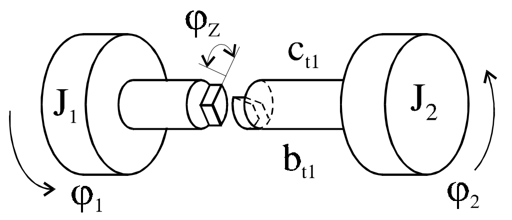

Fig. 1 Model of the crane mechanism with gap

The model characteristics that are non-linear are: the gap reduced to the wanted shaft, reduced rigidity, damping and mass model inertia moments. The greatest influence on non-linearity is done by the gap.

2. THE GAP IN CRANE MECHANISMS

The circular gap is the functional follower of the cogwheel. The gap in the cogwheel transmitters makes the major part of the mechanism gap. Multi-particle cylindrical cogwheel transmitter with straight cogs can be shown in (Fig. 2). We can assume that dynamic processes at the transmitters aren't followed by the irregular harnessing and energy dissipation i.e. a transmitter with a simple bonds is analyzed. The relation of the shaft and each cogwheel bearing is maked with an elastic spring with two main rigidity directions [2].

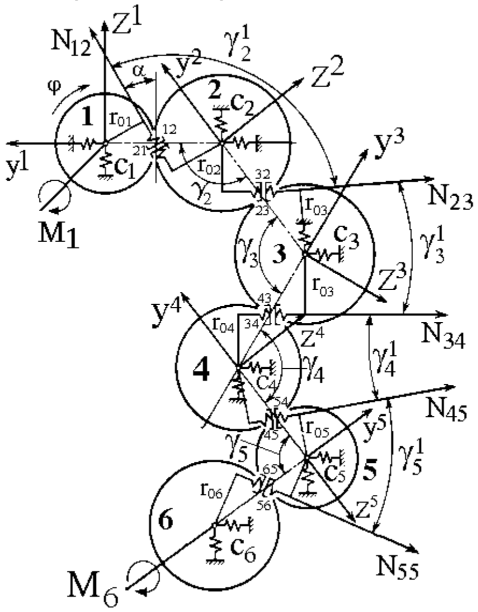

Fig. 2. The pattern of the multi-cylindrical transmitter with straight cogs

Each cogwheel can be presented as a dynamic subsystem with three degree freedom, described with coordinates y_m , z_m , ψ_m ;

where:

y_m , z_m - the inertia center position m cogwheel,

φ_m - the rotating angle of the m cogwheel,

$$\psi_m = r_{0m} \cdot \varphi_m ;$$

For the dynamic system of the cogged transmitter (Fig. 2) with elastic or simple degree bonds, that are separated into sub-systems, we define quasi-elastic bond characteristic $F_{k,k+1}(\delta_k)$, where δ_k represents one type function of the independent generalized cogwheel coordinates (subsystems) in a hitch. Subsystems are characterized by vectors of generalize coordinates of U_i order n_i. If we take multi-degree cogged transmitter for the elastic dynamic system, then each cog represents a subsystem with three degree freedom. Subsequently, each cog can be presented as a dynamic graph with three independent oscillators and the cogged pair as a graph made of two cogs. Therefore, bond equation is:

$$f_k = (y_i , z_i , \psi_i , \delta_k) = 0 \tag{1}$$

On the basis of previous the general form of differential movement equation of multi (n) cylindrical transmitter can be written in a form:

$$\Theta_i \cdot \ddot{U}_i + G_i \cdot U_i + \sum_{j=n} \tau_{ij} \cdot U_i = 0 \quad (2)$$

where: $U_i = (y_i, z_i, \psi_i)^T$ - is generalized cogwheel coordinates matrix,

$\Theta_i = \text{diag}[m_i, m_i, J_i \cdot r_{0i}^{-2}]$ - is cogwheel inertia characteristics matrix,

$G_i = \text{diag}[c_i, c_i, 0]$ - is cogwheel rigidity matrix.

To solve this equation system it is needed to do its linearization. Elastic bonds characteristics can be, with great accuracy, presented in a linear form which suits the quasi-elastic with a gap: $F_{k,k+1}(\delta_k) = \xi_k \cdot \delta_k$. Then:

$$\tau_{ij} = \sum_{k=1}^{n-1} \xi_k \cdot T_{ij}^{(k)} \;;\; T_{ij}^{(k)} = [T_{ji}^{(k)}]^T = \left(\frac{df_k}{dU_i}\right)^T \frac{df_k}{dU_j} \;;\; (i, j = 1, 2, \ldots n;\ k = 1, 2, \ldots m) \quad (3)$$

Therefore, in equation (2) matrix $G_i + \xi \cdot T_{ii}$ represent characteristics of the local elasticity of some subsystems within the general model (system), and matrix T_{ij} characterizes the common structure of the mutual subsystem elastic bonds. It is known that the dissipation of energy appears at work, which is particulary presented at multi-degree transmitters. As the energy dissipation in the cogged transmitters characterized by small damping coefficient values, the damping character of the separate cogs can also be presented in linear resistance form.

Analogous to the conservative model (4), taking into consideration damping, differential equations of the dissipative model movement multi (n) cylindrical transmitters with straight cogs can be presented in the form of matrix-vector equations:

$$\Theta_i \ddot{U}_i + B_i \dot{U}_i + G_i U_i + (\xi_{i-1} T_{ii}^{(i-1)} + \xi_i T_{ii}^{(i)}) U_i + \xi_{i-1} T_{i,i-1}^{(i-1)} U_{i-1} + \xi_i T_{i,i+1}^{(i)} U_{i+1} = 0 \quad (4)$$

where: $B_i = \text{diag}[b_i, b_i, 0]$ - cog damping matrix.

Because of the non-linear cog transmitters characteristics enforced by the gap between two cogs, we use the hitch pattern in the form of quasi-elastic bond with partial characteristics $F_{k,k+1}(\delta_k)$ of elastic bond with the gap type (Fig. 3) for the calculation. Therefore, 1 and 3 graphs of the elastic hitch characteristic (k,k+1) represent rigidity coefficient ξ_k, and part 2 characterized with zero coefficient value ξ_k. The value $2\gamma_{k0}$ of the characteristic $F_{k,k+1}(\delta_k)$ represent summary gap in hitch k, k+1.

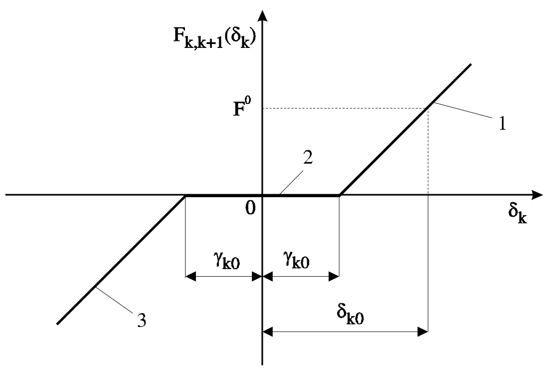

Fig. 3 Partial-linear characteristics of the cog gap

The dynamic manifestation of different mechanic systems is studied in differential coordinates. We can assume that the working point ($k,k+1$) hitch has coordinates δ_{k0} , $F_{k,\ k+1}^{0}$. At the dynamic deformations ($k,k+1$) hitch the position $P(i,j)$ of the shown points of the characteristics in the moment of leap (transition from branch i to the j branch characteristics) have the following values:

$$P(1,2) = P(2,1) = (\delta_k^{(1)}, -F_{k,k+1}^{0})$$
$$P(2,3) = P(3,2) = (\delta_k^{(2)}, -F_{k,k+1}^{0}) \quad (5)$$
$$\delta_k^{(1)} = \gamma_{k0} - \delta_{k0}; \delta_k^{(2)} = -\gamma_{k0} - \delta_{k0}$$

Quasi-elastic bond has a partial-linear characteristics $F_{k,k+1}(\delta_k)$ that represent elastic bond characteristics with a gap [2, 4] and the previous loads (Fig. 3):

$$F_{k,k+1}(\delta_k) = \begin{cases} \xi_k \delta_k \ \text{za} \delta_k > \delta_k^{(1)} \\ \xi_k(\gamma_{k0} - \delta_{k0}) \text{za} \delta_k^{(1)} \geq \delta_k \geq \delta_k^{(2)} \\ 2\xi_k \gamma_{k0} + \xi_k \delta_k \ \text{za} \delta_k < \delta_k^{(2)} \end{cases} \quad (6)$$

where: $\delta_k^{(1)} = \gamma_{k0} - \delta_{k0}$; $\delta_k^{(2)} = -\gamma_{k0} - \delta_{k0}$;

δ_k - deformation k quasi-elastic bond in relation to its deformed state δ_{k0};

$2\gamma_{k0}$ - summary gap value.

Taking into consideration movement direction of the presented point, the leap moments (5) we can focus controlling the values of changeable (differential) coordinates $\delta_k^* = \delta_k - \delta_{k0}$, and its extract in time $\dot{\delta}_k^* = \dot{\delta}_k$. In that case marking the expression $P^*(i,j)$ as interparameter relation δ_k^* and $\dot{\delta}_k^*$, certain leaps an characteristics $F_{k,k+1}(\delta_k)$ have the following interrelation:

$$P^*(1,2) = (\delta_k^{(1)}, \dot{\delta}_k < 0);\ P^*(2,3) = (\delta_k^{(2)}, \dot{\delta}_k < 0);$$
$$P^*(3,2) = (\delta_k^{(2)}, \dot{\delta}_k > 0);\ P^*(2,1) = (\delta_k^{(1)}, \dot{\delta}_k > 0);$$

At the non-linear oscillations analysis, extract control $\dot{\delta}_k^*$ for movement direction determining at the leap moment at the characteristic $F_{k,k+1}$ (δ_k) is rare. It appears if δ_k^* coordinate values in the beginning of the current calculation step, coincide (with given accuracy) with one of its leap values, in accordance with (5). In all other calculation simulations the moment and the leap direction on $F_{k,k+1}(\delta_k)$ characteristic are determined with comparative values δ_k^* in the beginning of the current calculation step and its leap values in accordance with (5), that determines the current step of the working characteristic calculation $F_{k,k+1}$ (δ_k).

Non-linear dynamic model of the multi-cog transmitter in a complex form (4) model can be described (i = 1, 2, ... n):

$$\Theta_i \cdot \ddot{U}_i + B_i \cdot \dot{U}_i + G_i \cdot U_i + v_{i-1,i} \cdot \xi_{i-1}(T_{i,i-1}^{(i-1)} \cdot U_{i-1} + T_{ii}^{(i-1)} \cdot U_i) +$$
$$+ v_{i1} \cdot \xi_i (T_{ii}^{(i)} \cdot U_i + \xi_i \cdot T_{i,i+1}^{(i)} \cdot U_{i+1}) = \sum_{j=i-1}^{i} [(1-v_{j1})H_{j1}^{(i)} + (1-v_{j1}-v_{j2})H_{j2}^{(i)}] \tag{7}$$

where: $H_{j1}^{(i)} = -\left(\frac{df_j}{dU_i}\right)^T \xi_j \cdot \delta_j^{(1)}; H_{j2}^{(i)} = -2 \cdot \left(\frac{df_j}{dU_i}\right)^T \xi_j \cdot \gamma_{j0}$ - are disturbed functions.

Full number indices v_{i1}, v_{i2}, (i=1, 2, … , n–1), that organize the current structure of the partial-linear model (7), are determined, depending on branches 1, 2 or 3 characteristic $F_{k,k+1}(\delta_k)$ respectively:

$$v_{j1} = 1, v_{j2} = 0(\delta_i^* > \delta_i^{(1)}); v_{j1} = 0, v_{j2} = 1(\delta_i^{(1)} \geq \delta_i^* \geq \delta_i^{(2)}); v_{j1} = 1, v_{j2} = -1; (\delta_i^* < \delta_i^{(2)}) \tag{8}$$

In the separate model on the concentrated masses, that are in respect to U_i , U_{i+1} constant disturbances have influence $H_{i1}^{(j)}, H_{i2}^{(j)}$ (j = i, i+1), that are determined by right parts of i and i+1 differential system equation (7), when index v_{i1}, v_{i2} values correspond to 2 curve characteristic $F_{k,k+1}$ (δ_k). That model structure is kept until the following leap moment of $F_{k,k+1}$ (δ_k) characteristic of one transmitter cogging, when a new model change corresponding to the current structure indices value v_{i1}, v_{i2} comes.

The importantnon-linear model characteristics of the multi cog transmitter for the numeric analysis are that boundary leaps in the number of characteristics $F_{k,k+1}(\delta_k)$, (i = 1, 2, n-1), are described in relation to independent subsystems of linear second order equations. Generally, at the numeric analysis of non-linear multi cylindric cogged transmitters the sequence of leaps on the characteristic $F_{k,k+1}(\delta_k)$ and the current quasi-linear model structure (7) of the transmitter can be determined on the basis of (8) rule of structure indices.

Linearized dynamic model of the multi-cylindrical cogged transmitters with slope coggs and simple bonds can be built in analogy with the mentioned approach for the cylindrical cogged transmitters with straight cogs.

3. Elastic Bonds Model Characteristics

The majority of reduced inertia characteristics at most driving mechanisms has a constant value (inertia coefficient m_i and J_i). However, at mechanisms for reach change that is realized by changing the elevation arrow angle at the rotating cranes, the positional inertia arrow moment and the load is changed with distance square according to the crane axe ($m_i \cdot r^2$). Therefore, the dynamic analysis of these mechanisms gets complicated. The reduction of inertia characteristics, for translatory and rotation masses of the mechanisms is done according to the expression:

$$J_R = \sum_i \left[m_i \left(\frac{\dot{x}_i}{\dot{\varphi}_R} \right)^2 + J_i \left(\frac{\dot{\varphi}_i}{\dot{\varphi}_R} \right)^2 \right] = \sum_i \left[\frac{m_i}{i_i^2} R^2 + \frac{J_i}{i_i^2} \right] \tag{9}$$

The elastic bonds between the reduced masses are also characterized with rigidity coefficient and damping. Their form is non-linear and depends on the very mechanism type.

The rigidity represents dependance between the load (force or moment) and the provoked deformity (linear or angle). The rigidity characteristic can have linear or most often non-linear stream. We can also differentiate progressive, degressive and combined dependance (Fig. 4). In cases of load decrease need it is done by the help of degressive element characteristic (for example elastic links with rubber elements). The progressive characteristic can be found at load lifting mechanisms because of the steel rope, the rigidity of which depends on its lenght and is changed at load lifting and landing. For the practical calculations non-linear curves are changed with broken straight lines. The element rigidity of crane mechanisms can be determined by experimental or analytical method. For practical analysis rigidity coefficient is most often made constant. At the dynamic modelling a reduction of rigidity coefficient values should be done at a certain spot in the equivalent model. Depending on the complexity of the real system and its bonds we can differentiate numerical, parallel and combined elastic bond hitch.

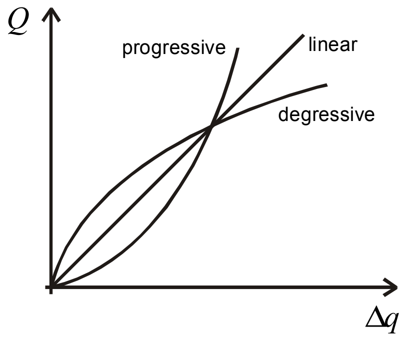

Fig. 4. The load change in deformation function

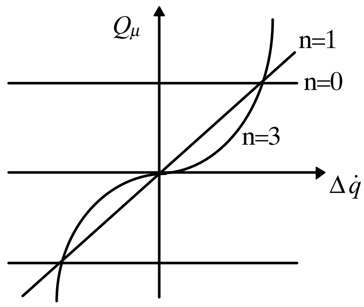

Fig. 5. The resistance change of the viscosity friction

Damping depends on the type and material characteristic, character and load intensity, deformity, speed, amplitude, frequency, temperature etc. This damping in mechanism elements causes dissipative resistance i.e. influences the shortening of the lasting time and the intensity of the oscillatory movement and load. The resistance can be the dry friction resistance in proportion to the normal pressure and viscosity friction resistance in proportion to the speed degree.

The dry friction resistance force or Coulomb slide friction force is:

$$F_{\mu} = -\mu \cdot F_N \cdot (\text{sign } \dot{q}) \tag{10}$$

where: μ - friction coefficient,

F_N - normal force,

$(\text{sign } \dot{q}) = \begin{cases} +1 & \dot{q} > 0 \\ -1 & \dot{q} < 0 \end{cases}$ - multiplier that has opposite sign from speed direction.

The viscosity friction resistance (Fig. 5) is determined:

$$Q_{\mu} = -b \cdot \Delta\dot{q}^{n} \cdot (\text{sign}\ \dot{q}) \tag{11}$$

where: b - damping coefficient,
$\Delta\dot{q}$ - relative movement speed,
$n = 0,\ 1,\ 3$ respectively for constant, linear and non-linear damping resistance change (Fig. 5).

In real oscillatory systems damping resistance usually act simultaneously and practically, can't be differentiated. As some resistance have very small influence on the damping process, the very influence of one dominant resistance to damping is often considered at modelling. At the dynamic modelling only the influence of the dominant damping resistance is being accepted, so at crane driving mechanism models the influence of viscosity friction (n = 1) is most often considered for the acceleration and braking period where in the slow down period after braking the dry friction resistance is considered.

4. Crane mechanisms work simulations and the conclusion

For the analysis of the dynamic behaviour of crane mechanisms an original computer program for their work simulation is developed [1]. Non-linear elastic bond characteristics are linearized as well as the external disturbances that influence model masses. By Runge-Kutta method application differential equations are solved and certain load dependances are got shown in the elastic bond torsion moment. The received simulations greatly coincide with experimental recordings which confirms the adequacy of the elasto-kinetic model at solvind of these problems and justification of model characteristics linearization.

In (Fig. 6) experimental recordings of safety brake braking at load landing at one redundant mechanism and also an adequate computer simulation. The coincidence of the first several amplitudes of shaft torsion moment is evident, according to its values and also the time of performance. The only difference is with the minimal value of the first amplitude oscillation moment.

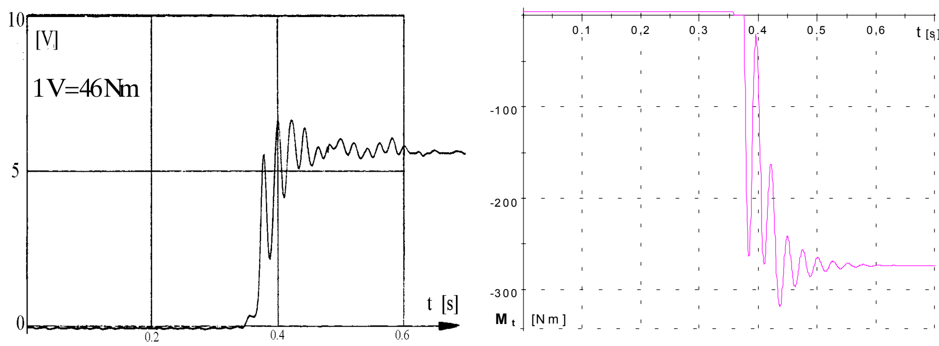

Fig. 6 Shaft mechanism torsion moment change: a) experimental recording, b) simulation

The great complexity and non-linearity of elastic bond characteristics of crane mechanism models allows its linearization which accomplishes great simplification of model equations [1,4]. The equations can be then got by direct integration (Laplace transformations) or numerically. The received simulations coincide with the experimental recordings which confirms the adequacy of the used methods.

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NELINEARNE ELASTIČNE VEZE U MODELIMA DIZALIČNIH MEHANIZAMA

Saša Marković, Zoran Marinković

U radu je prikazano modeliranje dizaličnih mehanizama tj. predstavljanje nelinearnih karakteristika modela. Dizalične mehanizme svodimo na elasto-kinetičke modele sa konačnim brojem koncentrisanih masa i elastičnim vezama izmedju njih. Zazor kao glavni uzrok nelinearnosti se linearizuje i svodi na parcijalno-linearnu karakteristiku. I ostali parametri modela, krutost elastične veze, prigušenje, momenti inercije redukovanih masa modela, oblik spoljašnjih poremećajnih funkcija su nelinearnog oblika ali se zamenjuju većim brojem linearnih funkcija koje omogućavaju analitičko rešavanje postavljenih diferencijalnih jednačina kretanja. Tako dobijena rešenja daju veoma bliske rezultate sa eksperimentalnim zapisima čime se opravdava linearizacija navedenih funkcija.