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RECTILINEAR GUIDING OF THE COUPLER POINT REALIZED BY SOME FOUR-BAR LINKAGES WITH AN ELASTIC COUPLER

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Abstract. *The paper deals with two rigid-body four-bar linkages, where the coupler point is capable of being guided on an approximately rectilinear path, and their equivalent counterparts which have an elastic coupler. It considers the rectilinear guiding of the point located at the end of the coupler (the Hoecken four-bar linkage) and in the middle of the coupler (the Watt four-bar linkage). Two models have been applied: the model of an elastic simple beam and the model of an hybrid compliant mechanism. The differences between the realized paths and the exact rectilinear paths of the rigid-body mechanisms, as well as their counterparts with an elastic coupler, have been also presented.*

RIGID-BODY FOUR BAR LINKAGES WITH AN ELASTIC COUPLER FOR RECTILINEAR GUIDING

The coupler point C of the Hoecken and Watt four-bar linkages (Figure 1), being acted by the horizontal force F, can be guided on an approximately rectilinear path. The displacement Δx_C = 1mm and the rotation angle of the input crank $\Delta\varphi = 5^o$ have been adopted for both above-mentioned rigid-body four-bar linkages. Using the method of position analysis of mechanism we can calculate respective link lengths of the above-mentioned four-bar linkages in order to satisfy the demands about displacement Δx_C and rotation angle $\Delta\varphi$ (Table 1). Minimal difference between the realized path and the exact rectilinear path has been denoted with the deviation Δy_C.

Table 1. The dimensions of the rigid-body four-bar linkages for rectilinear guiding

	$a = \overline{A_0A}$ [mm]	$b = \overline{B_0B}$ [mm]	$c = \overline{AB}$ [mm]	$d = \overline{A_0B_0}$ [mm]	$\overline{AC}$ [mm]	$\overline{BC}$ [mm]	φ [°]	Δy_C [μm]
Hoecken	8.594	21.486	21.486	17.188	42.971	21.486	177÷182	0.7
Watt	12.281	12.281	12.772	25.791	6.385	6.385	311÷316	0.4

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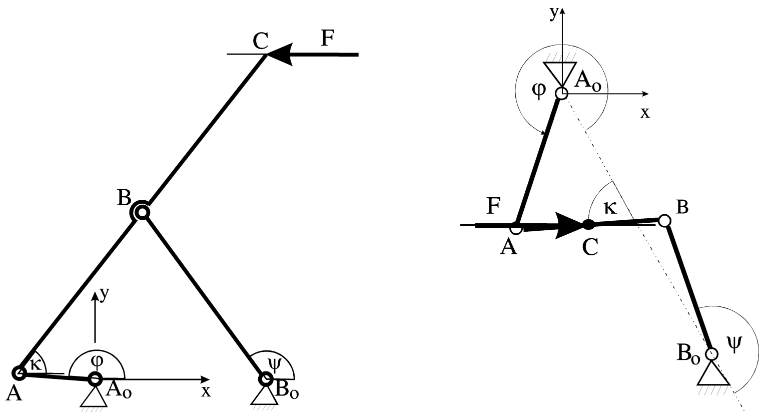


a) b)

Fig. 1. Rigid-body four-bar linkages for rectilinear guiding: a) Hoecken; b) Watt

If the coupler link is elastic, it can be modeled as a beam overhanging one support (Hoecken four-bar linkage) and simple beam (Watt four-bar linkage) . The input crank and the follower are assumed to be rigid.

The position of the coupler point C of the Hoecken four-bar linkage can be calculated using the relations:

$$x_C = a\cos\varphi + 2c\cos\kappa + w\cos\left(k - \frac{\pi}{2}\right) \tag{1a}$$

$$y_C = a\sin\varphi + 2c\cos\kappa + w\sin\left(k - \frac{\pi}{2}\right) \tag{1b}$$

where:

$$w = \frac{2F_w c^3}{3B} \tag{2}$$

$$B = EI_z \tag{3}$$

$$F_w = -F\sin\kappa \tag{4}$$

The position of the coupler point C of the Watt four-bar linkage can be calculated using the relations:

$$x_C = x_C^R\cos\frac{\pi}{3} + y_C^R\sin\frac{\pi}{3} \tag{5a}$$

$$y_C = y_C^R\cos\frac{\pi}{3} - x_C^R\sin\frac{\pi}{3} \tag{5b}$$

where:

$$x_C^R = a\cos\varphi + \frac{c}{2}\cos\kappa + w\cos\left(k - \frac{\pi}{2}\right) \tag{6a}$$

$$y_C^R = a\sin\varphi + \frac{c}{2}\sin\kappa + w\sin\left(k - \frac{\pi}{2}\right) \tag{6b}$$

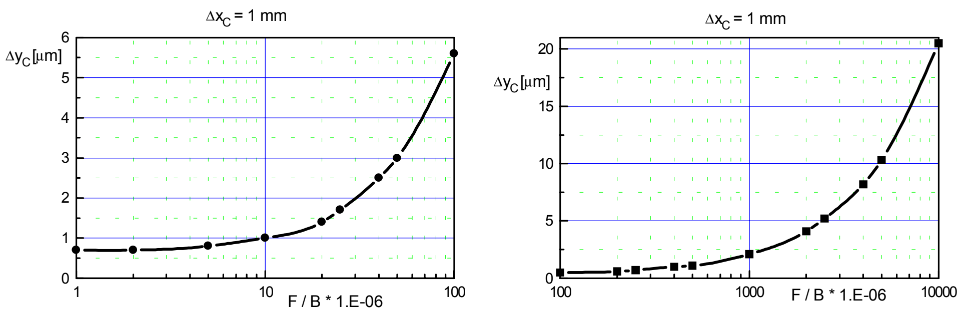
$$w = \frac{F_w c^3}{48B} \tag{7}$$

$$F_w = F\sin\left(k - \frac{\pi}{3}\right) \tag{8}$$

The displacement Δx_C and the deviation Δy_C of the coupler point C of the model with an elastic coupler have been shown in Table 2 and Figure 2. Flexural rigidity EI_z has been denoted by B.

Table 2. The displacement Δx_C and the deviation Δy_C of the coupler point C of the Hoecken and Watt four-bar linkage with an elastic coupler

Hoecken four-bar linkage			Watt four-bar linkage		
F/B [mm^{-2}]	Δx_C [mm]	Δy_C [mm]	F/B [mm^{-2}]	Δx_C [mm]	Δy_C [mm]
$1\cdot10^{-4}$	1.0184	0.056	$1\cdot10^{-2}$	1.0037	0.0205
$5\cdot10^{-5}$	1.0092	0.003	$5\cdot10^{-3}$	1.0018	0.0103
$4\cdot10^{-5}$	1.0074	0.0025	$4\cdot10^{-3}$	1.0015	0.0082
$2.5\cdot10^{-5}$	1.0046	0.0017	$2.5\cdot10^{-3}$	1.0009	0.0052
$2\cdot10^{-5}$	1.0037	0.0014	$2\cdot10^{-3}$	1.0007	0.0041
$1\cdot10^{-5}$	1.0018	0.0010	$1\cdot10^{-3}$	1.0004	0.0021
$5\cdot10^{-6}$	1.0009	0.0008	$5\cdot10^{-4}$	1.0002	0.0011
$4\cdot10^{-6}$	1.0007	0.0008	$4\cdot10^{-4}$	1.0001	0.0010
$2.5\cdot10^{-6}$	1.0005	0.0007	$2.5\cdot10^{-4}$	1.0001	0.0007
$2\cdot10^{-6}$	1.0004	0.0007	$2\cdot10^{-4}$	1.0001	0.0006
$1\cdot10^{-6}$	1.0002	0.0007	$1\cdot10^{-4}$	1.0000	0.0005
0	1.0	0.0007	0	1.0000	0.0004



a) b)

Fig. 2. The deviation of the coupler point C (Δy_C) of the Hoecken (a) and Watt four-bar linkage (b) with an elastic coupler

The horizontal force F has greater influence on the accuracy of rectilinear guiding of the coupler point realized by Hoecken four-bar linkage with an elastic coupler. Therefore is Watt four-bar linkage with an elastic coupler more suitable for rectilinear guiding of the coupler point.

HYBRID COMPLIANT LINKAGES FOR RECTILINEAR GUIDING

Above mentioned model can be applied for greater displacements of the coupler point. For smaller displacements of the coupler point (the displacements in the domain of the micromechanisms) it is more suitable to apply a model of a hybrid compliant mechanism. Compliant mechanisms gain their mobility due to the relative flexibility of their joints. Compliant mechanisms can provide many benefits in the solution of design problems. They are desirable because they may have less wear, weight, noise and clearance than their rigid-body counterparts. By reducing the number of required interconnections the reliability of a design can be improved. Although there are many advantages, the inclusion of compliance provides several challenges in mechanism analysis and design. Nonlinearities introduced by the large deflection of members further complicate the analysis of compliant mechanisms. The field of compliant mechanisms is expected to continue to grow as materials with superior properties are developed. These mechanisms may be made, for example, from modern-day plastics by an injection molding process that gives them the desired resiliency and strength.

Hybrid compliant linkages can be designed as the counterparts of the rigid-body Hoecken and Watt four-bar linkages. Figure 3 shows the model of the hybrid compliant Hoecken and Watt four-bar linkages.

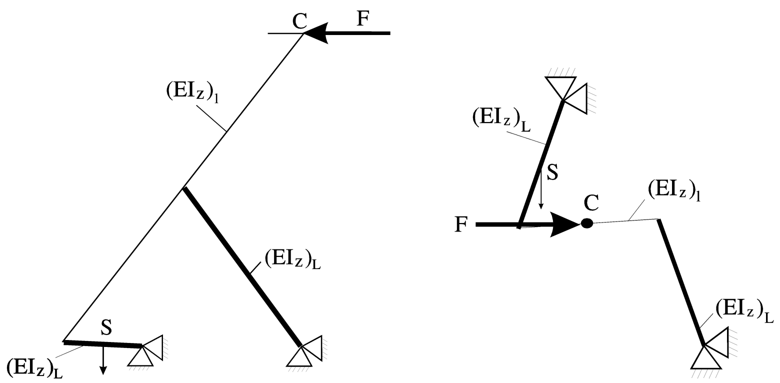


Fig. 3. The hybrid compliant Hoecken four-bar linkage (a) and the hybrid compliant Watt four-bar linkage (b)

Flexural rigidity of the flexible section of the mechanism (the coupler) and flexural rigidity of the relatively rigid section of the mechanism (the input crank and the follower) have been respectively denoted with $(EI_z)_l$ and $(EI_z)_L$, where $(EI_z)_L >> (EI_z)_l$. We introduce a structural displacement at the point S, located in the middle of the input crank, which is equivalent to a rotation of the rigid-body input crank of $\Delta\varphi = 5^o$, in order to get approximately horizontal displacement $\Delta x_C = 1$mm.

The calculation has been performed by Finite Element Method (FEM). Two-dimensional elastic beam (Fig. 4) has been chosen as a characteristic element type for the four-bar linkage with an elastic coupler. The element has three degrees of freedom at each node: translations in the nodal x- and y- directions and rotation about the nodal z-axis.

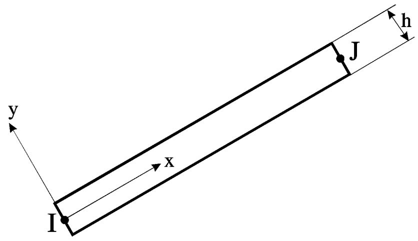


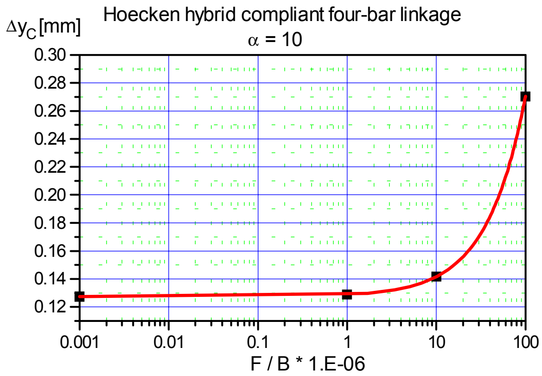
Fig. 4. Two-dimensional Elastic Beam

The deviation of the coupler point C (Δy_C) for horizontal displacement Δx_C=1mm of the hybrid compliant model have been shown in Table 3 and Figure 5. The rigidity ratio has been denoted by α:

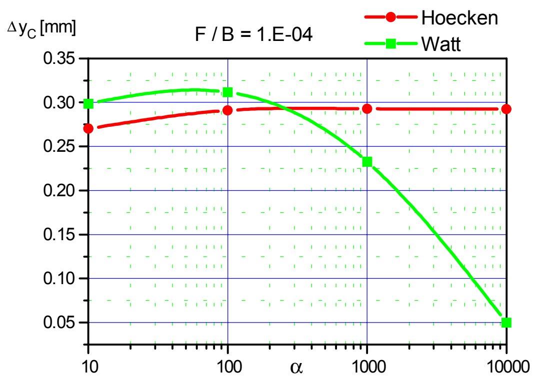
$$\alpha = \frac{(EI_z)_L}{(EI_z)_l} \quad (9)$$

α	Hoecken four-bar linkage			Watt four-bar linkage		
	F/B [mm^{-2}]	Δy_C [mm]	$\Delta\varphi$ [^{o}]	F/B [mm^{-2}]	Δy_C [mm]	$\Delta\varphi$ [^{o}]
10	$1\cdot10^{-4}$	0.2704	2.7531	$1\cdot10^{-2}$	0.31304	5.0114
	$1\cdot10^{-5}$	0.1416	3.3797	$1\cdot10^{-3}$	0.29982	5.4609
	$1\cdot10^{-6}$	0.1288	3.4499	$1\cdot10^{-4}$	0.29883	5.5119
	0	0.1273	3.4553	0	0.29872	5.5176
100	$1\cdot10^{-4}$	0.2909	2.6122	$1\cdot10^{-2}$	0.30693	5.8500
	$1\cdot10^{-5}$	0.1761	3.1616	$1\cdot10^{-3}$	0.31109	5.9073
	$1\cdot10^{-6}$	0.1647	3.2214	$1\cdot10^{-4}$	0.31161	5.9158
	0	0.1634	3.2283	0	0.31161	5.9158
1000	$1\cdot10^{-4}$	0.2929	2.8444	$1\cdot10^{-2}$	0.22596	8.2258
	$1\cdot10^{-5}$	0.1796	3.4446	$1\cdot10^{-3}$	0.23200	8.2423
	$1\cdot10^{-6}$	0.1684	3.5113	$1\cdot10^{-4}$	0.23272	8.2480
	0	0.1671	3.5167	0	0.23278	8.2480
10000	$1\cdot10^{-4}$	0.2926	5.2671	$1\cdot10^{-2}$	0.05636	16.18
	$1\cdot10^{-5}$	0.1793	6.5132	$1\cdot10^{-3}$	0.050169	16.1917
	$1\cdot10^{-6}$	0.1679	6.6328	$1\cdot10^{-4}$	0.04956	16.1974
	0	0.1667	6.6515	0	0.0495	16.200

Table 3. The deviation of the coupler point C (Δy_C) for horizontal displacement Δx_C=1mm of the hybrid compliant Hoecken and Watt four-bar linkages



a)



b)

Fig. 5. The deviation of the coupler point C (Δy_C) for horizontal displacement $\Delta x_C = 1$mm of the hybrid compliant Hoecken and Watt four-bar linkages

The deviation Δy_C of the Hoecken hybrid compliant four-bar linkage increases with the force increasing (Figure 5a). However, the deviation Δy_C of the Watt hybrid compliant four-bar linkage has been changing a little bit with the force increasing.

Figure 5b shows comparisons of the deviation Δy_C of the Hoecken and Watt hybrid compliant four-bar linkage for different values of rigidity ratio α. The smallest deviation Δy_C can be got by using the Watt hybrid compliant four-bar linkage with the great rigidity ratio α. However, for $\alpha = 10000$ we have got too great angular displacement $\Delta\varphi > 16^o$. The angular displacement $\Delta\varphi$ of the compliant mechanisms should not be greater than 10^o.

CONCLUSION

This paper discusses properties of some rigid-body four-bar linkages for rectilinear guiding and their equivalent counterparts with the elastic coupler. It considers the rectilinear guiding of the point C located at the end of the elastic coupler (the Hoecken four-bar linkage) and in the middle of the coupler (the Watt four-bar linkage).

If the coupler link is elastic, it can be modeled as a beam overhanging one support (Hoecken four-bar linkage) and simple beam (Watt four-bar linkage) . The input crank and the follower are assumed to be rigid. This model can be applied for greater displacement of the coupler point. The horizontal force F, that acts on the coupler point to be guided, has greater influence on the accuracy of rectilinear guiding of the couple point realized by Hoecken four-bar linkage with an elastic coupler. Therefore is Watt four-bar linkage with an elastic coupler more suitable for rectilinear guiding of the couple point for this model.

For smaller displacements of the coupler point (the displacements in the domain of the micromechanisms), the four-bar linkage with an elastic coupler can be modeled as an hybrid compliant mechanism. For this model calculations have been performed using the Finite Element Method (FEM). The smallest deviations of the coupler point C (Δy_C) of the Hoecken hybrid compliant four-bar linkage have been got for small values of rigidity ratio and horizontal force. The smallest deviations of the coupler point C (Δy_C) of the Watt hybrid compliant four-bar linkage have been got for great rigidity ratio α. The influence of the horizontal force value is negligible for the Watt hybrid compliant four-bar linkage.

The presented hybrid compliant mechanisms could not achieve the rectilinear guiding accuracy of the rigid-body linkages with the elastic coupler. However, compliant mechanisms are desirable because they may have less wear, weight, noise and backlash than their rigid-body counterparts. The lack of accuracy in rectilinear guiding of compliant counterparts discussed in this paper could be explained by nonlinearities introduced by the large deflection of the links.

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PRAVOLINIJSKO VOĐENJE TAČKE SPOJKE NEKIH POLUŽNIH ČETVOROUGLOVA SA ELASTIČNOM SPOJKOM

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U radu su prikazana dva polužna četvorougla sa krutim članovima, čija jedna tačka spojke može da bude vođena po približno pravolinijskoj putanji, kao i njima ekvivalentni mehanizmi koji imaju elastičnu spojku. U radu se razmatra vođenje tačke na kraju spojke kod Hoecken-ovog polužnog četvorougla i vođenje tačke na sredini spojke kod Watt-ovog polužnog četvorougla. Primenjena su dva modela: model elastične proste grede i model hibridnog gipkog mehanizma. U radu su takođe prikazana odstupanja između realizovanih i tačnih pravolinijskih putanja mehanizama sa krutim članovima, kao i njima ekvivalentnih mehanizama sa elastičnom spojkom.