

ABOUT SOME VARIOUS INTERPRETATIONS OF THE FATIGUE CRITERION AT LOW NUMBER OF STRAIN CYCLES

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Abstract. *Based on stress-strain analysis the stabilized hysteresis loops with the corresponding elastic and plastic amplitudes as well as cyclic stress-strain curve are defined. They represent the stable cyclic behavior under the total strain amplitudes. With available experimental fatigue data based on some various treatments, the fatigue criterion constants are estimated which values are compared with those estimated according to Manson's, Coffin's and Martin's suggestions. It is also shown that a general agreement don't exist and that the reliable values of these constants could be obtained only from fatigue test data. In some cases the suggested functions do not represent the fatigue criterion through all fatigue life region, when it is necessary to determine a limited part of this region where the agreement is satisfactory.*

1. INTRODUCTION

The whole destruction process is possible to separate in two phases; the first one is a pure fatigue phenomena, from the beginning to the fatigue crack initiation, and the second one is the crack growth with the fatigue of remaining cross section. Here will consider only the first phase, t.i. fatigue process from the cyclic beginning to the initial fatigue crack; the crack growth is studied in the Fracture mechanics. So, the fatigue criterion describes the end of the first destruction phase when the initial fatigue crack begins.

The fatigue phase can be further separated in two domains; one is that for which the cyclic stresses are relatively low with dominant elastic strains and high numbers of cycles to initial crack, called shortly as HCF; the other is that for which the cyclic stresses and strains are relatively high with existing both elastic and plastic strains and low numbers of cycles to initial crack, called shortly as LCF. The transition from LCF to HCF occurs at about $(0.5-1.0) \cdot 10^5$ cycles.

The fatigue crack origin in HCF region could be explained by the existing of micro plastic strains and the fatigue criterion includes only the elastic strain or corresponding stress. It is given even in nineteen century by well known Wöhler's or $S - N$ curve or by

Basquin's relation expressed through the elastic strain or corresponding stress as function of the number of cycles to fatigue failure [1]. The corresponding criterion in LCF region cannot be satisfactorily expressed only through elastic strains, but the plastic strains are to be included into consideration because of their dominant influence on fatigue behavior. The first forms of fatigue criterion in LCF region are given for 5-6 decennium by Orowan, Manson and Coffin [2,3,4]. Up to the moment this criterion has developed and improved, but some various treatments in its interpretation exist. Some of such interpretations is here described and tested on known experimental data.

2. CYCLIC ELASTIC AND PLASTIC DEFORMATION BEHAVIOR OF MATERIAL IN LCF REGION

The experimental results have shown that in the LCF region the fatigue process under constant total (elastic plus plastic part) strain amplitudes corresponds the most commonly [5,7,8,9]. During such cyclic straining of smooth specimens, the changes of nominal stress and plastic strain are also cyclical and much greater than the corresponding micro plastic changes. Because of that, the fatigue process in LCF region is quantitatively different than in HCF region. Only the existing of macro plastic strain amplitudes is the reason why the fatigue life in LCF region is many times shorter than in HCF region, where they don't exist and the macro elastic strain amplitudes are some smaller. Therefore, the macro plastic strain amplitudes have dominant effect on LCF process. Because of the uniform stress distribution in the cross-section of the smooth specimen, the mentioned changes of stress and plastic strain amplitude are caused only by the cyclic behavior of material. Under cyclic strains with constant total strain amplitudes the corresponding stress amplitudes could remarkably increase, decrease or not to change itself. Then the deforming behavior of material is: cyclical hardening (Fig. 1.a,c), cyclical softening (Fig. 1.b,d) and cyclic neutral, respectively.

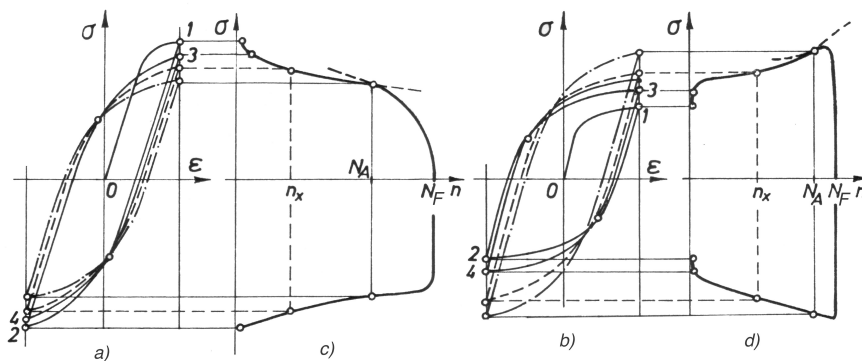


Fig. 1. Cyclic deforming, hysteresis loops, cyclic softening and hardening

3. THE REPRESENTATION OF THE CYCLIC STRESS-STRAIN FATIGUE PROCESS AND CYCLIC STRESS-STRAIN CURVE

During the cyclic straining, independently what happens - the softening or hardening, the distribution of the real micro stresses is not uniformly even at all points of cross section of smooth specimen because of polycrystalline structure of materials. For that reason, the distribution of these real stresses is different from the nominal stress one. In that way the actual stress, but only in one or in a certain number of points in cross section, could arise, up to any critical value and at that time failure occurs only here. For specimen in whole, this failure means an initial fatigue crack. Its technical detection as a geometrical value is usually possible only when crack length or crack area reaches a value which is remarkable for test equipment, for example 0.1 to 1mm or 0.01 to 1mm², respectively [7,8]. So, it is wellknown definition for technical fatigue crack.

Therefore, if beginning strain amplitude is too small, the real stress at any point cannot arise up to the critical value and the initial crack cannot occur. The maximum nominal stress is then equal to the endurance limit when the maximum real stress value at any or at some points is equal to the critical stress value.

For materials with cyclic hardening, the eventual beginning macro plastic strain can completely disappear during the fatigue process and then only micro plastic strains remain to act. In the case of cyclic hardening these remained micro plastic strains can not cause a fatigue crack. This is the reason which can explain the fact why the endurance limit for some materials is greater than the yield point stress for materials with cyclic hardening behavior (for example - stainless austenitic high alloyed steels, also steel Č4572, JUS mark).

To each cycle corresponds one hysteresis loop which is changeable during the cyclic straining (Fig. 1a, b). A represent hysteresis loop could be assumed as a middle one, which corresponds to the current number of cycles n_x equal to one half of the number of cycles to the initial crack, t.i. $n_x = N_A / 2$. This hysteresis is also called the stabilized or steady state hysteresis loop and it represents the cyclic deformation behavior during the fatigue process. On the base of its size and shape, the represent elastic and plastic strain amplitudes are estimated.

The loci of the tips of the stabilized hysteresis loops from fatigue processes under various total strain amplitudes define the cycle stress-strain curve (Fig. 2).

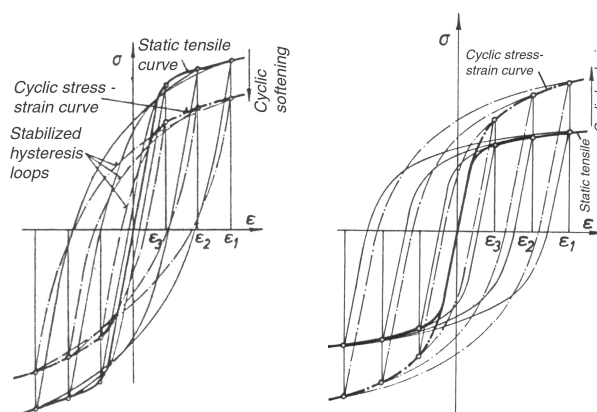


Fig. 2. Cyclic stress-strain curves

Therefore, in the case of cyclic softening, this curve is located lower than the static stress-strain curve and above it, in the case of cyclic hardening. If these two curves mutually intersect, the intersection point defines the stress and strain amplitudes at which the material shows a neutral cyclic behavior. Based on a known cyclic stress-strain curve, it is possible for each total strain amplitude to estimate both the stabilized hysteresis loop and the representative amplitudes of elastic and plastic strain.

4. A DEFINITION OF FATIGUE INITIAL CRACK AND ITS PRACTICAL DETECTION DURING FATIGUE PROCESS

It is interesting to estimate the curves of the extreme values of stress amplitude versus current number of cycles n , the one for maximum - tensile and the other for minimum - compression stress values (Fig. 1,c,d). It is the loci of the tops of the hysteresis loops during the cyclic straining under constant total strain amplitude. In a fatigue process, without any fatigue or any other type of cracks, a corresponding stable state relation between these two curves establishes. This represents the flow of the extreme value of inside resistance of material including all irregularities and errors in the material structure, but without any fatigue crack.

When the crack originates and if its cyclical growth begins, then a more rapid falling per cycle of absolute values of extreme stresses in tension than in compression direction exists for cyclic softening [10,11]. For cyclic hardening it is the case when the growth of extreme stress values begins rapidly to fall in the tension than in compression direction. In compression direction the areas of crack mutually contact and the specimen shows practically the same deformation resistance as well as without any crack. The beginning of these different changes in tension and in compression direction means that they are not caused by the mechanical behavior inside of material, but definitely because of the loss of completeness - integrity condition of body, i.e. because of a crack existing. Therefore, this fact can be used for detecting of an initial crack and for determination of number of cycles N_A when it originates during the fatigue tests.

Such definition of the initial crack is not based on its geometrical size, but only from different behavior of the specimen in tension and in compression direction. It is much easier to detect the existence of an initial fatigue crack when the decreasing of tensile stresses for cyclic softening has a noticeable amount, for example 1% in spite of such non-decreasing of compression stresses (Fig. 1c). For cyclic hardening such detection of fatigue crack is also possible when the tensile stress growth shows a falling of 1% related to corresponding further compression stress growth (Fig. 1d). For example, the steel Č5432 shows the cyclic softening and a crack grows up to the failure for a long time, but in opposite, the steel Č4572 shows the cyclic hardening where crack increases for a short time. The corresponding number of cycles $N_{A,1\%}$ for steel Č5432 is greater than N_A for an average amount of 14%, for lower strain amplitude and 28%, for the higher one, but for steel Č4572 none of remarkable difference exists [10].

After the crack initiation, further cyclic straining causes the growth of crack and simultaneously a more intensive continuation of fatigue in the instantaneous actual cross section exists. After a critical reduce of the remaining cross section, a total failure finally occurs. The corresponding number of cycles needed for the crack growth until the failure,

is designated as N_B , so that the total number of cycles needed to produce the total failure N_F , is: $N_F = N_A + N_B$.

5. FATIGUE CRITERION - TOTAL STRAIN AMPLITUDE VERSUS NUMBER OF CYCLES TO CRACK INITIATION

The main parameters in low cycle fatigue process are the elastic and plastic strain amplitude, as well as the corresponding stress amplitude the estimation of which values is based on stabilized hysteresis loops, t.i. on cyclic stress-strain curve.

The strains values in LCF region can be even significantly great and then it is more competent to use the true strain δ' and true stress σ' than their corresponding engineering nominal value ε and σ . Therefore, the true strain is

$$\delta' = \int_{l_0}^l \frac{dl}{l} = \ln \frac{l}{l_0} = \ln(1 + \varepsilon) \quad (1)$$

where l_0 is the beginning unit length, $\varepsilon = (l - l_0) / l_0$ is the nominal strain.

Further, the experimental results have shown that in the plastic range the volume of material remains approximately constant throughout the deforming process, thus

$$l/l_0 = A_0/A = (1 + \varepsilon) \quad (2)$$

where A_0 and A are the original and actual cross sectional area, respectively. The true stress for acting force F is:

$$\sigma' = \frac{F}{A} = \frac{F}{A_0} \cdot \frac{A_0}{A} = \sigma(1 + \varepsilon) \quad (3)$$

where $\sigma = F/A_0$ is the nominal stress. The corresponding true strain values at static failure by unit length l_f

$$\delta'_f = \ln l_f / l_0 = \ln A_0 / A_f = \ln(1/(1 - Z)) = -\ln(1 - Z) \quad (4)$$

known as true fracture ductility; A_f is true cross section area and Z is the coefficient of contraction at static failure. Under the static failure force F_f the true fracture strength is:

$$\sigma'_f \equiv R_R = F_f / A_f \quad (5)$$

The true elastic strain at static tensile failure is

$$\delta'_{el,f} = R_R / E \quad (6)$$

where E is the modulus of elasticity.

By plotting the such separated elastic and plastic strain amplitudes versus number of cycles to fatigue crack initiation N_A (previously to number of cycle to the total failure N_F , because of such test data presentation) on a log-log diagram, the data for majority of materials approximately lay to two straight lines with the slopes b and c , respectively. Its equations are:

$$\varepsilon_{el,a} = \Delta\varepsilon_{el}/2 = B \cdot N_A^b \quad (7)$$

for elastic strain amplitude, in the literature known as Basquin's equation [1], and

$$\varepsilon_{pl,a} = \Delta\varepsilon_{pl}/2 = C \cdot N_A^c \quad (8)$$

for plastic strain amplitude, known as Manson-Coffin's equation [3, 4]. Here B and C , as well as the exponents b and c , are the material constants.

The both equations relate to the same criterion which is in the first case expressed with elastic strain amplitude and in the second case with the plastic one. Because of too small influence of elastic strains in LCF region, the fatigue crack criterion can be much favorably expressed through corresponding plastic strain amplitudes. Manson determined the exponent value as a material constant based on available fatigue test results, firstly by means of N_F , $c = -1/3$ and later with N_A , $c \approx -0,6$. So, this means that the exponent value depends on treatment of fatigue life which is expressed in number of cycles either to final failure, in the first case, or to crack initiation, in the second case. The constant C should be determined as the ordinate at $N_A = 1$ based on the fatigue tests results for material applied.

According to Coffin, the exponent c has a universal constant value $-1/2$ for all materials and constant C should be determined by using the static tensile test only as a fatigue one so that for $N_A = 1/4$ and true fracture ductility $\varepsilon_{pl,a} = \delta'_f$, t.i. from Eq.(8) by using $c = -1/2$, it follows. $C = \delta'_f/2$.

These various interpretations lead to the different values for criterion constants. Because of too great difference of exponent values, it was also necessary to separate the whole destruction process in two phases in which the destruction criteria are also different. In the first phase, there is the micro destruction of material structure and in the second one, the macro destruction of body integrity exists. Based on that, the treatment of pure fatigue process in the first phase has its fully physics meaning. The corresponding criterion should be treated only at this phase with an explicit fixing of boundary between both phases. In that sense, the determination of fatigue crack initiation and corresponding number of cycles is very important [10, 12].

Because of the problem by separation of total strain on its elastic and plastic part, these criteria are to be expressed as one function:

$$\varepsilon_a = \varepsilon_{el,a} + \varepsilon_{pl,a} = BN_A^b + CN_A^c \quad (9)$$

This relation, known as Basquin-Manson-Coffin's equation, represents the fatigue criterion expressed through total strain amplitude as function of number of cycle to fatigue crack initiation in LCF region, (Fig. 3a), and corresponds to $S - N_A$ curve in HCF region.

It should also to note that the experimental fatigue data for some materials don't show a good agreement with Basquin-Manson-Coffin's relations in all region of fatigue life. A such example is steel Č4572, JUS mark (X10 CrNiTi 18 9, DIN mark) which mostly shows cyclic hardening. The corresponding experimental data for elastic and plastic strain amplitude versus number of cycles to fatigue crack don't lay on straight lines in log-log plots for all LCF region (Fig. 3b). In these cases it is necessary to estimate a limited region of fatigue life, where such relations could exist. For this steel that limited region

may be from 10^3 to 10^5 cycles. This means that the original Basquin-Manson-Coffin's criterion is not a general one for all materials.

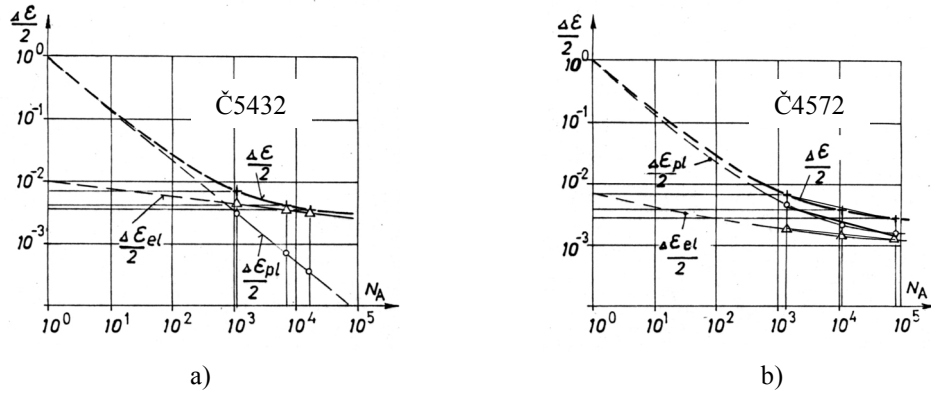


Fig. 3. Elastic, plastic and total strain amplitudes versus number of cycles to fatigue crack

Assuming the simplified form of hysteresis loop with a linear work hardening, Martin has suggested an energy criterion, using that only this work hardening w per cycle is damage one (Fig. 4), [6]. So, according to this criterion, the total damage work absorbed in N_A cycles is equal to the corresponding work hardening of a static tensile test W_{static} , t.i.

$$N_A \cdot w = W_{static} \tag{10}$$

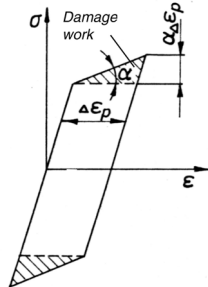


Fig. 4. Work hardening per cycle as damage work according to Martin

By interpreting the static tensile test with $N_A = 1/2$ and $\Delta\epsilon_{pl} = \delta'_f$, it will be

$$N_A \cdot \alpha \cdot (\Delta\epsilon_{pl})^2 = \frac{1}{2} \alpha \cdot \delta'_f{}^2 \tag{11}$$

where α is the slope of the work hardening straight line. By using $\Delta\epsilon_{pl} = 2\epsilon_{pl,a}$, it follows

$$\epsilon_{pl,a} = \frac{\Delta\epsilon_{pl,a}}{2} = \frac{\sqrt{2}}{4} \delta'_f \cdot N_A^{-0.5} \tag{12}$$

6. TESTING OF RESULTS ACCORDING TO VARIOUS INTERPRETATIONS
OF THE CRITERION WITH TRUE FATIGUE TEST DATA.

The values of all criterion constants is the best to estimate based on corresponding data obtained from fatigue tests with various total strain amplitude. In the absence of fatigue data, it is possible the value of constant C to estimate based only on static tensile test data and for the exponent c to use the recommended values according to Coffin and Martin.

Besides of this, the values of all constants is possible to estimate based on fatigue test data by using also the static tensile data as a fatigue one with the number of cycle to failure, t.i. to crack initiation $N_F = N_A = 1/4; 1/2; 1$ and the corresponding amplitudes of elastic and plastic strains: $\epsilon'_{el,f} = R_R/E$, $\epsilon'_{pl,a} = \delta'_f$ (In Fig. 3.a, b, the static failure is illustrated only for $N_A = 1$).

Martin has compared the values for constant C for some various materials and concluded that his hypothesis shows a better agreement with fatigue test values than assumption according to Coffin's hypothesis, for tests at room temperature but vice versa, for tests at elevated temperature. In some cases the errors were considerably great [6].

The general value of exponent $c = -1/2$ according to Coffin's and Martin's hypotheses also differs from the test data value. The recommendation of exponents values as the slope ranges of two mentioned straight lines on log-log plot is: $b = -(0.05...0.15)$ and $c = -(0.5...0.8)$ [9].

For steel JUS Č5432 (30CrNiMo8 -DIN mark), for which the fatigue test data show a good agreement with Basquin-Manson-Coffin's criterion, the static tensile characteristics are: $R_R = 2405 \text{ N/mm}^2$, $E = 2.12 \cdot 10^5 \text{ N/mm}^2$, $Z = 0.66$ [8, 10, 11, 12]. Then the true strain at failure is

$$\delta'_f = -\ln(1 - Z) = -\ln(1 - 0.66) = 1.079.$$

The true elastic strain at failure is

$$\delta'_{el,f} = R_R / E = 2405 / 2.12 \cdot 10^5 = 0.0113.$$

According to Coffin's assumption the constant $C = \delta'_f / 2 = 1.079/2 = 0.539$.

According to Martin's hypothesis the constants are:

$$C = \frac{\sqrt{2}}{4} \delta'_f = 0.354 \delta'_f = 0.354 \cdot 1.079 = 0.382 \text{ and } c = -0.5.$$

If for steel Č5432 the fatigue crack criterion is determined according to the initial crack beginning, t.i. according to number of cycles to crack initiation N_A , and based both on fatigue and on static tensile test data by using $N_A = 1/4$, the constants are: $C = 0.394$ and $B = 0.0096$, and exponents: $b = -0.121$, $c = -0.727$ [10].

According to the other definition of crack beginning by 1% decrease of tensile stresses $N_{A1\%}$ [10], the constants are: $B = 0.0055$, $b = -0.062$, $C = 1.55$ and exponent $c = -0.860$.

By omission of the static tensile test data and by using only pure fatigue tests data based on criterion for beginning of the fatigue crack, the corresponding values for constants are: $B = 0.0055$, $b = -0.062$, $C = 0.850$, $c = -0.800$.

All of these results as well as results obtained by using both fatigue and tensile data for $N_A = 1/2$ and 1 are exhibited in the next table.

	Only static tensile tests data		Both N_A fatigue and tensile tests data			Only fatigue tests data	
	Coffin $N_A=1/4$	Martin $N_A=1/2$	$N_A=1/4$	$N_A=1/2$	$N_A=1$	$N_{A,1\%}$	N_A
b	–	–	–0.121	–0.126	–0.136	–0.062	–0.062
B	–	–	0.0096	0.0104	0.113	0.0055	0.0055
c	–0.5	–0.5	–0.727	–0.777	–0.828	–0.860	–0.800
C	0.539	0.382	0.394	0.652	1.079	1.550	0.850

The results of criterion constants obtained from pure fatigue test data based on two various interpretation of fatigue crack beginning are given in the last two columns. In the both cases the constants b and B have the same values, but the values for constants c and C differ, especially for constant C . These different values show that these criterion constants depends on the crack initiation criterion, t.i. on various definitions of fatigue crack initiation and that an accurat interpretation of fatigue crack beginning is very significant. Because of that, for determined constants values it should be also cited the criterion used for crack initiation. Of course that the values in the last column better interpret the criterion constants for pure fatigue phase because the corresponding values for $N_{A,1\%}$ criterion include also a part of the number of cycles from second destruction phase.

A comparison of the corresponding constants values obtained from both N_A fatigue and tensile tests data for $N_A = 1/4; 1/2; 1$, shows that they mutually differ. The differences are: small for the constant values for b and B , for the constant c few greater and for the constant C the greatest. Their further comparison with those in the last column, shows also the differences: for constants b and B abote twice, and for constants c and C a smaller. So, the constants values for c and C for pure fatigue test are on the middle of difference between those constants values for $N_A = 1/2$ and $N_A = 1$.

The comparison of the constants values according to Coffin and Martin with those values for pure fatigue test show relative great differences both for c and for C . The many times greater absolute values for constants c and C than for constants b and B show only that the plastic strain amplitudes have much more influence in LCF region than the elastic one.

7. CONCLUSION

The estimation of fatigue test constants depends on the stress-strain treatment and the definition of fatigue crack initiation. By separation the total strain amplitude into its elastic and plastic strain amplitude, the influences of plastic strains on fatigue process is much more powerfully than the elastic ones. The estimation of the values of constants depends on the definition of fatigue crack initiation, because it is boundary between the phase of pure fatigue and the phase of fatigue crack growth.

Instead of classical geometrical definition, a more appropriate definition based on beginning of more rapid change per cycle in tensile extreme stresses than in compression ones during the fatigue process, has been given and in that sense the Basquin-Manson-Coffin's criterion is tested. Besides the analysis of the hypotheses according to Coffin and Martin shows that in all of cases the estimation of constants values based only on the static tensile test data is not possible. This fact can be understood by the various physics character of the static and the fatigue destruction process. The special problem is the

claim of these two hypotheses according to which the value of exponent $c = -1/2$ for all of the materials, what the fatigue test data have also denied.

The true constant values can be determined only in a family of fatigue tests with a careful estimation of the numbers of cycles to crack initiation and the elastic as well as plastic strain amplitudes based on stable hysteresis loops. If the fatigue test data for some materials don't show a good agreement with the criterion equations, it is necessary to estimate a limited region where a better coincidence exist.

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O NEKIM RAZLIČITIM INTERPRETACIJAMA KRITERIJUMA ZAMORA PRI MALOM BROJU CIKLUSA DEFORMACIJA

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Na osnovu naponsko deformacione analize definisane su stabilizovane histereze, odgovarajuće amplitude elastične i plastične deformacije kao i ciklična kriva napon-deformacija koja u odnosu na statičku krivu zatezanja opisuje stabilizovano ciklično ponašanje u zavisnosti od amplitude ukupne deformacije. Sa raspoloživim eksperimentalnim rezultatima dobijenim različitim postupcima, određene su vrednosti konstanata kriterijuma zamora i upoređene sa onim vrednostima određenim prema preporukama Manson-a, Coffin-a i Martin-a. Pokazano je da opšta saglasnost ne postoji i da se pouzdane vrednosti ovih konstanata mogu odrediti jedino iz rezultata ispitivanja zamaranjem. U nekim slučajevima predložene funkcije ne predstavljaju kriterijum zamora u celoj oblasti zamornog veka kada je nužno odrediti ograničeni deo ove oblasti gde postoji zadovoljavajuća saglasnost.