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CALCULATION OF DESTRUCTION MECHANICS PARAMETERS AND TEMPERATURE OF DISSIPATIVE HEATING BY SINGULAR THREE-DIMENSIONAL FINITE ELEMENTS IN CONDITIONS OF NONLINEAR VISCOELASTIC DEFORMATION

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Abstract. *For sizing of constructions from weak compressible materials difficulties are appeared, arising with use of a traditional finite element method. That difficulties are connected with properties of rigidity matrix, the moment scheme of finite elements with threefold approximation function of displacement fields, component of deformations and function of volume change is developed* [1].
One of the most widespread defects is the crack. From analytical solutions it is known, that about top of a crack the distribution of stresses and deformations has a feature $1/\sqrt{r}$*, where r - distance from a point of a construction up to top of a crack. To calculate constructions with a crack it's necessary to simulate that feature of deformations and stresses using special finite elements. As special finite elements are applicable singular triangular prismatic finite elements and singular parallelepiped finite elements* [2,3].
The power approach of determination of factors of stress intensity for designs from viscoelastic material is submitted in the work. The component of J-integral and factors of stress intensity are meanings by the method of equivalent volumetric integration. The method of definition of dissipative temperature and warm sources is offered with the decision of a heat conduction task in nonlinear statement. The algorithm of definition of warm sources is developed with the decision of the task by step-by-step iterative methods. For solution of nonlinear problems we shall take advantage of a modified method of Newton - Cantorovich in a combination to a method of an integration on the parameter of a load, or on the parameter of a strain with verification of the equations of balance.

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The given method is realized on PC IBM within the framework of the computer complex 'MIPEJIA', intended for the decision of tasks of the mechanics of the deformed rigid body and mechanics of destruction elastomer and composite constructions. The factors of stress intensity of the trapezoid shock-absorber with the crack are determined with the given complex. The solution of a problem in nonlinear setting conducts to change not only a value of factors of stress intensity, but a also character of their distribution.

The influence of the crack on distribution of dissipative temperature is considered. It is possible to make a conclusion of the analysis of obtained results , that the availability of a crack and the account of nonlinearity has an effect for change of character of distribution and value of temperature of dissipative heating..

All constructions have some defects. One from the most widespread defects is the crack. From analytical solutions it is known, that about top of a crack the distribution of stresses and deformations has a feature $1/\sqrt{r}$, where r - distance from a point of a construction up to top of a crack.

One of the most universal methods for solving the problem of mechanics of a deformable hard body and destruction mechanics is the finite element method. To calculate constructions with a crack it's necessary to simulate a feature of deformations and stresses $1/\sqrt{r}$ using two methods: 1.condensation of a grid of finite elements about top of a crack; 2.application of special finite elements. The first method results in increase of an amount of the solving equations and time of the calculation. Second - with constant amount of the equations allows to increase accuracy of calculations. As special finite elements are applicable singular triangular prismatic finite elements and singular parallelepiped finite elements [1, 2].

For solving the problem of the mechanics of destruction elastomer constructions with cracks it is necessary to take into account viscoelastic properties of a material, which are described with the hereditary theory Boltzmann-Volterra:

$$\sigma^{ij}(t) = C^{ijkl}\left[\varepsilon_{kl}(t) - \int_0^t R(t-\tau)\varepsilon_{kl}(\tau)d\tau\right], \qquad (1)$$

where σ^{ij} - component of a stress tensor,

C^{ijkl} - component of an elastic constants tensor,

ε_{kl} - component of a deformation tensor,

t - time,

$R(t-\tau)$ - incremental nucleus of relaxation.

Generally the front of a crack represents a space curve. The significances of power parameters of a destruction mechanics are calculated on a surface of a small handset surrounding front of a crack. The coordinate system of a crack is shown on Fig. 2. The power parameters of a destruction mechanics(component of a J-integral J_k and intensity of a release of energy G_{III}) for viscoelastic material are under the formulas:

$$J_k(t) = \frac{1}{\Delta}\lim_{e\to 0}\int_{A_e}(W(t)n_k - \sigma_{ij}(t)\frac{\partial u_i(t)}{\partial x_k}n_j)dA, \ (k=1,2) \qquad (2)$$

$$G_{III}(t) = \frac{1}{\Delta} \lim_{e \to 0} \int_{A_e} (W_{III}(t) n_1 - \sigma_{3j}(t) \frac{\partial u_3(t)}{\partial x_1} n_j) dA, \tag{3}$$

where Δ - length of front of crack's section,
W - density of deformation energy,
W_{III} - density of deformations of transversal shift's energy,
e - diameter of a handset enclosing a section of front of a crack,
n_j - component normal to an element of an outline dA,
A_e - surface of a handset enclosing a section of front of a crack.

The dependence between power parameters of a destruction mechanics and factors of stress intensity for viscoelastic material has the following form:

$$K_{I,II}(t) = \frac{1}{2}\sqrt{H}(\sqrt{J_1(t) - J_2(t) - G_{III}(t)} \pm \sqrt{J_1(t) + J_2(t) - G_{III}(t)}), \tag{4}$$

where H - effective module of the elasticity equal to the module of the elasticity E for a plane stress state and equal $E / (1 - \nu^2)$ for plane deformation.

The solution of a nonlinear viscoelastic problem is usually reduced to solution of the system of the linearized equations with step-by-step iterative algorithms. For solution of nonlinear problems we shall take advantage of a modified method of Newton-Cantorovich in a combination to a method of an integration on the parameter of a load, or on the parameter of a strain with verification of the equations of balance. Such approach allows to guarantee accuracy of solution.

The temperature field in a construction is determined with help of the equation of stationary heat conduction:

$$\mathrm{div}(\lambda_q \mathrm{grad} T) + \omega_0 = 0 \tag{5}$$

where λ_q - thermal conductivity of a material,
T - temperature,
ω_0 - specific power of internal sources.

For solution of the given equation it is necessary to set boundary conditions, which are expressed as convective heat exchange between a surface of a body and environment

$$-\lambda_q(T)\frac{\partial T(t,z^i)}{\partial n} = \alpha(T)[T(t,z^i) - \theta(t,z^i)], \tag{6}$$

where $\alpha(T)$ - heat transfer coefficient,
$\theta(t,z^i)$ - temperature of the environment.

The function of sources of heat generation with allowance for nonlinearities of deformations and stresses will look like:

$$\omega_0 = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} \sigma_{ij} \frac{\partial \varepsilon_{ij}}{\partial t} dt . \tag{7}$$

For solution of a problem of heat conduction by iterative step-by-step algorithms the specific power of internal sources of heat generation on n-th iteration is determined by relation:

$$\omega_0^{(n)} = \omega_0^{(n-1)} + \Delta\omega_0^{(n)}, \tag{8}$$

where

$$\omega_0^{(n\text{-}1)} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \sigma_{ij}^{(n-1)} \frac{\partial \varepsilon_{ij}^{(n-1)}}{\partial t} dt, \tag{9}$$

$$\Delta\omega_0^{(n)} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \left(\sigma_{ij}^{(n-1)} \frac{\partial \Delta\varepsilon_{ij}^{(n)}}{\partial t} + \Delta\sigma_{ij}^{(n)} \frac{\partial \varepsilon_{ij}^{(n-1)}}{\partial t} + \Delta\sigma_{ij}^{(n)} \frac{\partial \Delta\varepsilon_{ij}^{(n)}}{\partial t} \right) dt \tag{10}$$

Problem. The trapezoid shock-absorber with a square hole (Fig.1).

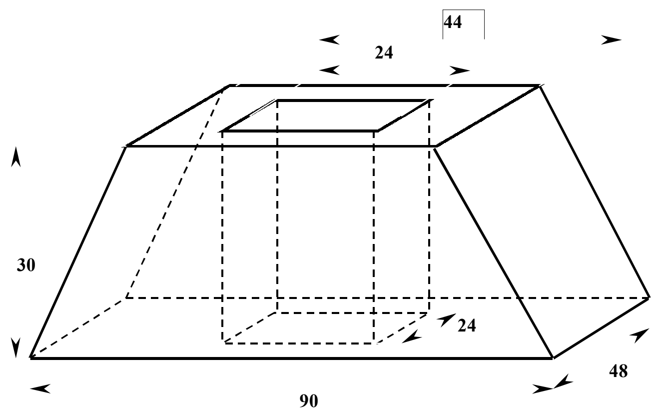

Fig. 1. The general view of a trapezoid shock-absorber.

The sizes of the shock-absorber are represented on a Fig. 1. The crack was simulated in a place of operation of the greatest expanding stresses for specific scheme of loading (Fig. 2). A metal slice was vulcanizated to the rubber shock-absorber from above. Below shock-absorber leans on the absolute rigid basis, the friction between the basis and lower plane of the shock-absorber was not taken in the account.

The mark of rubber is 2959. The elastic constants of materials: the shift module of rubber $G_r = 1.76$ MPa, Poisson's constant of rubber $\nu_r = 0.495$, elasticity module of metal $E_m = 2.0 \times 10^5$ MPa, Poisson's constant of metal $\nu_m = 0.3$. A load P = 1 MPa. Viscoelastic property of rubber are described by the Boltzman-Volterra's hereditary theory . As a nucleus of relaxation we shall take advantage of a Y.N. Rabotnov`s nucleus. Reological characteristics of a nucleus for the given mark of rubber: $\alpha = -0.6$; $\beta = 1.06$; $\chi = 0.64$. On Fig. 3, 4 the distribution of factors of stress intensity along front of a crack is shown.

For a solution of a problem of heat conduction the following parameters of a material were used: thermal conductivity $\lambda_q = 0.293$ Wt/(mK), frequency $\omega = 5$ Hz and indicated factors of convective heat transfer $H_1 = 5240$ 1/m, $H_2 = 40$ 1/m, . On a surface of a rubber element of the shock-absorber there is a convective heat exchange to a metal slice and air.

The distribution of temperature of dissipative heating on a length of the shock-absorber in a plane parallel to the basis on distance of 18 mm from it is represented on Figs 5, 6.

The problem was decided in linear and nonlinear viscoelastic settings on the limits of calculating complex "МІРЕЛА".

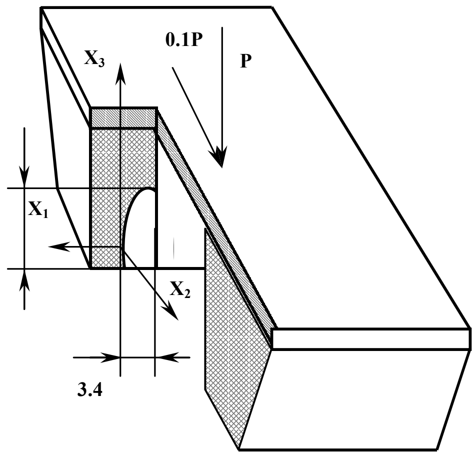

Fig. 2. The trapezoid shock-absorber with a crack.

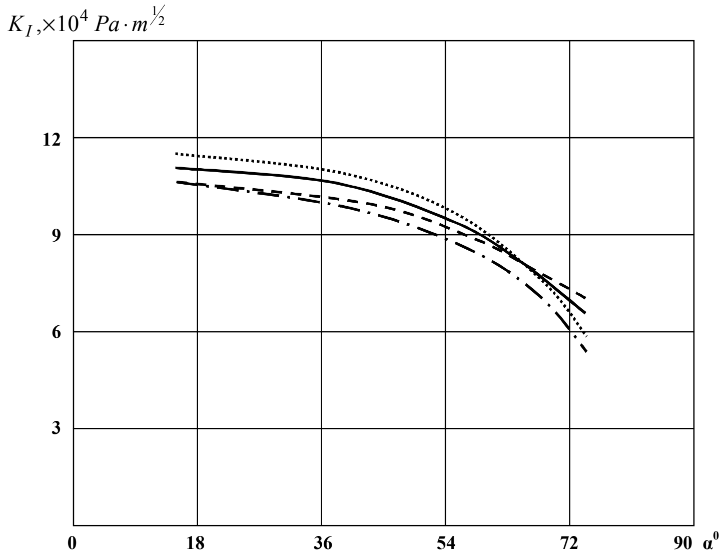

Fig. 3. The distribution of factors of stress intensity K_I along front of a crack:

——— - linear elastic solution;
– – – - linear viscoelastic solution;
— · – - nonlinear elastic solution;
.... - nonlinear viscoelastic solution.

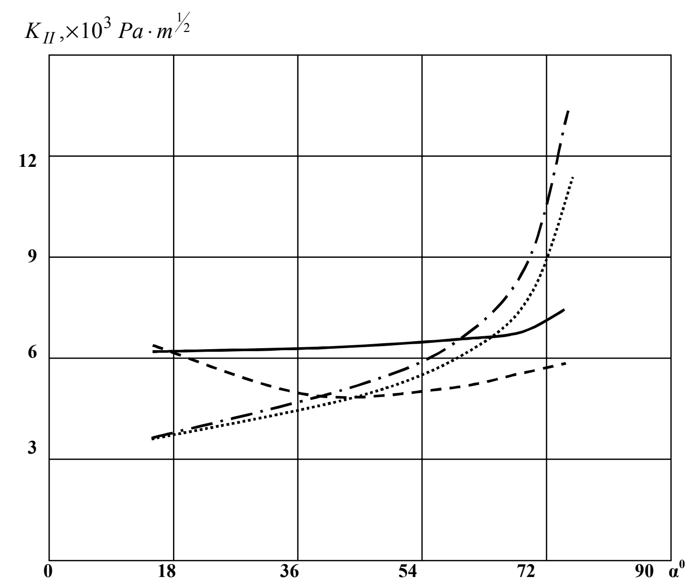

Fig. 4. The distribution of factors of stress intensity K_{II} along front of a crack:

——- linear elastic solution;
– – –- linear viscoelastic solution;
—· –- nonlinear elastic solution;
……… - nonlinear viscoelastic solution.

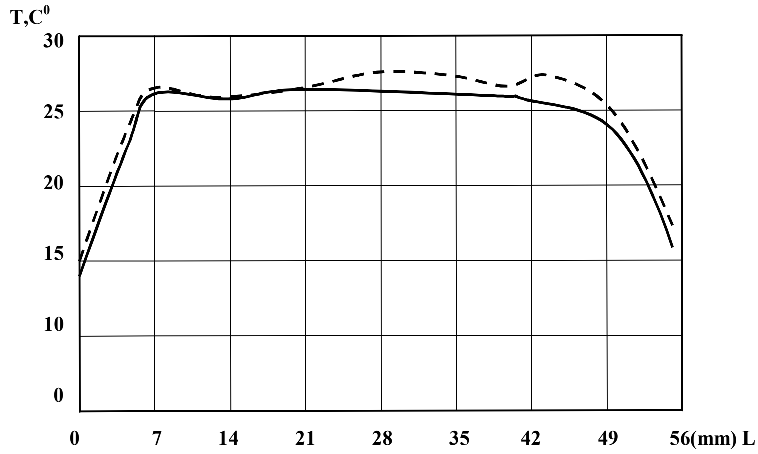

Fig. 5. The distribution of temperature of dissipative heating on a length of the shock-absorber (linear solution):

– – – – – - with a crack;
———— - without a crack.

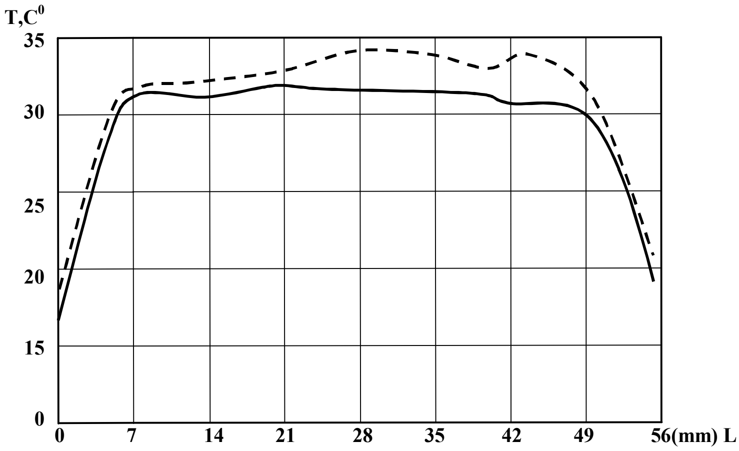

Fig. 6. The distribution of temperature of dissipative heating on a length of the shock-absorber (nonlinear solution):

- - - - - - **with a crack;**

—— - **without a crack.**

It is possible to make a conclusion of the analysis of obtained results, that the availability of a crack and the account of nonlinearity has an effect for change of character of distribution and value of temperature of dissipative heating (on the average on 10-15%).

From the indicated solutions it is evident, that the account reological characteristics of a material for the account of the trapezoid shock-absorber with a crack results in change of the value of factors of stress intensity. The solution of a problem in nonlinear setting conducts to change not only a value of factors of stress intensity, but a also character of their distribution.

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PRORAČUN PARAMETARA MEHANIKE LOMA I TEMPERATURE DISIPATIVNOG ZAGREVANJA POMOĆU SINGULARNIH TRODIMENZIONIH KONAČNIH ELEMENATA U USLOVIMA NELINEARNE VISOKOELASTIČNE DEFORMACIJE

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Pri proračunu konstrukcija od slabo kompresibilnih materijala pojavljuju se teškoće pri primeni tradicionalne metode konačnih elemenata. Teškoće su povezane sa svojstvima matrice krutosti, isto tako momentna šema konačnih elemenata sa trostrukom aproksimacijom funkcije polja deformacije, komponente deformacija i funkcija promene zapremine su izvedene.

Takođe je ponuđen i metod za određivanje disipativne temperature i toplih izvora pri rešavanju problema toplotne kondukcije sa nelinearnim graničnim uslovima. Za rešavanje nelinearnih problema primenjen je modifikovani metod Newton-Cantorovich-a.