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STRUCTURAL STABILITY OF THE PLANETARY REDUCTOR NONLINEAR DYNAMICS PHASE PORTRAIT

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Abstract. *Some results of numerical experiments on the planetary reductor nonlinear dynamics, and some properties in the nonlinear dynamics phase portrait in the area around singular saddle points and fixed points of the dynamical processes are presented in this paper.*
This paper deals with the study of the configuration of the equilibrium positions of planetary reductors and their structural stability. The chosen reductor has its potential and kinetic energy as well as the modified form of the potential energy, by means of which the equilibrium positions are determined. The paper, also, gives the numerical experiment for the real planetary reductor, the dependence graphic between potential energy and the generalized coordinate and the dependence graphic between total energy of the system and the generalized coordinate. In addition, the paper presents the integral curves in the phase plane. The conclusion is drawn about the equilibrium positions and configurations under the dynamic conditions as well as about the equilibrium positions stability.
By using numerical experimental results on the planetary reductor nonlinear dynamics and phase portraits with total energy surfaces in phase space, we show that it is very important to know qualitative own properties of nonlinear dynamics change with small or slowchanging critical systems parameters to the structural stability of phase portrait.

Key words: *Nonlinear dynamics, planetary reductor, phase portrait, equilibrium positions, structural stability, numerical experiment, total energy surface, singular points, homoclinic trajectories.*

1. INTRODUCTION

The deviation of the system from the equilibrium position can be regarded as a kind of disturbance and the oscillatory motion can be regarded as the perturbed state of the

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system equilibrium. During the change of the parameter values of the dynamic system, the number of the equilibrium positions and their stability can be altered. Such changes of nonlinear system, which are the consequence of the system parameter change, are subject of the bifurcation theory [6].

The equilibrium stability criteria of a body were postulated by E. Torricelli, and the theorem was postulated by J.L. Lagrange. According to this theorem a conservative system has a stable equilibrium position if the potential energy of the system has an isolated minimum in the given balance position. The theorem was proved by G. Lejeun-Dirichlet. E.I.Routh developed these ideas further on, and Liapunov finally made the fundamental contribution to the theory of stability (see [32], [7] and [8]).

This paper studies the dynamic equilibrium positions configuration of the dynamic model of the planetary reductor under the conditions of the disturbance interaction which are the consequence of the real planetary reductors function. For such conditions, "disturbances" are presupposed, the model is made and the potential energy in the function of generalized coordinates is determined. According to Dirichlet's criteria, the equilibrium positions in relation to the arrangement of "mass disturbances" is determined; and then those positions are determined in relation to the values of the given equilibrium positions characteristics. Further on, the geometric analysis of such a model is performed. The kinetic and the total energy of the system are determined. Numerical experiment is made for the particular planetary reductor as well as the dependence diagrams between potential energy and generated coordinates and the 3-D dependence diagrams between total energy and generalized coordinates. Phase plane contains the projections of integral curve.

2. THE "DISTURBANCE" FACTORS IN THE PLANETARY REDUCTOR WORK

2.1. Coupling of gear teeth

The disturbance forces appear only in the real planetary reductors with the elements that are not produced accurately and the elements that have finite rigidity. The combination of these two factors leads to the load asymmetry of the coupled elements of the planetary reductors and to the appearance of disturbance forces.

Characteristics of reductor systems are impulsive forces that are created in the gear tooth profile [2]. These are the forces that are generated upon the entrance of the gear coupling, the foras that are generated as a consequence of the periodic change of the gear-tooth number change in the coupling, periodical moments, the forces that appear at the same time with these two or more factors.

The gear teeth coupling process goes on at the same time with the collision of greater or smaller intensity. These collisions contribute to a greater number of internal dynamic forces and gear vibrations. Besides, collision is the fundamental source of disturbance energy in the system. The gear collision takes place when the gear-teeth enter the coupling, but it can take place when the gear-teeth leave the coupling. Resonance in the couple gears action process happens with the gear-teeth with high repetition frequency. During flanks rolling, rough spots, bumps e.t.c. are colliding as well.

Considering the fact that the gear-teeth with a certain mistake of the steps and the elastic deformations the position of the first point of connection differs from the theoretical position, that point is not on the line of the connection and the direction of the connection is changed, i.e. the relative motion speed of one gear-tooth in relation to another becomes obvious. So, during the contact of the gear-teeth there appear some strokes that have to be analyzed taking into account the immediately measured collision speed. The size of the collision velocity (speed) in the gear coupling depends on both the mistake of the coupling

$$f_{sp} = f_{kor} + w \tag{1}$$

and the velocity (speed) and kinematic parameters of the gears

$$V_z = \omega_z \delta_2' \sqrt{2 f_{sp} / \delta_{sp}} \tag{2}$$

where f_{kor} - stands for the difference of the basic steps of the coupled gears, w - elastic deformation of the gear-teeth that were coupled, ω_z - angular velocity (speed) of the gear-teeth, f_{sp} - reduced radius of the profile curve of the gear-teeth that were coupled. $\frac{1}{\delta_{sp}'} = \frac{1}{\delta_1'} + \frac{1}{\delta_2'}$, δ_1', δ_2' - radii of the profile curve of the gear-teeth at the point of connection.

The additional parts δ_1, δ_2 represent the result of the statically deformation of the gear-teeth and the mistake steps [2]: -characterizes premature entrance of the gear-teeth in the coupling (Figure 1 A-A) till the theoretical line of coupling (head-on collision), late coming of the gear-teeth out of the coupling at the end of the two-pair coupling.

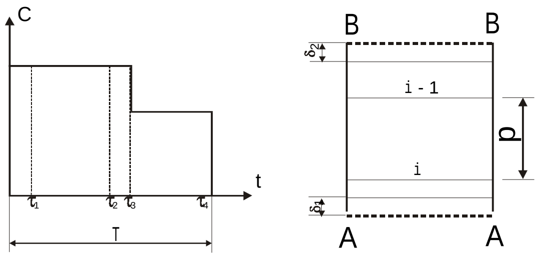

Fig. 1. Presentation of late coming of the gear-teeth out of the coupling at the end of the two-pair coupling

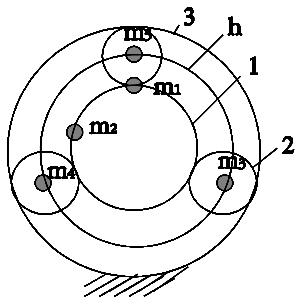

Fig. 2. Model of the Planetary reductor

The period of coupling can be decomposed into four phases, at the end of which the fixed moments of time are suitable τ_1, τ_2, τ_3 τ_4. For the begining of the counting, we take the moment of the entrance of the i-th gear-tooth into the coupling in the point A. At the moment $t = \tau_1$ that pair of gear-teeth reaches the theoretical line of the coupling, when $t = \tau_2$ the gear-teeth begin to leave the coupling and when $t = \tau_3$ the gear-teeth leave the coupling and the pair I-1 remains. During the time $\tau_3 < \tau < \tau_4$ one pair coupling is realized. During the period $t = \tau_4 = \tau$, the next pair of gear-teeth $I + 1$ enters the coupling and the same process repeats.

The basic characteristic of the gearing is the fact that after the meeting of the gear-teeth profile in the point that is not the theoretical point of meeting, the normal component of the velocity decreases, and the different values of velocity change randomly and they equal zero in the moment when the contact point of gear-teeth enters the coupling.

On the basis of the information given above, we can say that in the coupling gears action (if the gear-teeth are real) the following disturbance factors are involved: periodical impulses which are the consequence of the gear-teeth collision in the contact point, periodical inertia-forces which are the consequence of the reduced velocities values from v_z to 0, periodical forces which are the consequence of the gear-teeth number change in the coupling and the rotating couples that the gearing receives. The last factor is realized only if the rigidity change in coupling phases is not important for the production in the periodical oscillation system, in that case it can be regarded as the additional disturbance force.

2.2. The gear-connections

This gear-connection is one of the fundamental elements of the planetary reductors so that the performing ability of the system as well as the degree of the equal load on parallel branches of the transmission forces depends on the production accuracy and the construction. By means of gear-connections the connection of the central gears can be realized.

The basic sources which create oscillations are: ill-balance of elements of rivet, and total error in pace of cog-teeth of a wheel.

3. Configuration of dynamic equilibrium positions

In consideration of disorders which are the characteristic of work of the planetary reductor, we made the simple model on a Figure 2, where are: 1 - sun gear, 2 - a satellite, 3 - epicycle and h - the support (porter) of satellite, and with the added masses m_i are presented those disorders (mass perturbance and debalances) which, we said before, can be the consequence of many factors.

The disbalance of masses can have different positions, and that depends on which disorder we want to analyze and give it certain importance. On the picture 2 we give an arrangement of mass-particles - masses so that with mass-particle m_1 we presented disorder as a consequence of irregular connection of sun gear with the satellite, by mass-particle m_2 is presented disorder because of incorrect production and installation of sun gear, with mass-particles m_3 and m_4 are presented disorders because of uneven transfer of loads, because of incorrect production and installation of satellite and with mass m_5 is presented disorder because of uneven transfer of loads of the satellite.

Potential energy of the planetary system in accordance with adopted physical model (Fig. 2) will be:

$$E_p = m_1 g r_1 (1+\cos\varphi_1) + m_2 g r_1 (1+\sin\varphi_1) + (m_3+m_4) g r_h (1-\cos\varphi_h) + m_5 g r_h (1+\cos\varphi_h) \quad (3)$$

where are: r_1, r_h - the basic radiuses of sun gear and support of the satellite, φ_1 and φ_h - the generalized coordinates (the rotation displace - angles of sun gear and support of the satellite).

For the mechanical system with holonomic and stationary constraints, the position of which is defined with generalized coordinates q_1, q_2,......q_n., as it is known, that in equilibrium position, all generalized forces Q_k, in that kind of system, are equal to zero.

$$Q_1 = 0, \ldots\ldots\ldots\ldots Q_n = 0 \quad (4)$$

If generalized forces depend on coordinates q_i and velocities $q_i^{'}$, for arbitrary position, in which system can be in equilibrium position, it's correct enough to put into the equation (4) the value for $q_i^{'} = 0$ and solve obtained equation through $q_1,......q_n$.

For conservative forces $Q_k = -\partial E_p / \partial q_k$, where E_p is potential energy of system, equation (4) have a form:

$$\frac{\partial E_p}{\partial q_1} = 0..............................\frac{\partial E_p}{\partial q_i} = 0 \tag{5}$$

With solving of this system of equations in relation to $q_1,........,q_i$, we are determing the values of generalized coordinates, with which the system can be in equilibrium positions. There can be more than one such position, in which equilibrium, can be stable, and in other can't be stable.

If at the equilibrium position, the function of force of the conservative holonomic scleronomic system, according to Lejevn-Dirichlet's stability theorem, have a maximum (potential energy have a minimum) then the equilibrium position of the system is stable, in opposite it is unstable or indifferential.The criterion of stability towards that can be written:

$$\frac{\partial E_p}{\partial \varphi_i} = 0 \ ; \qquad \frac{\partial^2 E_p}{\partial \varphi^2} = \begin{cases} > 0 \ \text{stable} \\ = 0 \ \text{indefferential} \\ < 0 \ \text{unstable} \end{cases} \tag{6}$$

In our case is:

$$\frac{\partial E_p}{\partial \varphi_i} = 0 \quad \text{for} \quad \varphi_1 = \frac{\pi}{4}, \frac{5}{4}\pi, \frac{9}{4}\pi, ...$$
$$\varphi_h = 0, \pi, 2\pi, ...$$

towards this the first derivative of potential energy is equal to zero under the different position of mass-particles Figure 3.

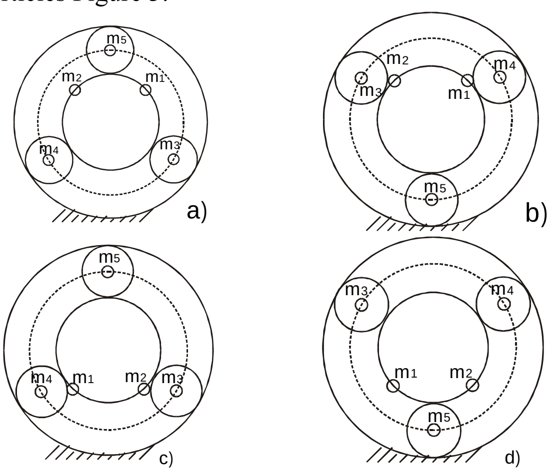

Fig 3. Masses Configuration of equilibrium positions of the planetary reductor dynamic model

For this positions of mass-particles the characters of equilibrium positions are shown in table 1.

Table 1. The positions of mass-particles the characters of equilibrium positions

Case	φ_1	φ_h	On condition	$\frac{\partial^2 E_p}{\partial \varphi_i^2}$	character of the the equilibrium position
I Picture 3a	1/4π	0	$m_5>m_4+m_3$	<0	unstable
			$m_3+m_4>m_5+\sqrt{2}\ (m_1+m_2)r_1$	>0	stable
			$m_3+m_4=m_5+\sqrt{2}\ (m_1+m_2)r_1$	$=0$	indifferential
II Picture 3b	1/4π	π	$m_5<m_4+m_3$	<0	unstable
			$m_5>m_4+m_5+\sqrt{2}\ (m_1+m_2)r_1$	>0	stable
			$m_5=m_4+m_5+\sqrt{2}\ (m_1+m_2)r_1$	$=0$	indifferential
			$m_5=m_4+m_3$	<0	unstable
III Picture 3c	5/4π	0	$m_5<m_4+m_3$	>0	stable
			$m_5>m_4+m_5+\sqrt{2}\ (m_1+m_2)r_1$	>0	unstable
			$m_5=m_4+m_5+\sqrt{2}\ (m_1+m_2)r_1$		indifferential
			$m_5=m_4+m_3$	>0	stable
IV Picture 3d	5/4π	π	$m_5>m_4+m_3$	>0	stable
			$m_3+m_4>m_5+\sqrt{2}\ (m_1+m_2)r_1$	<0	unstable
			$m_3+m_4=m_5+\sqrt{2}\ (m_1+m_2)r_1$	$=0$	indifferential

4. Local geometric analysis of dynamics of planetary reductor

Modern ideas about nonlinear dynamics are often presented in geometric terms or graphical presentation in the form of pictures. For example, the motion of system can be presented in the phase plane $(\varphi, \dot{\varphi})$. In these graphical presentations, time isn't implicit, but a time process is used to describe the motion of the system on phase trajectories with the change of phase coordinates.

More generally if we want to examine stability of the nonlinear systems, before all, we have to determine the equilibrium positions of the corresponding conservative system, and then consider the motion of the nonlinear system around every equilibrium position. By changing the value of parameters of the dynamic system the number of equilibrium positions and it's stability changes. The local motion of one nonlinear system in a neighborhood of equilibrium position is characterized with characteristics typical for the linearized systems.

Conservative system is very often a rough approximation of real nonlinear systems, which can only give the satisfying results to a certain point (see [7] and [8]). The important characteristic property of conservative systems is that the total energy of the system is constant, and does not depend on motion time.

The kinetic energy of adopted model of the planetary reductor in the gravitational field, presented on the Figure 2, is:

$$E_k = \frac{1}{2}[I_1\dot{\varphi}_1^{\,2} + (m_1+m_2)r_1^{\,2}\dot{\varphi}_1^{\,2} + 3M_2 r_h^{\,2}\dot{\varphi}_h^{\,2} + (m_3+m_4+m_5)r_h^{\,2}\dot{\varphi}_h^{\,2} + 3I_2\dot{\varphi}_2^{\,2}] \quad (7)$$

where are: I_1,I_2 - the axiall moments of masses inertia of the sun gear and a satellite, r_1,r_h -

the basic radiuses of sun gear and support of the satellite, $\dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_h -$ angles velocity of sun gear, of satellite and support of the satellite.

We can make the change

$$\dot{\varphi}_2 = i_{21}^{h}\dot{\varphi}_1 + i_{2h}^{1}\dot{\varphi}_h \tag{8}$$

where are i_{jk} - transmitted relations (reductor ratio).

Transmitted relations team gear of examined reductor are 4,42 and with consideration on that, we can make exchange $\varphi_1 = 3{,}42\varphi_h$. After these two exchanges we get that the expression for the kinetic energy is:

$$E_k = \frac{1}{2}[11{,}696 I_1 + 11{,}696(m_1 + m_2) r_1^2 + 3M_2 r_h^2 + (2m_{34} + m_5) r_h^2 + \\ + 35{,}1 I_2 \left(\frac{r_1}{r_2}\right)^2 + 3I_2 \left(\frac{r_h}{r_2}\right)^2 + 20{,}52 I_2 \frac{r_1 r_h}{r_2^2}] \dot{\varphi}_h^2 \tag{9}$$

The basic parameters of this planetary reductor are: $r_1 = 4{,}0375$cm, $r_2 = 4{,}675$cm, $r_h = 8{,}7125$cm, $M_2 = 4{,}246$kg, $I_1 = 0{,}2728$Ncms^2, $I_2 = 0{,}46398$Ncms^2, $I_3 = 17{,}9$Ncms^2, $m_1 = 0{,}5$kg, $m_2 = 0{,}4$kg, $m_3 = 0{,}3$kg, $m_4 = 0{,}2$kg, $m_5 = 0{,}1$kg.

For these parameters we get the finally total energy of the system and it is:

$$E = 0{,}2363\dot{\varphi}_h^2 + 0.198[1 + \cos(3{,}42\varphi_h)] + 0{,}1584[1 + \sin(3{,}42\varphi_h)] + \\ + 0{,}0855(2 - \cos\varphi_h) + 0.2563(1 + \cos\varphi_{h1}) \tag{10}$$

For initial conditions the constant of the integration is:

$$E = \frac{1}{2} I \dot{\varphi}_0^2 + E_p(\varphi_0) \tag{11}$$

and it is constant for all the time of motion, until coordinate and velocity always have to satisfy that the total energy of the system is constant, and does not depend on motion time. Phase trajectories in phase plane present curve of constant energy of the system. Singular points are on the places where the phase velocity is equal to zero.

As in the singular points forces Q_k is equal to zero, we can see that the singular points of integral curves are showing in the points with the x-axis φ_1 for which the potential energy have extremal values. This we can see considering Figures 4, 5 and 6 and Figures 15, 16 and 17.

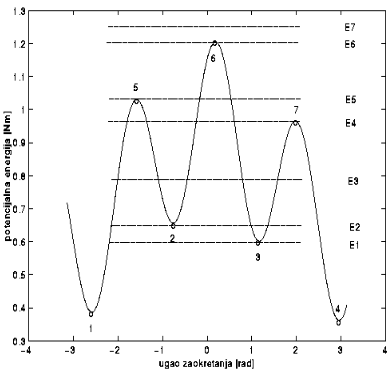

Fig. 4. Potential energy in the function of the generalized coordinate of a planetary reductor

On a Figure 4 points 1, 2, 3, 4 and 5 - responding to the stable equilibrium positions and on phase portrait (Fig. 6, points 1, 2, 3, 4 and 5) are singular points of the type of stable center, and points 6, 7, 8 and 9- responding to the unstable equilibrium positions and in a phase plane (Fig. 6, points 6, 7, 8 and 9) are the singular points of the type of unstable saddle.

On a Figures from 7 to 17, the surface of the total energy given with expression (7) and integral curve (phase tracks) in a phase space, in the Descartes rectangular's coordinate system (φ,φ,E)

for different initial conditions and all total energy of the system which make possible motion, are presented.

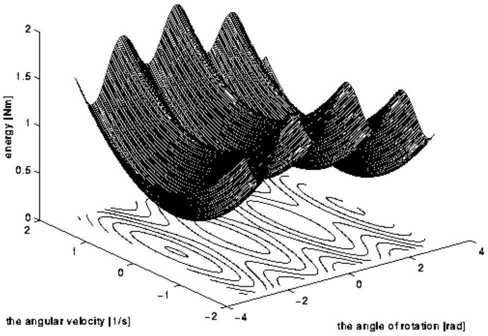

Fig. 5. The surface of total system energy in a exstended phase space (φ,φ,E) of a planetary reductor

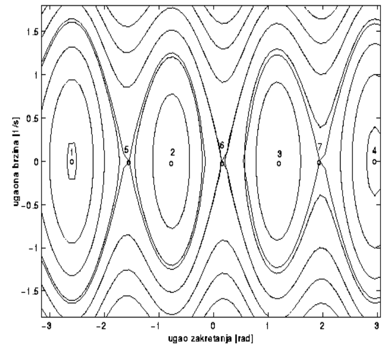

Fig. 6. Family of curves of the constant energy of a planetary reductor

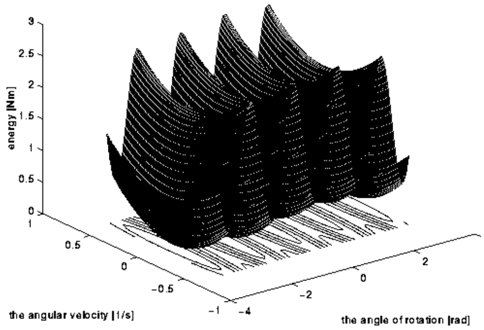

Fig. 7. The surface of total system energy in a exstended phase space (φ,φ,E) of a planetary reductor (A)

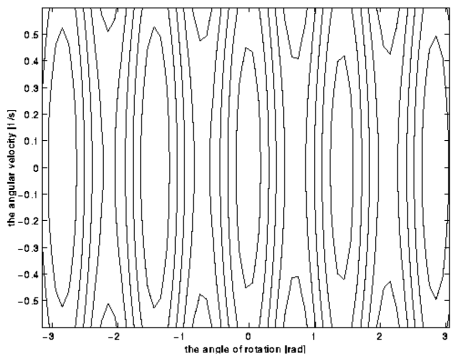

Fig. 8. Phase portrait and constant total energy curves of the nonlinear dynamics of a planetary reductor (A)

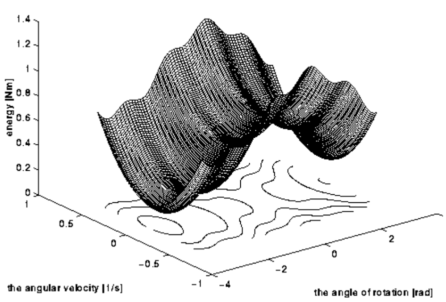

Fig. 9. The surface of total system energy in a exstended phase space (φ,φ,E) of a planetary reductor (B)

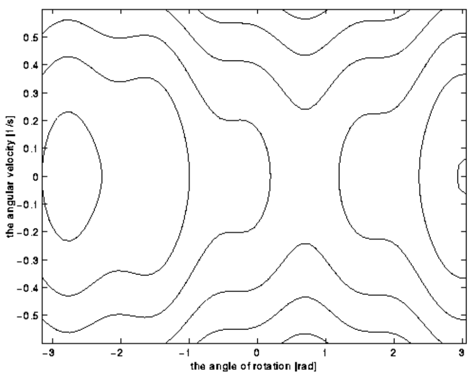

Fig. 10. Phase portrait and constant total energy curves of the nonlinear dynamics of a planetary reductor (B)

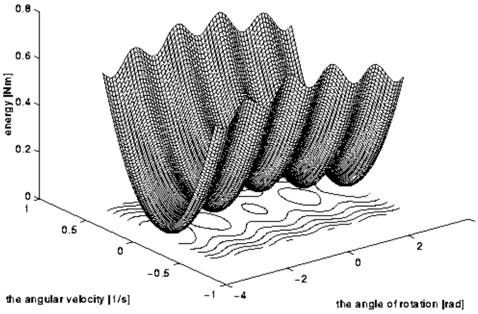

Fig. 11. The surface of total system energy in a exstended phase space (φ,φ,E) of a planetary reductor (*C*)

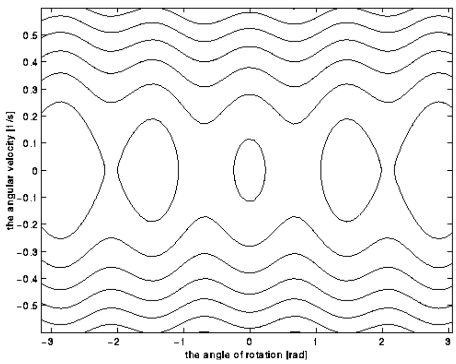

Fig. 12. Phase portrait and constant total energy curves of the nonlinear dynamics of a planetary reductor (*C*)

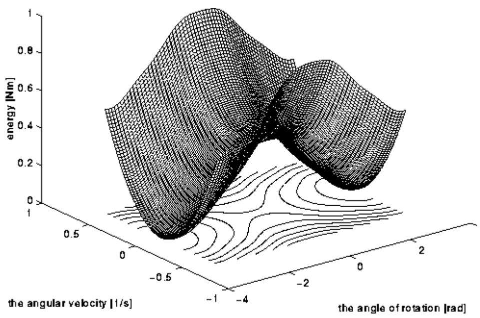

Fig. 13. The surface of total system energy in a exstended phase space (φ,φ,E) of a planetary reductor (*D*)

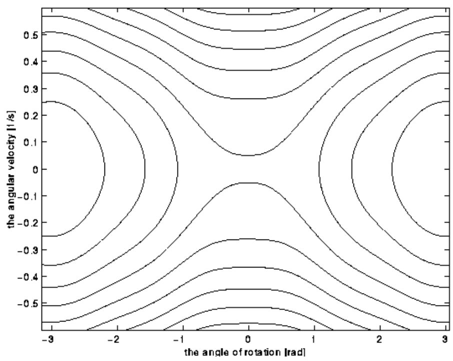

Fig. 14. Phase portrait and constant total energy curves of the nonlinear dynamics of a planetary reductor (*D*)

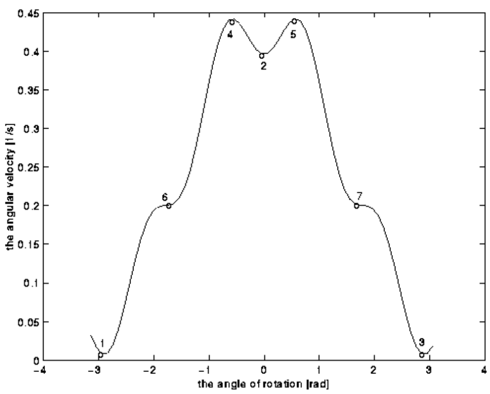

Fig. 15. Potential energy in the function of the generalized coordinate of a planetary reductor (*E*)

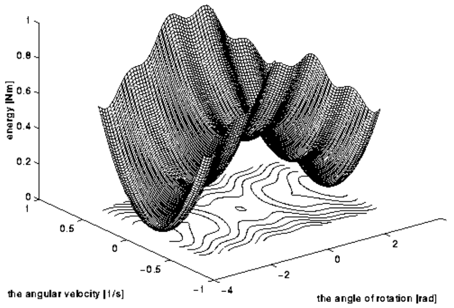

Fig. 16. The surface of total system energy in a exstended phase space (φ,φ,E) of a planetary reductor (E)

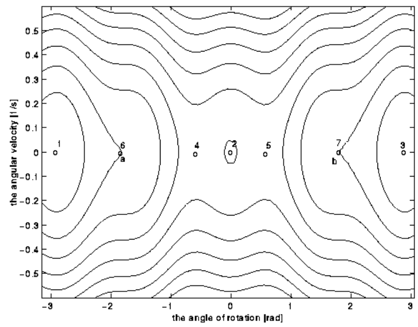

Fig. 17. Phase portrait and family of curves of the constant energy of a planetary reductor (E)

On a Figure 15 points 1, 2, 3 - correspond to stable equilibrium positions and on a phase portrait (Figure 17, points 1, 2, 3) are singular points of the center type, and a points 4, 5 - correspond to unstable equilibrium positions and in a phase plane (Figure 17, points 4, 5) are singular points type of instable saddle, points 6 and 7 correspond to unstable equilibrium positions, and in a phase plane (Figure 17, points 6, 7) are the places with peaks on the phase trajectory.

The Figure 7 and 8 was obtained for the parameters $m_1 = 0{,}1$kg, $m_2 = 0{,}2$kg, $m_3 = 0{,}3$kg, $m_4 = 0{,}4$kg, $m_5 = 0{,}5$kg; Figure 9 and 10 for $m_1 = 0{,}5$kg, $m_2 = 0{,}4$kg, $m_3 = 0{,}03$kg, $m_4 = 0{,}02$kg, $m_5 = 0{,}01$kg; Figure 11 and 12 for $m_1 = 0{,}1$kg, $m_5 = 0{,}05$kg; Figure 13 and 14 for $m_1 = 0{,}5$kg, $m_5 = 0{,}01$kg; Figure 15, 16 and 17 for $m_1 = 0{,}5$kg, $m_3 = 0{,}03$kg, $m_4 = 0.02$ for this parameters we get the curve of the constant energy on a Figure 15 too.

On Figures from 4 to 17 it's shown that if function of potential energy E_0 has minimum, than closed integral curves go over in singular point, which is a stable center. When the $E > E_0$ then we have closed integral curves which are around the singular point type of stable center. This singular points are stable center, because the potential energy in that point has a minimum (Figure 4 and Figure 17), and because of that these closed phase trajectories around stable equilibrium positions give a periodic motion.

On places where E have a maximum, integral curves have four *branches (sectors),* which are going through the singular point type of saddle, and the x-axis of which corresponds to the x-axis of generalized coordinates of maximum potential energy. Then we are talking about asymptotic motion through that homoclinic point corresponding to the unstable equilibrium position or about aperiodic unstable motion.

When the function of potential energy has a ridge point in which the first and second derivatives are equal to zero, in other words, when the tangent on the curve of constant energy is "horizontal" in that point, so that an integral curve in a phase plane as we can see has a top (peak) on x-axis which corresponds to ridge point. This is presented on a Figure 17 and the equilibrium positions to which these points (a) and (b) correspond are unstable.

5. CONCLUDING REMARKS

With the change of sizes of mass's disorders of the dynamic system the number of equilibrium positions and their stability changes. This change of nonlinear systems, which is the consequence of changing parameters of the system, is object of theory of the bifurcation. That meaning of parameters, which make the changes of qualitative and topologic characteristics of motion, are named critical or bifurcation values.

With change of values of disturbed parameters m_i the point of equilibrium position (Fig. 13) decomposes on four points (Figure 5). On "dynamic language" the unique center is going over to two saddles in a central coordinate ($\varphi_i = 0$) and two centers. The bifurcations of that kind have names, and we can find, in the russian literature, bifurcations type "VIL".

With different values of parameters we can see the potential energy is with two isolated points and the changes of their positions (Fig. 5, 7 and 11). For different parameters values there are curves with peak in which the equilibrium position is unstable (Fig. 16).

If we want to have small oscillations around the equilibrium position, and if we want to have real supposition about small oscillations we need that potential energy in an equilibrium position has minimum and it can be the stable equilibrium position.

According to the position "of disorders" which are presented in cases Figure 3 we can see that beside stable equilibrium positions we have both unstable and indifferent positions, which don't depend only on position "of the mass's disorders" of planetary reductor, but also on value relations of some parameters.

It depends on initial conditions whether the motion will be periodic, and what will be the character of phase trajectories.

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STRUKTURNA STABILNOST FAZNOG PORTRETA NELINEARNE DINAMIKE PLANETARNOG PRENOSNIKA

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Rezultati numeričkog eksperimenta nad nelinearnim dinamikom planetarnog prenosnika su prikazani u ovom radu. Svojstva faznog portreta nelinearne dinamike planetarnog prenosnika u oblasti singularnih tačaka tipa sedla su izučavana i svetlu dinamičkih položaja ravnoteže sistema. Za studiranje strukturne stabilnosti faznog portreta nelinearne dinamike planetarnog prenosnika korišćena je analiza stacionarnih tačaka krivih potencijalne energije, ukupne energije sistema, kao i odgovarajuće grafičke prezentacije površi ukupne energije sistema, i njihove transformacije na promene parametara planetarnog prenosnika.

Iz analize su izvedeni zaključci o promeni kvalitativnih svojstva i osetljivosti strukturne stabilnosti faznog portreta na male promene nekih parametara sistema.

Ključne reči: *Nelinearna dinamika, planetarni reduktor, fazni portret, položaji ravnoteže, strukturna stabilnost, numerički eksperiment, površ ukupne energije sistema, singularne tačke, homokliničke trajektorije.*