FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol.1, N° 7, 2000, pp. 879 - 886

LINEARIZATION AND SOLVING OF DIFFERENTIAL MOTION EQUATIONS OF CRANE DRIVING MECHANISMS

UDC 621.87

Zoran Marinković, Saša Marković, Dragan Marinković

Mechanical Engineering Faculty University of Niš
Str. Beogradska 14, 18000 Niš, Yugoslavia

Abstract. *The work deals with discrete elasto-kinetic model of finite mass number and its system of non-homogenous differential second order equations modelling successfuly crane driving mechanisms. Generally speaking the model parametres, i.e. inertia characteristics, elastic bonds (damping, rigidity and gap) and external impulses (electromotor, brakes, movement resistance etc.) are non-linear. It was proved that these characteristics in practical estimations are being linearized, i.e. changed with constant values or piecewise linear characteristics. By this for practical analysis and estimation cases there is no significant influence on results accuracy, which was confirmed by comparive certain simulations with experimental recording.*

1. THE INTRODUCTION

Cranes as complex transport machines, in accordance with their application, construction and location, deal with not only the carrying structure and managing equipment but also a certain number crane mechanisms to be able to perform certain operations and movements for load space transfer. Basically, these mechanisms can be grouped into four groups: for load lifting, for translatory movement, for circular movement (rotation) and for the reach change. For the description and analysis of these mechanisms different models and certain approaches to their work simulation are being used, that also depend on the complexity and kinetic mechanism structure, research aims and the required accuracy of the obtained results. One of the aims of the research and crane driving mechanisms work analysis are movement rules of the specific masses and load change within the elastic bonds between the masses. One mechanism movement is usually being observed that consists of acceleration period, stagnant movement and braking.

The adequate results in these analysis are given by the equivalent elasto-kinetic model with a finite number of discrete masses, elastic bonds between them and external

Received August 15, 2000
Presented at 5th YUSNM Niš 2000, Nonlinear Sciences at the Threshold of the Third Millenium, October 2-5, 2000, Faculty of Mechanical Engineering University of Niš

disturbances. As these masses in the transitional working regimes i.e. in acceleration and braking periods, perform forced oscillatory movement their abstract mathematic model represents a system of non-homogenous differential second order equations.

In the work this type of model with its characteristics is analysed, where they are by the rule non-linear. It was shown that they can be linearized in practical estimations with an aim of a simplified and easier mathematic model, and at the same time not to influence the accuracy of the received results.

2. ELASTO-KINETIC MODEL AND ITS CHARACTERISTICS

Complex real crane driving mechanisms are by modelling being replaced with an equivalent model, that most often in the dynamic analysis represents an oscillatory chain with a finite number of discrete masses (fig. 1) [1]. This is an elasto-kinetic model with n mass known inertia characteristics (a_i) and n-1 elastic bonds defined with damping characteristics (b_1), rigidity (c_1) and gap (Δ_{zi}). Certain masses are affected by external disturbances (Q_{si}) that derive from electromotor or brakes depending on the work regime within one movement (acceleration period, stagnant movement and braking) and on movement resistance. This type of model can be torsic, linear or mixed, which depends on whether it is consisted of only rotating, only translatory or both of the masses in accordance with the drive mechanism purpose and modelling purposes, i.e. research.

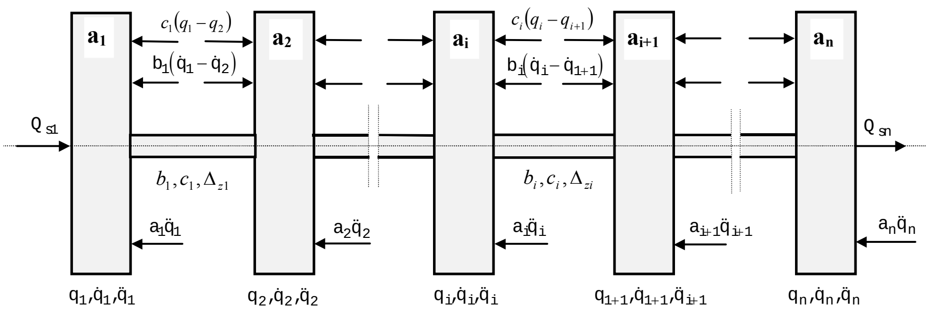

Fig. 1. Equivalent-oscillatory model with n degree freedom (unbound chain system)

Modelling also involves setting of an abstract mathematic model for a mechanism movement as the masses are performing small forced damping oscillations in the transitional work regimes (acceleration and braking periods), the simplified mathematic simulation leads to the system of non-homogenous differential second order equations, presented in the matrix shape as [1]:

$$\| a \| \cdot \{\ddot{q}\} + \| b \| \cdot \{\dot{q}\} + \| c \| \cdot \{q\} = \{Q_s\} \ , \tag{1}$$

where:

$\|a\|$ - the matrix of $n \times n$ size the inertia system characteristic, i.e. diagonal mass matrix and inertia translatory (m_i) and rotative (J_i) mass moments,

$\|b\|$ - the matrix $n \times n$ size the damping characteristics (linear b_{li} and torsic b_{ti}),

$\|c\|$ - the matrix rigidity characteristics (linear c_{li} and torsic c_{ti}) analogue to the previous,

$\{Q_s\}$ - generalized external impulses vector, i.e. moments M_i and forces F_i, that derive from electromotor, brakr and movement resistance $Q_{si} = f(\dot{\varphi})$, $Q_{si} = f(t)$ or Q_{si} = const.,

$\{\ddot{q}\}, \{\dot{q}\}, \{q\}$ - generalized acceleration, speed and center mass positions (linear x_i and torsic φ_i generalized coordinates) vectors.

The majority of inertia characteristics at the crane driving mechanisms have a constant value (m_i and J_i coefficient inertia). However with changing the elevation arrow inertia and load are decreased with the distance square in contrast to the crane axe ($m_i \cdot r^2$). Therefore the dynamic analysis of these mechanisms is becoming greatly complicated.

The characteristics of the elastic bonds between the concentrated masses, i.e. damping, rigidity and gap are charactarized with non-linearity. The damping-friction resistance ($Q_{\mu i} = M_{\mu i}$ or $Q_{\mu i} = F_{\mu i}$) can be presented in fig. 2 and analytical descriptions depending on the relative speed, as [1]:

$$Q_{\mu i} = -b_i \cdot \Delta\dot{q}^n \cdot (sign\,\dot{q}), \quad (2)$$

where:

b_i - is damping coefficient for i elastic bond, which is constant,

$\Delta\dot{q}$ - relative mass movement speed in between there is elastic bond,

n - exponent n = 0, 1, 3 for a constant, linear and non-linear change of damping resistance,

sign $\dot{q}$ - multiplier with +1 values when the speed is $\dot{q} > 0$ and −1 when $\dot{q} < 0$.

In the analysis it is usually accepted that there are only resistance to viscose friction (in proportion to the speed n = 1) and that damping characteristic (coefficient b_i) always has a constant value, i.e. $Q_\mu = b_i \cdot \Delta\dot{q}$.

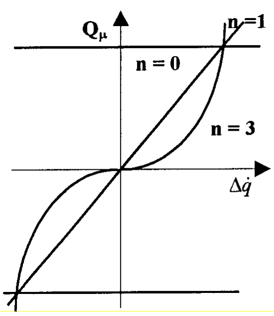

Fig. 2. Resistance damping (friction) change presentation

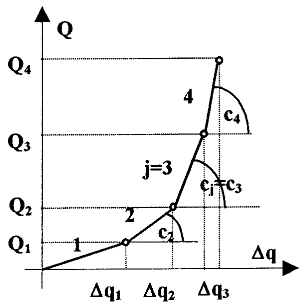

Fig. 3. Broken regressive load change

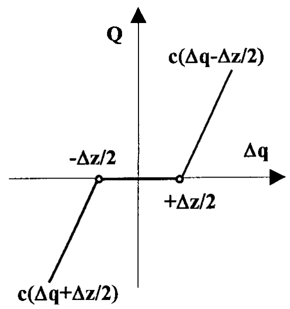

Fig. 4. Piecewise linear gap characteristic

Rigidity characteristics c and gap Δz of the elastic bond are characterized by extreme non-linearity. In order to get simple mathematic models (linear differential movement equation) these non-linear characteristics are represented by a few linear dependances, i.e. are replaced with a broken line (partial linear characteristic) as it is presented in fig. 3 and 4.

Fig. 3 shows loads change Q in the elastic bond depending on the non-linear regressive (increasing) rigity characteristic ($c_{j+1} > c_j$). However, this dependancy can be shown by a degressive (declining $c_{j+1} < c_j$) characteristic. The influence of this two non-linearity types of elastic bond to the load value and deformation is different. In the first case with deformation increase load increases rapidly, where in the second the increse is slown to great extent. This fact can be used in cases of load decrease need by the application with degressive characterisric (for example elastic links wth rubber parts etc.). The first case could be founds at mechanisms for load lifting because of the steel rope, the rigity of which pepends on its cength that changes at load lifting and landing. The analytic load dependancy Q with known values of partial rigidity coefficient c_j = const., according to fig. 3 is described in the following relation [1]:

$$Q = c_j \cdot \Delta q + Q_{j-1}(1 - c_j / c_{j-1}) \text{ for } j = 1, 2, 3, ... \quad (Q_0 = 0, c_0 = 0). \tag{3}$$

Fig. 4 represents the load change Q in the elastic bond depending on gap Δ_z (parctial-linear characteristic), with an analytic descrition [1, 2]:

$$Q = \begin{cases} c \cdot (\Delta q + \Delta_z/2) & for \quad \Delta q \le -\Delta_z/2 \\ 0 & for \quad -\Delta_z/2 \le \Delta q \le +\Delta_z/2 \\ c \cdot (\Delta q - \Delta_z/2) & for \quad +\Delta_z/2 \le \Delta q \end{cases} \tag{4}$$

External disturbances have an extreme non-linear characteristic that originate from asynchronous electromotors (further motor) in the acceleration period, i.e. during its setting in drive until achieving stationary work regime. Figure 5 and 6 show these changes, respectively, for the sliding-ring motor that sets in drive with more lines by the help of resistors and in the electrical circuit of the rotator and the cage motor that is directly set in drive (a natural characteristic). These changes are given in angle function speed ($\alpha = M_M/M_N = f(\dot{\varphi})$ or $M_N = f(\dot{\varphi})$). In the first case the moment change is skipped (so called "saw diagram") which can be inconvenient from the aspect of element load of crane driving mechanisms and in the other it is continual. As it has been stated many times before, here is linearization done by the exchange of non-linear line parts of setting asynchronous motors with rectilinear segments. This is especially obvious at the last, fifth line of the sliding-ring motor (fig. 5) and the whole change of the cage motor (fig. 6) each, i.e. j segment with previously stated pictures is analytically described with line equation with two points, with known coordinates, such as [1,2,3,4]:

$$\begin{aligned} M_{Mj}(\dot{\varphi}) &= \frac{\alpha_{j+1} - \alpha_j}{\dot{\varphi}_{j+1} - \dot{\varphi}_j} M_n \cdot \dot{\varphi} + \left(\alpha_j - \frac{\alpha_{j+1} - \alpha_j}{\dot{\varphi}_{j+1} - \dot{\varphi}_j} \dot{\varphi}_j \right) M_n = \\ &= \frac{\Delta\alpha_j}{\Delta\dot{\varphi}_j} M_n \cdot \dot{\varphi} + \left(\alpha_j - \frac{\Delta\alpha_j}{\Delta\dot{\varphi}_j} \dot{\varphi}_j \right) M_n = k_{Mj} M_n \cdot \dot{\varphi} + p_{Mj} M_n , \end{aligned} \tag{5}$$

where:

$\alpha_j, \dot{\varphi}_j$ and $\alpha_{j+1}, \dot{\varphi}_{j+1}$ - coordinates of the starting (j) and final (j+1) point on the j-line (segment),

$k_{Mj} M_n = (\Delta\alpha_j / \Delta\dot{\varphi}_j) M_n$ - slope coefficient j line with $\dot{\varphi}$ - axe ($\Delta\alpha_j = \alpha_{j+1} - \alpha_j$, $\Delta\dot{\varphi}_j = \dot{\varphi}_{j+1} - \dot{\varphi}_j$),

$p_{Mj} M_n = (\alpha_j - k_{Mj}\dot{\varphi}_j) M_n$ - segment of j line on $\alpha = M_M / M_n$ - axe, (M_n - the called motor moment).

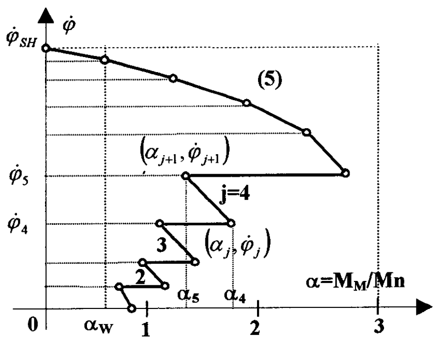

Fig. 5. "Saw diagra" of the sliding ring motor

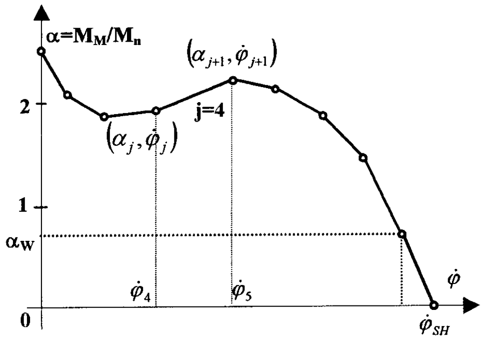

Fig. 6. The natural characteristic of the cage motor

The development of the braking moment is most often modelled as a constant value. However, in these analysis temporal functions are being used i.e. exponential for electromagnetic brake and the limited function of the slope ascent for the hydraulic brake. Moreover, for the movement resistance some constant values are accepted. With this modelled brake influence and movement resistance, mathematic models are not significantly complicated and it is possible to get their solutions in a closed shape [1,2].

3. Torsic two mass model, movement equations and simulations

The researches have shown that for determine and analysis of the element movement law (discrete masses) and shaft load - elastic bonnds of the masses (Fig. 1) at crane driving mechanisms torsic elasto-cinetic model can be used successfully wth two deoree freedom movement (Fig. 7) [1,2,3,4]. It is reseived whenth characteristics of the real system are reduced to thr drive mechanism shaft on which torsion moment change is asked $M_t = T(t) = f(t)$. Consists of drive mass with inertia moment J_1 before the reduction spot, driven mass J_2 behind the reduction spot and elastic bond (referential shaft) beween mass defined characteristis: rigidity c_1, damping b_1 and gap φ_z. The characteristics of the model are by the procedure linearized, i.e. these are constant values coefficients. Furthermore, by this model will be considered and simulated only the acceleration period of the crane driving mechanisms wth asynchronous motors. The matematic mass movement model under motor influence $M_1 = M_M = (\dot{\varphi})$ along the j-line (Fig. 5 and 6), to which movement resistance is opposed $M_2 = -M_W = \text{const.}$, is [1,2,3,4]:

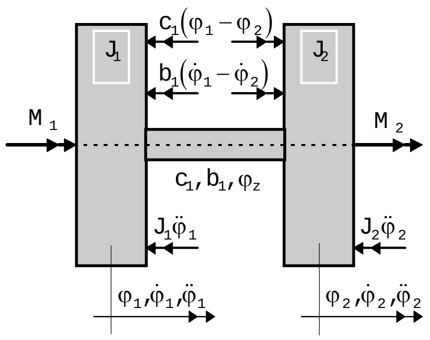

Fig. 7. Elasto-kinetic model with two masses

$$\begin{aligned} J_1\ddot{\varphi}_{1j} + b_1(\dot{\varphi}_{1j} - \dot{\varphi}_{2j}) + c_1(\varphi_{1j} - \varphi_{2j}) = M_1 = M_{Mj}(\dot{\varphi}_{1j}) = k_{Mj}M_n \cdot \dot{\varphi}_{1j} + p_{Mj}M_n \\ J_2\ddot{\varphi}_{2j} - b_1(\dot{\varphi}_{1j} - \dot{\varphi}_{2j}) - c_1(\varphi_{1j} - \varphi_{2j}) = M_2 = -M_W = const. \end{aligned} , \quad (6)$$

where beside the explained values:

$\varphi_{ij}, \dot{\varphi}_{ij}, \ddot{\varphi}_{ij}$ - generalized angle road coordinates, speed and acceleration of drive (i = 1) and driven (i = 2) mass for j - line the setting motor moment.

The solution of the system from two plain non-homogenous linear differential second order equations for the known starting conditions gives the movement rules of each mass $\varphi_{1j} = f_{1j}(t)$ and $\varphi_{2j} = f_{2j}$ (t). However, as the change of the twisting moment of the elastic bond between masses $M_t = T(t) = c_1\cdot(\varphi_1 - \varphi_2) = c_1\cdot\Delta\varphi(t)$, is being observed it is, by help of adequate transformation system to come to the following non-homogenous linear differential third order equation of deformation change $\Delta\varphi(t) = (\varphi_1 - \varphi_2)$ [1,2,3]:

$$A_{0j} \cdot \Delta\dddot{\varphi}_j(t) + A_{1j} \cdot \Delta\ddot{\varphi}_j(t) + A_{2j} \cdot \Delta\dot{\varphi}_j(t) + A_{3j} \cdot \Delta\varphi_j(t) = B_{0j}, \quad (7)$$

wehere the following replacements are introduced:

$$A_{0j} = 1,\ A_{1j} = 2\delta_0 - \rho_{1j},\ A_{2j} = \omega_0^2 - 2\delta_2\rho_{1j},\ A_{3j} = -\rho_{1j}\omega_2^2,\ B_{0j} = \rho_{1j}h_2 \text{, i.e.}$$

$$\begin{aligned} 2\delta_0 = b_1/J_1 + b_1/J_2,\ \omega_0^2 = c_1/J_1 + c_1/J_2,\ \rho_{1j} = k_{Mj}/J_1, \\ 2\delta_2 = b_1/J_2,\ \omega_2^2 = c_1/J_2,\ h_2 = M_2/J_2 = -M_W/J_2 \end{aligned} .$$

This type of differential equations for (7) the known starting conditions can be solved in the closed shape by Laplac transformation method [1] or numeric Runge-Kutta method [2]. The general solution in the closed shape, i.e. the twisting moment change of the elastic bond (referentialshaft of the crane driving mechanism) $M_{tj} = T_j(t) = c_1\cdot\Delta\varphi_j(t)$, at the motor moment development along j-line will be [1,2,3,4]:

$$\begin{aligned} M_{tj} = T_j(t) = c_1 \cdot \Delta\varphi_j(t) = M_{1j} + M_{2j} \cdot \exp(\alpha_{0j}(t - t_{0j}))\ \big| + \\ + M_{3j} \cdot \exp(\beta_{0j}(t - t_{0j})) \cdot \cos(p_{0j}(t - t_{0j}) - \psi_{0j}), \end{aligned} \quad (8)$$

where:

M_{1j}, M_{2j}, M_{3j} - constant, that depend on the model characteristics, external spars and starting conditions [1],

$\alpha_{0j} = k_{Mj}M_n / J_R$, $J_R = J_1 + J_2$,

$\beta_{0j} = -(b_1J_R / 2J_1J_2 - \alpha_{0j}J_2 / 2J_1) = -(\delta_0 - \delta_{Mj})$, $p_{0j} = (\omega_0^2 - \beta_{0j}^2)^{1/2}$,

ψ_{0j} - phase angle and t_{0j} - starting time.

The received twisting moment change $M_t = T(t)$ (8) consists of two parts:low and high-frequency components. The first one is up to the middle value of the moment around which the second oscillates (the product e and cos function). Highfrequency component is harmonious change with the adequate overall damping β_{0j}, that depends on the system damping b_1, the setting motor line slope $k_{Mi}M_n$ and on the inertia moment of driven and drive mass i.e. from their relation (J_2/J_1).

To learn the effect of the suggested linearization differential equation movement, especially the one that is applied at the setting moment of the asynchronous motor, an algorithm is made and two computer programs for crane driving mechanisms simulation work is developed. One of the programmes takes the ready solutions of the moment in the closed shape (8) [1], and the other uses Runge-Kutta method [2] for solving a differential equation.

Fig. 8 shows a parallel simulation and experimental recording of the twisting moment change of the referential shaft of a laboratory device in the acceleration period under the influence of the sliding-ring motor with five setting lines (fig. 5) [1,3]. Fig. 9, also shows this change at the output shaft of the crane mechanism for movement wth cage motor (fig. 6) [1,4]. The first change is slopped, and the second is continual for the type of asynchronous motor. The comparison of simulations and experimental recording in both cases confirms its big coincidence, which is sufficient for practical application. By this it is justified and popularized the methodology of linearization of dynamic model characteristics (the asynchronous motor setting line).

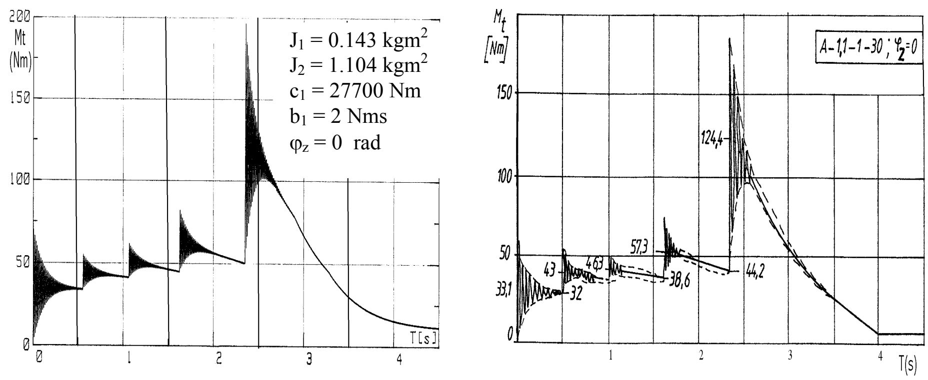

Fig. 8. Simulation and experimental recording of the twisting moment change $M_t = T(t)$ of the referential shaft of a laboratory device in the acceleration period with sliding-ring motor [1,3]

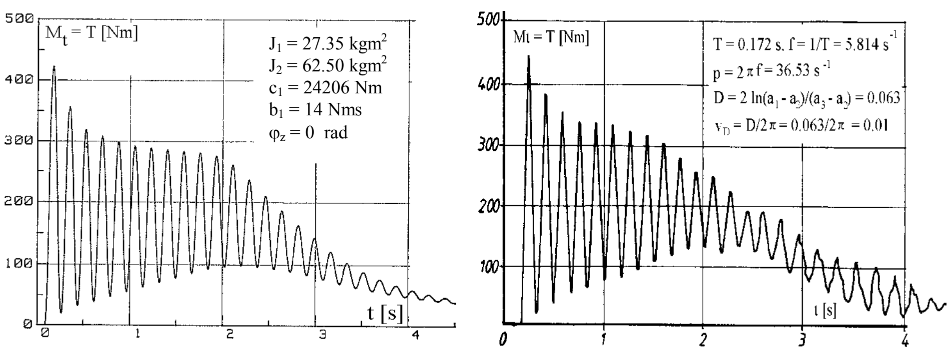

Fig. 9. Simulation and experimental recording of the twisting moment change $M_t = T(t)$ of the output shaft of the crane mechanism in the acceleration period with cage motor [1,4]

4. Conclusion

The basic conclusion is that complex elasto-kinetic model, that describes and simulates the work of crane driving mechanism, can be simplified by applying the procedure of linearization of its characteristics. This is especially valid for the impulses, i.e. in case of representing the asynchronous motor setting line by partial-linear characteristics.Justification and affirmation of this procedure is approved by appropriate simulation and experiments, whose accordance is great and satisfying for practical analysis and calculations.

References

1. Marinkovic Z.: *Probable-statistical model of the life-time calculation of the crane driving mechanism elements*, (in Serbian), Ph. D. thesis, Mechanical Faculty University of Nis, Nis, 1993.
2. Markovic S.: *The research of the influence of the gap size on the dynamic loads at crane mechanisms*, (in Serbian), Master thesis, Mechanical Faculty University of Nis, Nis, 1999.
3. Marinkovic Z., Mijajlovic R., Jovanovic M., Markovic S., Djordjevic T.: *Modeling and smulation of acceleration periods at lifting driving mechanisms with sliding rings*, (Part one and two), XIV International conference on material handling and warehousing, Proceedings, Belgrade, 1996., pp 2.37 ÷ 2.42 and pp 4.48 ÷ 4.87.
4. Marinkovic Z., Jevtic V.: *Modeling and simulation of operation of crane mechanisms with cage electric motors*, (in Serbian), Tehnika–Masinstvo 49 (2000) 1, pp M9 ÷ M15.

LINEARIZACIJA I REŠAVANJE DIFERENCIJALNIH JEDNAČINA KRETANJA POGONSKIH MEHANIZAMA KRANOVA

Zoran Marinković, Saša Marković, Dragan Marinković

Rad se bavi diskretnim elasto-kinetičkim modelom sa konačnim brojem masa i sistemom nehomogenih diferencijalnih jednačina koje uspešno modelišu pogonske mehanizme kranova. Uopšteno govoreći parametri modela, tj. inercione karakteristike, elastične veze (prigušenja, krutosti i zazori) i spoljašni poremećaji (elektromotor, kočnice, otpora kretanju) su nelinearni. Iz tog razloga ove karakteristike treba da budu linearizovane ili zamenjene konstantnim vrednostima po intervalima (tzv. stepenasti ili "stepwise" profil). Time se omogućava rešavanje jednačina kao i analiza praktičnih situacija bez značajnije greške što je za neke slučajeve potvrđeno upoređenjem sa eksperimentom.