FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol.1, N^{o} 7, 2000, pp. 871 - 878

NONLINEAR MATHEMATICAL MODEL OF THE CONDENSING STEAM TURBINE

UDC 621.134.5:519.87

Dragoljub Živković

Faculty of Mechanical Engineering, Beogradska 14, 18000 Niš
E-mail: dzivkovic@masfak.masfak.ni.ac.yu

Abstract. *In this paper a nonlinear mathematical model of the condensing steam turbine, which gives essential new possibilities compared with linear models, is presented. This model enables: researches of work regimes of the steam turbines in wide range of the loading, making simulators for education working staff, analysis starting, stopping and accidental regimes and solving other problems.*
Nonlinear mathematical model is built of some turbine element models (turbine rotor, regulating and stopping valve, steam rooms, group of stages) and steam reheater, in structural way. The above mentioned elements are presented like objects with disposed and concentrated steam parameters. Mathematical model is presented by the system of: nonlinear partial hyperbolic differential equations, ordinary nonlinear differential equations and algebraic equations.
System of equations is solved by method of characteristics and method of finite differences. Original software is developed for numerical simulation. The results of the simulations of the steam-turbine work regimes after protection system action and during impulsive response are presented in this paper.

1. INTRODUCTION

The last development of the steam turbines had two primary purposes: increases the power and efficiency. These purposes are entirely realized because the maximum values of the power and efficiency are attained.

The research work of the dynamic characteristics of the steam turbines in unsteady operations represents one of the most important tasks of the modern theory of turbomachines. The transient work regimes of steam turbines occur under the normal exploitation - during the start-up and stopping period, for the primary and secondary regulation of the power of the electric power system, and accident conditions [1,3,4].

Received August 15, 2000
Presented at 5th YUSNM Niš 2000, Nonlinear Sciences at the Threshold of the Third Millenium, October 2-5, 2000, Faculty of Mechanical Engineering University of Niš

Under these work regimes, the complicated thermal and hydrodynamic processes in fluid become evident - appearing and expanding of the pressure waves, condensation, evaporation, the expanding of front phase crossing and so on. There is an interaction between the fluid and hard structure of the steam turbine itself at the same time. During this, there might come to the production of such a great mechanical and thermal load of the turbine endangering its security [5,9]. Based on the above reasons, the analysis of transient work regimes of steam turbines represents one of the most important tasks of the contemporary theory of thermal turbomachines.

In this paper a nonlinear mathematical model of dynamic behavior of steam turbines, applicable for studying unsteady operation is presented. The model is a structural one based on the models of individual steam turbine elements connected it each other through the control borders with defined boundary conditions. The existing methods for numerical simulations are improved by taking into consideration the disturbing pressure waves of high amplitudes, caused by the valve action, and accumulative capability of wet steam rooms.

Like reference object is used the steam turbine Alsthom Atlantique 308.5 MW, at 5-th block of the thermal electric power station "Nikola Tesla" - A in Obrenovac installed.

2. Mathematical Model of the Condensing Steam Turbine

The beginning step in the steam turbines behaviour study the simplified mathematical model, which incorporates the relevant features of the turbine, has to be generated. The physical boiler-turbine system is complex and to gain insight into the behavior of the turbine this complexity must be reduced. The present objective is to examine only the turbine behavior and to regard the boiler as having infinite capacity, i.e. constant pressure and temperature is maintained regardless of the mass flow of steam demanded [6,10,12]. Other assumptions are:

- the condenser, like the boiler, has infinite capacity;
- the turbine is governed by a single throttle valve.

The model chosen to represent the steam turbine is shown in Fig.1 [13]. It consists of a throttle valve, followed by:

- the loop pipe between the throutle valve and the inlet nozzle of the turbine cylinder (1), length of pipe is L_1;
- group of stages HPT;
- reheat boiler pipe (3), length of reheat boiler pipe is L_2;
- groups of stages MPT and LPT;
- steam rooms (2, 4, 5, 6, 7, 8a, 8b, 9a 9b).

Mathematical model of the steam turbine is complex and can be presented by the following equations [11,13]:

- Differential conservation equations which describe processes in loop pipe (1) and reheat boiler pipe (3);
- Equations describing steam flow through the turbine stages;
- Differential equations which describe accumulative capability of the steam rooms;
- System equations of the initial and boundary conditions.

Differential conservation equations are:

- Continuity equation: $$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial x}+\rho\frac{\partial u}{\partial x}+\frac{\rho}{A}\left(\frac{\partial A}{\partial t}+u\frac{\partial A}{\partial x}\right)=0\,, \tag{1}$$

- Momentum equation: $$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{2f_{tr}}{D}\frac{u^3}{|u|}=0\,, \tag{2}$$

- Energy equation; $$\frac{\partial}{\partial t}\left(i+\frac{u^2}{2}\right)+u\frac{\partial}{\partial x}\left(i+\frac{u^2}{2}\right)-\frac{1}{\rho}\frac{\partial p}{\partial t}-\dot{q}-\frac{p}{\rho A}\frac{\partial A}{\partial t}=0\ . \tag{3}$$

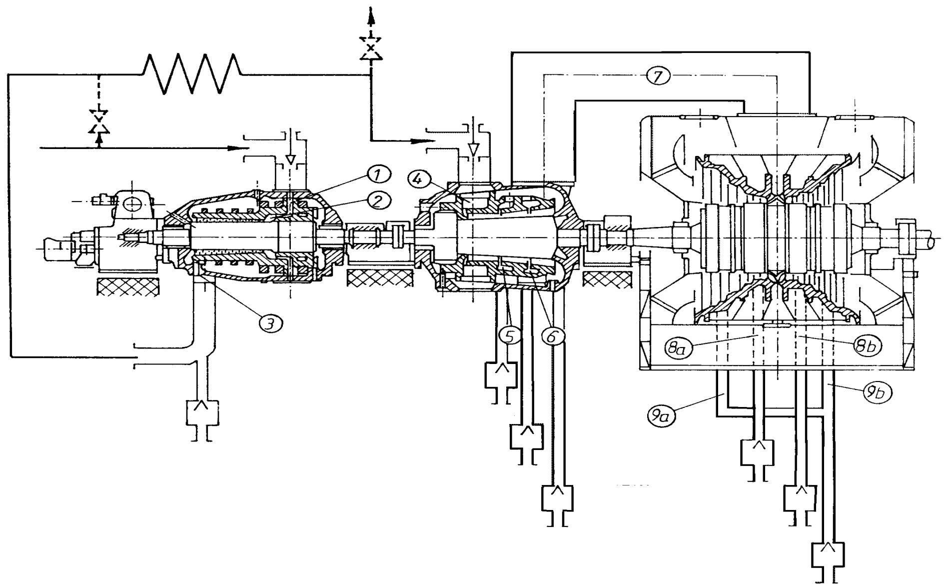


Fig. 1. Meridian section of the steam turbine CEM-Alsthom Atlantique 308.5 MW

Steam flow through the group of turbine stages [6,7,8,10]:

$$\frac{\dot{M}}{\dot{M}_0}=\sqrt{\frac{{\varepsilon_0}^2-{\varepsilon_z}^2-\sigma^{gr}(\varepsilon_0-\varepsilon_z)^2}{1-{\varepsilon_{z0}}^2-\sigma^{gr}(1-\varepsilon_{z0})^2}}\sqrt{\frac{1}{\tau_0}}\ , \tag{4}$$

where:

$$\varepsilon_0=\frac{p_0}{p_{00}};\ \ \varepsilon_{z0}=\frac{p_{z0}}{p_{00}};\ \ \varepsilon_z=\frac{p_z}{p_{00}};\ \ \tau_0=\frac{T_0}{T_{00}};\ \ \sigma^{gr}=\frac{{\varepsilon_*}^{gr}}{(1-{\varepsilon_*}^{gr})};$$

p_0, T_0, p_{00}, T_{00} - pressure and temperature at inlet of group of stages for any and nominal work regime;

p_z, p_{z0} - pressure at outlet of group of stages for any and nominal work regime;

$$\varepsilon_*^{gr} = \frac{\varepsilon_*^{st}(1-\varepsilon_*^{st})\varepsilon_{z0}}{\sqrt{\varepsilon_{st}^2(1-\varepsilon_*^{st})^2-(\varepsilon_{st}-\varepsilon_*^{st})^2(\varepsilon_{st}^2-\varepsilon_{z0}^2)}} \quad , \tag{5}$$

where:
ε_{st} - pressure ratio for the last stage and nominal regime;
ε_*^{st} - critical pressure ratio for the last stage;
ε_*^{gr} - critical pressure ratio for the group of stages.

Mathematical model must to take into account influence of the regenerative heaters of the water:

$$\frac{d\pi_j}{dt} = \frac{n}{M_{i0}}\pi_j^{\frac{n-1}{n}}(\dot{m}_{j-1}-\dot{m}_j-\dot{m}_E), \tag{6}$$

where: M_{i0} - mass of the steam in the volume "j"; π_j - pressure ratio p_j/p_{j0} ;
$\dot{m}_{j-1}$ - mass inflow to "*j*" steam room; $\dot{m}_j$ - mass outflow from "*j*" steam room;
$\dot{m}_E$ - mass flow of the subtracted steam.

Two boundary conditions that are important to the present analysis are those set by the throttle valve and by the nozzle. A relationship between particle and sonic velocities and throttle valve geometry can be derived by manipulation of the continuity equation, the momentum equation and the state equation of the steam. The boundary condition presented to the pipe by the turbine can be expected to be very similar to that presented by the simple nozzle. This is because the turbine consists of a series of nozzles cascaded one after the other. The simple nozzle has the effect of limiting the maximum permissible particle velocity (u) in the pipe. In fact, the value of u in the pipe just before the nozzle is practically independent of the pressure drop across the nozzle. The value of u set by the turbine can be found quite simply in practice and is equal to the particle velocity in the pipe under steady state conditions. This can be determined from the rate of mass-flow and the steam density in the pipe, all of which are known.

2.1. Accumulative possibility of the steam areas

Under this term is understood the capability of the steam area that the energy, accumulated in the steam, hot liquid and metal walls, can be used for evaporation of the liquid which is there. The new steam in the next sequence carries out additional operation [2,14].

The mass of one accumulative volume is:

$$\frac{dM_j}{d\tau} = \dot{m}_{j-1}-\dot{m}_j-\dot{m}_E \ , \tag{7}$$

where: $dM_j/d\tau$ - unsteady article of variation of steam mass in the volume "*j*" (Fig. 1).

The total accumulative action of one steam-water volume consists of accumulative action of: steam, hot liquid and metal parts. Differential equation for the total accumulative action is:

$$\frac{dM_j}{dp} = \frac{dM_p}{dp}+\frac{dM_{PW}}{dp}+\frac{dM_{PM}}{dp} \, , \tag{8}$$

where: $\frac{dM_P}{dp}, \frac{dM_{PW}}{dp}, \frac{dM_{PM}}{dp}$ - accumulative action of steam, hot water and metal parts,

$$\frac{dM_j}{dp} > 0 \text{ for } dp < 0.$$

For the change of steam mass due to pressure change:

$$\frac{dM_P}{dp} = -V_P \frac{d}{dp}\left[\frac{1}{v(p)}\right]. \tag{9}$$

Taking into consideration that in the volume with damp steam the value of polytropic exponent is nearly 1,0 (Bürkner, [2]), equation (9) gets this expression:

$$\frac{dM_P}{dp} = -\frac{V_P}{p_0 v_0}, \tag{10}$$

where: V_P - volume of steam area,
p_0, v_0 - pressure and specific volume in the steam area.

From energy balance at the pressure reduction at accumulative area, the differential equation gets this expression:

$$dM_{PW} \cdot r = M_W \cdot di', \tag{11}$$

where: M_W - mass of water.

By differentiation equation (11), we have:

$$\frac{dM_{PW}}{dp} = -\frac{M_W}{r(p)} \frac{di'(p)}{dp}, \tag{12}$$

where: M_W - water mass in the steam area.

Accumulative action of the metal walls can be expressed as:

$$\frac{dM_{PM}}{dp} = -\frac{1}{r(p)} \cdot m_M \cdot c_M \cdot \frac{dt_s(p)}{dp}, \tag{13}$$

where: m_M - mass of the metal,
c_M - average specific metal heat,
t_s - condensation temperature.

By the equation (8) – (13):

$$\frac{dM_j}{d\tau} = \left\{-\frac{V_P}{p_0 v_0} - \frac{M_W}{r(p)} \frac{di'(p)}{dp} - \frac{1}{r(p)} \cdot m_M c_M \frac{dt_s(p)}{dp}\right\} \frac{dp_j}{d\tau}, \tag{14}$$

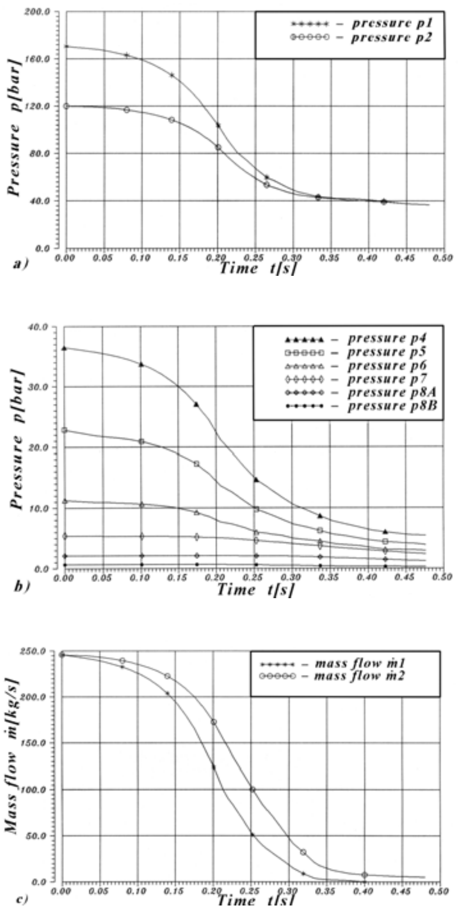
Differential equations, which describe accumulative capability of the steam rooms, are:

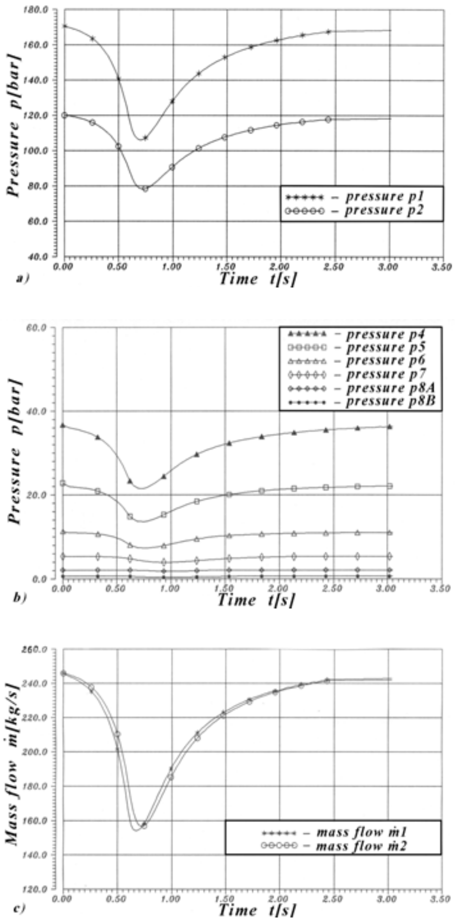
$$\frac{dp_j}{d\tau} = \frac{\dot{m}_{j-1} - \dot{m}_j - \dot{m}_E}{\left\{\frac{V_P}{p_0 v_0} + \frac{M_W}{r} \frac{di'}{dp} + \frac{1}{r(p)} m_M c_M \frac{dt_S(p)}{dp}\right\}_j}, \tag{15}$$

where: m_M - mass of the metal,
c_M - mean heat capacities,
V_P - volume of the steam room,
p_0, v_0 - pressure and specific volume,
M_W - mass of the water in the steam room.

3. THE RESULTS OF THE NUMERICAL SIMULATIONS

The computation has been done by using the method of characteristics and method of finite differences. The results of the numerical simulations of the turbine transient work regimes, after protection system action and during impulsive response, are presented at Fig. 2 and 3.





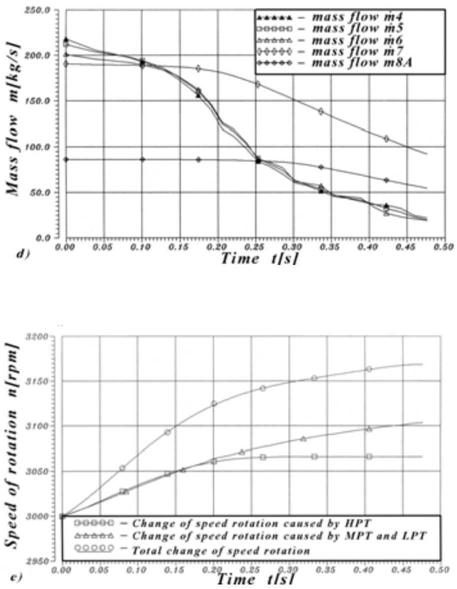


Fig. 2. The simulation results of the turbine work regime after protection system action

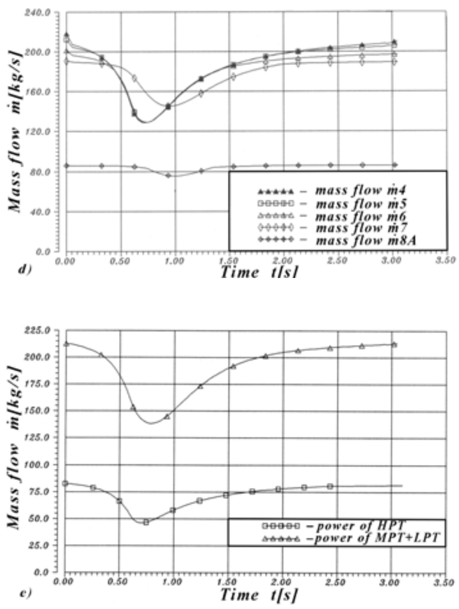


Fig. 3. The simulation results of the impulsive characteristic of the turbine

4. CONCLUSION

The main conclusions are:

- Pressure and mass-flow changes in the volumes of the high and middle-pressure turbines (HPT and MPT), after the valve action, are very fast. That means that HPT and MPT have good maneuver characteristics;
- Pressure and mass-flow changes in the volumes of the low-pressure turbine (LPT) are slower than in the volumes of HPT and MPT. That means that LPT has worse maneuver characteristics comparing with HPT and MPT;
- Accumulative possibilities of the steam areas effect slower pressure changes which is in the part of low pressure more obvious;
- The turbine responds faster to throttle valve openings than closures;
- The general response time of the turbine is roughly proportional to the magnitude of the throttle valve movement;
- For large disturbances the time-lag introduced by the reheater is too large to allow rapid control of the total turbine torque, and under these circumstances it would be preferable for the interceptor valve (in front of MPT) to move in unison with the throttle valve;
- Dynamic characteristics of the steam turbines after protection system action and during impulsive response are convenient.

REFERENCES

1. Bell R.D., Rees N.W., Lee K.B., *Models of large boiler-turbine plant*, "IFAC Symp. 1977. Autom. Control and Prot. Electr. Power Syst., Melbourne, 1977. Prepr. Pap.", Sydney, 1977., p.469-474.
2. Bürkner R., *Beitrag zur Berechnung des Überdrehzahl verhaltens von Sattdampf-turbosätzen bei Lasteinsenkung oder Lastabschaltung*, "Energietechnik", (6), 1975., p.247-254.
3. Condrac E.I., *Digital computer modelling of turbomachinery*, "2-nd Eur. Congr. Fluid Mach., Oil, Petrochem. and Relat. Ind. Conf., The Hague, 26-28 March, 1984.", p.93-98.
4. Drucks G., *Rechnerische simulation des dynamischen verhaltens eines kraftwerksblockes nach turbinenschnellschluss*, "VGB Kraftwerkstechn.", 1980., 60, № 1, p.23-27.
5. Grunwald B., Lewandowski J., Miller A., Plewa J., *Model matematyczny turbiny na pare, nasycona do badania dynamiki turbozespolu elektrowni jadrowej*, "Biul. inf. Inst. techn. ciep.", 1980., № 57, p.3-18.
6. Kirillov I.I., Avtomaticheskoe regulirovanie parovih turbin i gazoturbinnih ustanovok, "Mashinostroenie", Leningrad, 1988.
7. Kousal M., *Spalovaci turbíny*, Praha, SNTL, 1980.
8. Miller A., *Metoda wyznaczania krytycznego stosunku cisnien grupy stopni turbinowych*, "Archiwum Budowy Maszyn", (4), 1973., p.597-605.
9. Miller A., Lewandowski J., Grunwald B., *The mathematical model of condensing turbine for saturated steam installed in a nuclear power station*, "Pr. Inst. masz. przepl. PAN", 1976., № 70-72, p.477-490.
10. Samojlovič S.G., Trojanovsky M.B., *Peremennie i perehodnie re`imi v parovih turbinah*, "Energoizdat", Moskva, 1982.
11. Sheard G., Evans F.J., *Transient torque of steam turbines in power systems*, The Institution of Mechanical Engineers, Proceedings 1970-71, Volume 185, (27`71), London, p.641-653.
12. Traupel W., Thermische turbomaschinen, "Springer", Berlin, 1966.
13. Živković D., *Matematički model dinamičkog ponašanja parnih turbina nakon dejstva sistema zaštite*, YUTERM '90., Neum 1990., p.733-742.
14. Živković D., Laković S., Stefanović V., The Analysis of the Effects of Accumulative Capability of Steam Rooms on the Dynamic Characteristics of Wet Steam Turbines, CHISA '93., Prague, 1993.

NELINEARNI MATEMATIČKI MODEL KONDENZACIONE PARNE TURBINE

Dragoljub Živković

U radu je dat nelinearni matematički model kondenzacione parne turbine, koji omogućuje analizu rada u širokom opsegu opterećenja, simulaciju u edukacione svrhe, analizu starta, zaustavljanja i ispada turbine kao i mnogih drugih scenarija.

Nelinearni matematički model je struktuiran na bazi modela za pojedine elemente turbine i sadrži nelinearne hiperbolične parcijalne diferencijalne jednačine, obične nelinearne diferencijalne jednačine i algebarske jednačine.

Sistem jednačina je rešen metodom karakteristika i metodom konačnih razlike. Na bazi toga bila je moguća simulacija rada parne turbine pri raznim scenarijima.