

SYNTHESIS PROCEDURE OF PLANAR BAR LINKAGES IN INFINITESIMALLY CLOSE POSITIONS

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Abstract. *This paper presents a procedure of numerical method for kinematic synthesis of planar bar linkages in two or three infinitesimally close positions. The presented method has been used for resolving one practical case of mechanism for generating of prescribed function. The synthesis of mechanism is performed indirectly using inverse mechanism.*

1. INTRODUCTION

There are three customary tasks for kinematic synthesis: motion, path and function generation. *Motion generation* or *rigid body guidance* (Fig. 1a) requires that an entire body be guided through a prescribed motion sequence. The body to be guided usually is a part of "floating link" (not directly connected to the fixed link). The corresponding input (driving) link motion may or may not be prescribed.

In *path generation* (Fig. 1b) a point of a floating link is to trace a part defined with respect to the fixed frame of reference. If the path points are to be correlated with either time or input link positions, the task is called path generation with prescribed timing.

In *function generation* (Fig. 1c) the motions of input and output (driven) link are correlated by the prescribed function. Since any real mechanism has a finite number of dimension parameters it is not possible in general to obtain a mathematical exact solution but that the mechanism match given function, path or body positions at only a finite number of positions called *accuracy* or *precision points*. Between these points generated (actual) function $\Phi(x)$ deviates from the given (prescribed) mathematical function $F(x)$. Fig. 2a is a graph of arbitrary function $y = F(x)$. The kinematic synthesis task may be to design a linkage to correlate input or output such that as the input link moves by x , the output link moves by $y = F(x)$ for the range $x_0 < x < x_{n+1}$. Values of the independent parameter x_1, x_2, \dots, x_n correspond to the prescribed accuracy points P_1, P_2, \dots, P_n on the

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function $F(x)$ for the range x . The subscript j indicates the j -th prescribed position of mechanism; the subscript 1 refers to the *first* or starting prescribed position.

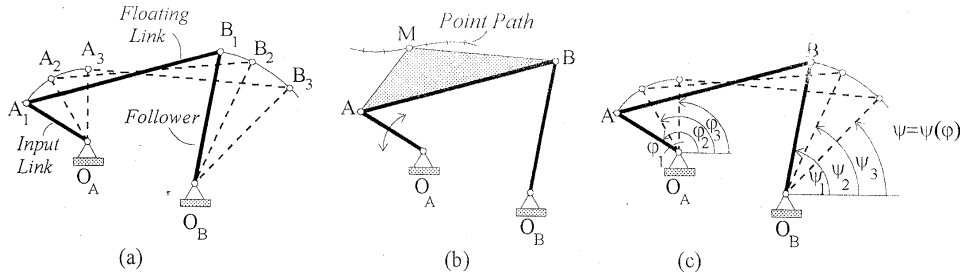


Fig. 1.

Structural error is defined as the difference between the prescribed (ideal) function $F(x)$ and actual function $\Phi(x)$ for a certain value of the input variable x (Fig. 2b)

$$\Delta y = F(x) - \Phi(x) \tag{1}$$

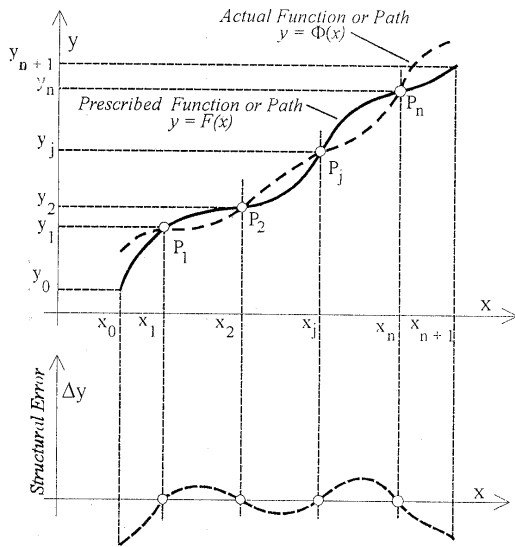


Fig. 2.

Structural error for path generation may be defined as the vector from the ideal to the actual path. Substitution of prescribed function by similar (approximate) function is called *approximation of function*. There are three methods of approximation: *first-order* approximation or *point approximation*, *higher-order* or *multiple point approximation* and combined *point-order approximation*. In first-order approximation, discrete points on the prescribed function are specified. The synthesised mechanism will generate a function (or path) that will coincide with the prescribed function at accuracy points P_1, P_2, \dots, P_n (Fig. 2). Structural error at these points will be zero, that is

$$\Delta y = F(x_j) - \Phi(x_j) = 0, \quad j = 1, 2, \dots, k \tag{2}$$

In some cases a mechanism is desired to generate not only a position but also the velocity, acceleration shock, and so on, at one accuracy point. In that case prescribed and actual function must have *higher-order of contact*¹. Two curves $F(x)$ and $\Phi(x)$ have n -th order of contact if they coincide at $n + 1$ infinitesimally close points A_1, A_2, \dots, A_{n+1}

¹ higher-order approximation of given function

represented by one *multiple accuracy point A*.

Two infinitesimally close points on the curve define the first derivative (velocity in Mechanics, slope of tangent in Mathematics), three infinitesimally close point define second-order derivative (acceleration or radius curvature), four points-third derivative (shock or rate of curvature change), and so on (see Fig. 3a). Regarding to this, the following conditions are to be fulfilled for a given multiple point $A(x_A, y_A)$:

$$\begin{aligned}
 F(x_A) &= \Phi(x_A) \\
 F'(x_A) &= \Phi'(x_A) \\
 &\vdots \\
 F^{(n)}(x_A) &= \Phi^{(n)}(x_A)
 \end{aligned}
 \tag{3}$$

that is

$$\begin{aligned}
 \Delta y(x_A) &= 0 \\
 \Delta y'(x_A) &= 0 \\
 &\vdots \\
 \Delta y^{(n)}(x_A) &= 0
 \end{aligned}
 \tag{4}$$

Moving away from multiple accuracy point, structural errors $\Delta y', \Delta y'', \dots, \Delta y^{(n)}$ start to increase, slightly in the beginning and then rapidly (Fig. 3b).

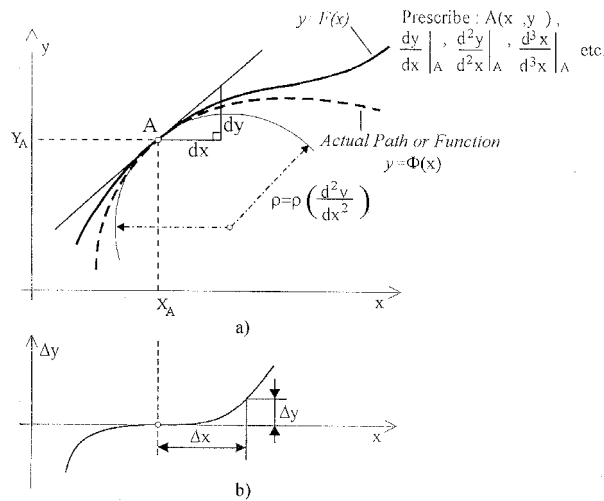


Fig. 3.

The combination of both point and order approximations is called point-order approximation. For example one might desire to prescribe a position and velocity at the accuracy point, only a position at a second point, and a position and acceleration at a third point.

2. CURVATURE THEORY

2.1. Osculating circle

Consider a general curve σ (Fig. 4a) and point A_0 on it. For finding the radius of curvature of σ at point A_0 , we take two points A_{-1} and A_1 on either side of A_0 . The perpendicular bisectors of the secants $A_{-1}A_0$ and A_0A_1 (Fig. 4a) intersect at a point O . With O as the centre and OA_0 as the radius we can always draw a circle passing through the 3(three) points A_{-1} , A_0 and A_1 . Now, let points A_{-1} and A_1 approach point A_0 ($\Delta\sigma = A_{-1}A_0 = A_0A_1 \rightarrow 0$). In the limiting case, the circle passing through three points as $\Delta\sigma \rightarrow 0$ is called *circle of curvature* or *osculating circle*. The radius of the osculating circle, ρ (Fig. 4b), is the *radius of the curvature* of curve σ at point A_0 and the centre of the osculating circle, O , is the *centre of curvature*. Thus the osculating circle has contact with curve σ at three (at least) infinitesimally closed points. The vector \vec{OA}_0 is called the *radius vector* of curvature at A_0 and is designated by $\vec{\rho}$.

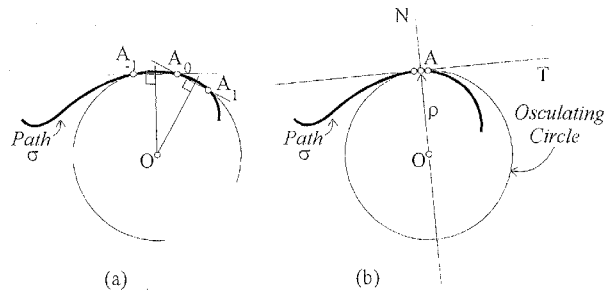


Fig. 4.

2.2. Polodes

Actual motion of a floating link with respect to the fixed (ground) link may (for analytical purposes) be replaced by the rolling motion of a moving polode against a fixed polode without sliding. The fixed polode p_Σ is connected to ground link and is always stationary, while the moving polode p_Π may be thought of as being rigidly attached to the moving link. The polodes may have different shapes such as ones shown in Fig. 5.

As the moving polode rolls on the fixed polode there is generally a single point of contact between them; this point is the instantaneous velocity pole P of the moving link with respect to ground.

So-called "relative polodes" can be constructed for the motion of any two links

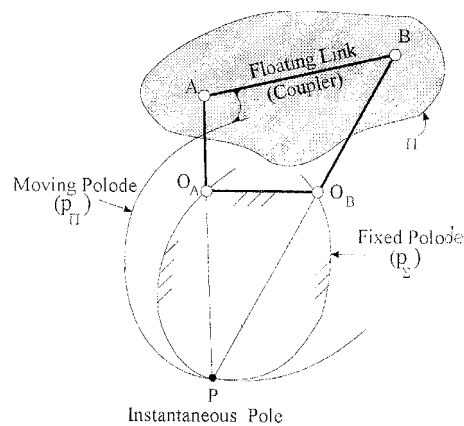


Fig. 5.

with respect to each other, i.e. not just for motion of a floating link such as the coupler AB of a four-bar linkage with respect to ground, but also for follower $O_B B$ with respect to the input crank $O_A A$.

2.3. Pole velocity

Fixed and moving polodes have one point in common, which is *instantaneous pole* P (Fig. 6). This is always the case, i.e., at any position of the moving plan Π associated with the floating link, so this common point travels in the fixed plane Σ along the fixed polode, and in the moving plane along the moving polode (Fig. 6a). The *pole velocity* \vec{u} is the instantaneous velocity with which the instantaneous pole P shifts position. So, the pole velocity is not the velocity of any given point (or material particle) but instead expresses how the instantaneous pole shifts along the fixed polode p_Σ .

Since pole velocity along the fixed polode equals that along the moving polode, fixed and moving polode have a common tangent t at P at any point of time (Fig. 6b). This common tangent is called the *pole tangent* of the observed position. Positive sense of the pole-tangent t is always opposite with respect to sense of vector \vec{u} .

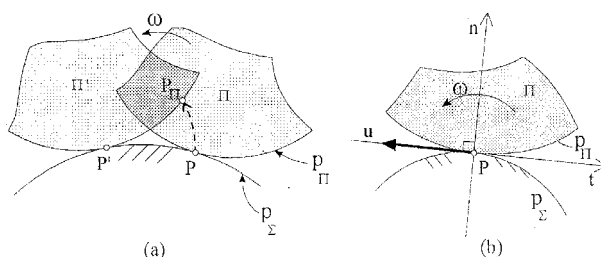


Fig. 6.

2.4. Polodes for the relative motion of the cranks in a four-bar function generator

For synthesis of functions generators (Fig. 1c) we regard the function generator as a special case of the guiding mechanism (Fig. 1a) using method of *kinematic inversion* about first position of input crank.

To this end, the input crank 1, Fig. 7 is made stationary (becomes ground link), ground link 4 becomes input crank whereas output crank (follower) 3 becomes floating link (coupler). The motion of link 3 is then constrained by rolling contact between the polodes for the relative motion of the two cranks.

Angular velocities of the inverted mechanism are

$$\begin{aligned}
 \omega_1' &= 0 \\
 \omega_2' &= \omega_2 - \omega_1 \\
 \omega_3' &= \omega_3 - \omega_1 \\
 \omega_4' &= -\omega_1
 \end{aligned} \tag{5}$$

According to Fig. 8

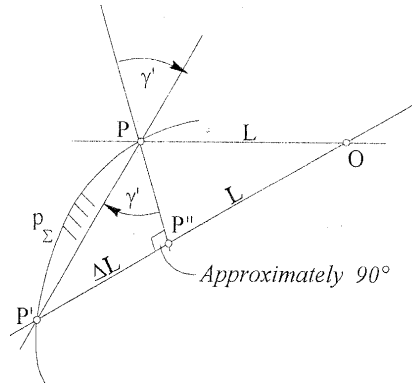


Fig. 8.

$$O\bar{P}'' = O\bar{P}$$

$$\Delta L \cong L \Delta \varphi \operatorname{tg} \lambda'$$

$$\Delta \varphi \rightarrow 0, \gamma' \rightarrow \gamma, \angle PP''P \rightarrow 90^\circ$$

$$dL = L d\varphi \operatorname{tg} \gamma$$

$$\frac{dL}{d\varphi} = L \operatorname{tg} \gamma = \frac{\lambda}{\lambda - 1} \rho_C \operatorname{tg} \gamma \tag{11}$$

Equating equations (10) and (11) we obtain the angle \$\gamma\$ between a perpendicular to the fixed link \$OC\$ and pole tangent \$t\$ (or between the fixed link \$OC\$ and pole normal \$n\$), Fig. 7

$$\begin{aligned} \frac{-\frac{d\lambda}{d\varphi} \rho_C}{(\lambda - 1)^2} &= \frac{\lambda \rho_C}{\lambda - 1} \operatorname{tg} \gamma \\ \gamma &= \operatorname{arctg} \frac{-\frac{d\lambda}{d\varphi}}{\lambda(\lambda - 1)} \end{aligned} \tag{12}$$

2.5. Inflection circle

Generally, all paths generated by the points of the moving plane \$\Pi\$ have a distinct radius and centre of curvature at every point of the path.

We define an *inflection point* as a point of the plane \$\Pi\$, which at the moment is going through a point of inflection of its path. Such points will have infinite radius of curvature (\$\rho = \infty\$) and zero normal acceleration (\$a_N = V^2/\rho = 0\$). The path of every inflection point as a point of the moving plane has second-order contact (three infinitesimally close points contact) with its path *circle of curvature* or *path osculating circle*, which is of zero curvature (i.e., of infinite radius), in other words, it is a straight line, coincident with the

path tangent T (Fig. 9). This property of the inflection point is often used by designers to design path generators for tracing an approximate straight line with three infinitesimally close accuracy points.

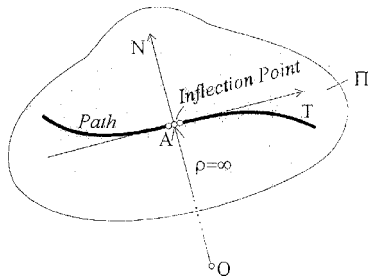


Fig. 9.

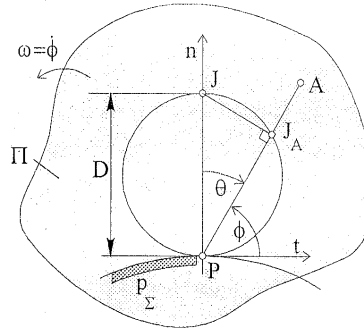


Fig. 10.

The instantaneous locus of all inflection points in the moving plane Π is a circle of diameter D (Fig. 10) touching the pole tangent t at the instantaneous pole P . This circle is called the *inflection circle*. The point J , the intersection of the normal at P (pole normal) and the inflection circle is called the *inflection pole*. The second intersection of the line PA with the inflection circle is also the projection of PJ onto PA . We may then write

$$r_J = D \cos \theta \tag{13}$$

where $r_J = PJ, D = PJ$.

Equation (13) is the scalar (polar) equation of the inflection circle.

2.6. Euler – Savary equation

Euler-Savary equation (ESE) allows exploitation of the properties of the inflection circle. Consider a plane Π moving with respect to a fixed plane Σ and let p_Σ and p_Π be, respectively, the fixed and the moving polodes of the motion.

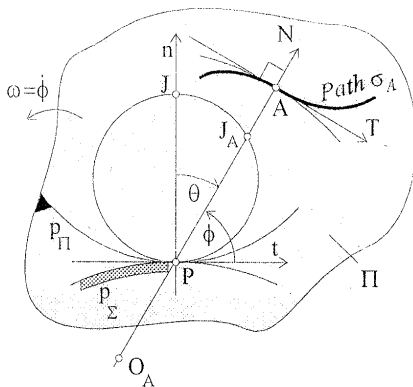


Fig. 11.

In Fig. 11 consider the following four points on the path normal N :

1. P , the instant pole
2. A , an arbitrary point of the moving plane
3. J_A , the inflection point
4. O_A , the centre of curvature of the path of A , described in fixed plane.

Also define the following three vectors all collinear with the line PA :

$$\vec{r} = P\vec{A}, \quad \vec{r}_J = PJ_A, \quad \vec{p}_A = O_A\vec{A}$$

The ESE correlates these vectors and thus provides a way to find any one of the four points

P , A , A_J , and O_A if the other three are known.

The ESE may be derived in several ways. Here it is one very effective.

The total acceleration of A may be written as

$$\vec{a}_A = (\vec{a}_A)_N + (\vec{a}_A)_T \quad (14)$$

or as

$$\vec{a}_A = \vec{a}_{J_A} + \vec{a}_{A^{J_A}} = (a_{J_A})_N + (\vec{a}_{J_A})_T + (\vec{a}_{A^{J_A}})_N + (\vec{a}_{A^{J_A}})_T = (\vec{a}_{A^{J_A}})_N + (\vec{a}_{J_A})_T + (\vec{a}_{A^{J_A}})_T \quad (15)$$

Equating right sides of equation (14) and (15) we have

$$(\vec{a}_A)_N = (\vec{a}_{A^{J_A}})_N$$

that is, in scalar notation

$$\begin{aligned} (a_A)_N &= (a_{A^{J_A}})_N \\ \frac{V_A^2}{O_A A} &= \omega^2 J_A A \\ \frac{(\omega \cdot PA)^2}{O_A A} &= \omega^2 J_A A \\ O_A A &= \frac{(PA)^2}{J_A A} = \frac{(PA)^2}{PA - PJ_A} \end{aligned} \quad (16)$$

or

$$\rho_A = \frac{r^2}{r - r_J} \quad (17)$$

The numerator $(PA)^2$ in equation (16) is always positive while denominator $(PA - PJ_A)$ may be positive ($PA > PJ_A$) or negative ($PA < PJ_A$). Subsequently, the radius of curvature may have positive or negative sign. Equation (16) reveals the fact that $O_A A$ and $J_A A$ are always laid off in the same sense along the line PA (O_A is always on the same side of A as J_A - Fig. 12).

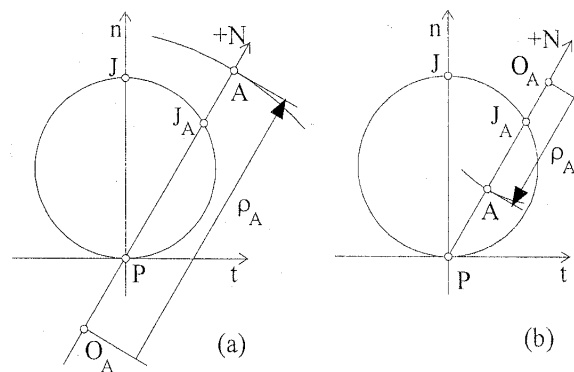


Fig. 12.

Thus, when J_A has been established, the sense of J_{AA} gives the sense of $O_A A$.

We can avoid "rule of sign" if we use "sign proof" complex – number notation of ESE

$$O_A \bar{A} = \frac{(PA)^2}{|P\bar{A} - P\bar{J}_A|} = e^{i \arg(P\bar{A} - P\bar{J}_A)} \quad (18)$$

or

$$\bar{\rho}_A = \frac{r^2}{|\bar{r} - \bar{r}_J|} e^{i \arg(\bar{r} - \bar{r}_J)} \quad (19)$$

To this end, the rectangular axes $O_\Sigma x$ and $O_\Sigma y$ fixed, in plane Σ are respectively considered as real (x) and imaginary (iy) axes, allowing vectors in planes Σ and Π to be expressed as complex numbers. A second form of ESE (see [3] for derivation)

$$P\bar{J}_A = P\bar{A} - \left(\frac{PA}{O_A A} \right)^2 O_A \bar{A} \quad (20)$$

or

$$\bar{r}_J = \bar{r} - \left(\frac{r}{\rho} \right)^2 \bar{\rho} \quad (21)$$

is applicable when points P , A and O_A are known and J_A is sought.

Complex-number form of ESE is well suited for automatic computation (without needing to use the "sign convention").

3. EXAMPLE

A four-bar linkage is to be designed (synthesised) so that, in design position, with the input crank motion rotating clockwise at a constant angular velocity of 20 rad/s, the output crank (follower) will have an angular velocity of 15 rad/s, counter clockwise and angular acceleration of 200 rad/s² counter clockwise. The distance between the crank pivots are 14 cm (Fig. E.1).

Solution

$$\omega_1 \Big|_{t_0} = -20 \text{ rad/s}$$

$$\omega_2 \Big|_{t_0} = \frac{d\psi}{dt} \Big|_{t=t_0} = 15 \text{ rad/s}, \quad \varepsilon_2 = \frac{d^2\psi}{dt^2} \Big|_{t=t_0} = -200 \text{ rad/s}^2$$

$$\psi = f(\varphi) = f(\omega_1 t) = \psi(t)$$

Function $\psi = \psi(t)$ will have specified value of first and second-order derivative if graphs (curves) at the instant considered have the same radius of curvature (three infinitesimally close points in common). It means that four-bar linkage is required to be synthesised in three accuracy points.

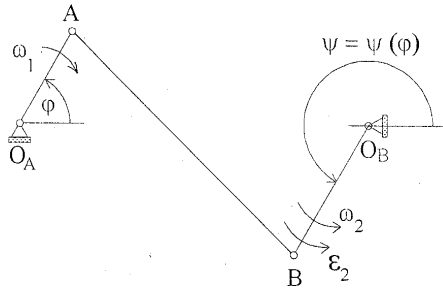


Fig. E.1

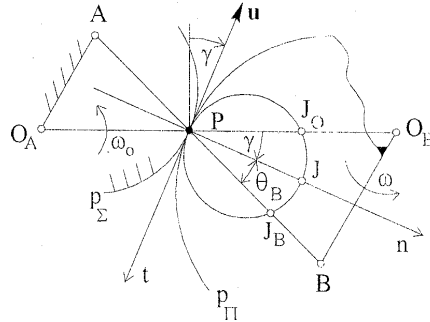


Fig. E.2.

- **Creation of the inverted mechanism**

In order to solve this problem we have to consider relative motion of the cranks, that is, the motion of the follower (output crank) with respect to the input crank. To this end we have to make input crank stationary ($\omega_1 = 0$), release the ground link $O_A O_B$ and impose angular velocity $\omega_0 = -\omega_1$ to it. That way we obtain so called "inverted mechanism", Fig. E.2. Floating link of this mechanism (crank $O_B B$) has instant angular velocity $\omega = \omega_2 - \omega_1$. Points B, O_B become moving pivots while points $O_A A$ are fixed pivots.

- **Instant velocity ratio**

$$\lambda \Big|_{t_0} = \frac{\omega_2 \Big|_{t_0}}{\omega_1 \Big|_{t_0}} = -\frac{15}{20} = -\frac{3}{4} = -0,75$$

- **Location of the instantaneous pole**

The point O_B belongs to both link $O_A O_B$ and $O_B B$. The link $O_A O_B$ rotates about O_A whereas link $O_B B$ rotates about instantaneous pole P . According to clause 2.5 and equations (8) we have

$$O_A P = \frac{\lambda}{(\lambda - 1)} O_A O_B = \frac{-0,75}{(-0,75 - 1)} \cdot 14 = 6 \text{ cm}$$

$$P O_B = O_A O_B - O_A P = 14 - 6 = 8 \text{ cm}$$

Notice: If length of the ground link $O_A O_B$ is not specified it can be assumed as $O_A O_B = 1$ since all similar mechanisms generate the same function.

- **The slope of the pole tangent**

According to equations (12) and Fig. E.2 we have

$$\gamma = \arctg \left[\frac{\frac{d\lambda}{d\varphi}}{\lambda(\lambda - 1)} \right] = \arctg \left[\frac{0,5}{-0,75(-0,75 - 1)} \right] = \arctg \frac{8}{21} = 20.85445^\circ$$

$$\gamma = 20.8544^\circ = 0.1158\pi \text{ rad}$$

but

$$\frac{d\lambda}{d\varphi} = \frac{d\lambda}{dt} \cdot \frac{dt}{d\varphi} = \frac{d\lambda}{dt} \cdot \frac{1}{\frac{d\varphi}{dt}} = \frac{\varepsilon_2}{\omega_1} \cdot \frac{1}{\omega_1} = \frac{\varepsilon_2}{\omega_1^2} = \frac{200}{(-20)^2} = 0,5$$

$$\frac{d\lambda}{dt} = \frac{d}{dt} \left(\frac{\omega_2}{\omega_1} \right) = \frac{\dot{\omega}_2 \omega_1 - \dot{\omega}_1 \omega_2}{\omega_1^2} = \frac{\dot{\omega}_2 \omega_1}{\omega_1^2} = \frac{\varepsilon_2}{\omega_1}$$

$$\frac{d\varphi}{dt} = \frac{d}{dt}(\omega_1 t) = \omega_1$$

We can now add the pole tangent to the layout.

- **Inflection point of O_B**

Pivot O_B of the inverted mechanism traces a circle with radius $O_A O_B$. Fixed pivot O_A is the centre of curvature. According to equation (20) we have (see Fig. E.2)

$$\begin{aligned} P\bar{J}_0 &= P\bar{O}_B - \left(\frac{PO_B}{O_A O_B} \right)^2 O_A \bar{O}_B = PO_B \cdot e^{i(\frac{\pi}{2} + \gamma)} - \left(\frac{PO_B}{O_A O_B} \right)^2 O_A O_B \cdot e^{i(\frac{\pi}{2} + \gamma)} = \\ &= \left[8 - \left(\frac{8}{14} \right)^2 14 \right] e^{i(\frac{\pi}{2} + \gamma)} = 3,4285 e^{i(0,6158\pi)} \\ P\bar{J}_0 &= 3,4285 \text{ cm} \end{aligned}$$

- **Diameter of the inflection circle**

According to equation (13) and Fig. E.2

$$D = PJ = \frac{PJ_0}{\cos \gamma} = \frac{3,4285}{\cos 20,8544^\circ} = 3,669 \text{ cm}$$

Having the point J located, and knowing that the inflection circle is tangent to the pole tangent, we can add the inflection circle for the relative motion to the layout.

- **Choice of the moving pivot B**

Since mechanism is required to be synthesised at three infinitesimally close positions (three accuracy points) any point of the moving plane associated with the floating link may be adopted as the moving pivot because the path of every point has three infinitesimally close point in common with the circle (second - order contact) – path of the moving pivot. Let a point B has polar co-ordinates

$$r_B = PB = 7 \text{ cm}$$

$$\theta_B = -10^\circ = -\frac{\pi}{18} \text{ rad}$$

- **Inflection point of B**

According to

$$PJ_B = D \cos \theta_B = 3,669 \cdot \cos(-10^\circ) = 3,669 \cdot 0,9848 = 3,613 \text{ cm}$$

$$P\bar{J}_B = PJ_B \cdot e^{i(\frac{\pi}{2} + \theta_B)} = 3,613 \cdot e^{i(\frac{4}{9}\pi)}$$

- **Coupler link length**

The point A is centre of curvature (fixed pivot) of the point B . Radius curvature length AB is the length of coupler link of origin mechanism (Fig. E.1). According to equation (18)

$$\vec{AB} = \frac{(PB)^2}{|\vec{PB} - \vec{PJ}_B|} e^{i \arg(\vec{PB} - \vec{PJ}_B)} = \frac{7^2}{|7 - 3,613|} e^{i \left(\frac{\pi}{2} + \theta_B \right)} = 14,467 e^{i \left(\frac{\pi}{9} \right)}$$

$$AB = 14.467 \text{ cm}$$

- **Length of the input crank**

$$a = O_A A = \sqrt{(O_A P - PA \cos \alpha_0)^2 + (PA \sin \alpha_0)^2} = \\ = \sqrt{(6 - 7,467 \cdot 0,85847)^2 + (7,467 \cdot 0,51285)^2} = 3,8513 \text{ cm}$$

where:

$$PA = AB - PB = 14,467 - 7 = 7,467 \text{ cm}$$

$$\alpha_0 = \gamma + \theta_B = 20,8544 + 10 = 30,8544^\circ$$

- **Length of output crank (follower)**

$$b = O_B B = \sqrt{(PO_B - PB \cos \alpha_0)^2 + (PB \sin \alpha_0)^2} \\ = \sqrt{(8 - 7 \cdot 0,85847)^2 + (7 \cdot 0,51285)^2} = 4,105 \text{ cm}$$

- **Angle of the input crank in the design position**

$$\varphi_0 = \arcsin \left(\frac{PA \sin \alpha_0}{O_A A} \right) = \arcsin \left(\frac{7,467 \cdot 0,51285}{3,8513} \right) = \arcsin(0,99433) = 83,894^\circ$$

CONCLUSION

Exposed method for synthesis of planar bar linkages in infinitesimally close positions belongs to the approximate algebraically methods. The method provides good results when mechanism (input crank) has limited range of motion. It is established that structural error seldom exceeds 1% when input cranks rotates within 25%.

As for the mechanisms for path generation, the method is applicable only if the mechanism is required to generate a segment of circle with specified radius of curvature, including segment of straight line.

Synthesis in the infinitesimally close positions is not applicable to motion-generator mechanisms (rigid-body guidance). Mathematical model of synthesis doesn't contains systems of non-linear equations requiring development of numerical methods for solution by computer. Of course, computer is an advantage but in this case all calculations can be made by the scientific calculator.

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POSTUPAK SINTEZE RAVNIH POLUŽNIH MEHANIZAMA U BESKONAČNO BLISKIM POLOŽAJIMA

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Izložena je procedura numeričke metode za kinematičku sintezu ravnih polužnih mehanizama u dva i tri beskonačno bliska položaja. Prikazana metoda je iskorišćena za rešavanje jednog praktičnog slučaja mehanizma za generisanje zadate funkcije. Sinteza mehanizma je izvršena indirektno korišćenjem inverznog mehanizma.