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DYNAMICS AND IDENTIFICATION OF ROTATING MACHINE SYSTEM FOR INVESTIGATION OF THE AIR TYRES

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Abstract. *In this paper the experimental-numerical method of determining, dynamic parameters i.e. mass moments of inertia and coefficients of motion resistance of the multimass rotating systems and energy provided to the examined aeroplane wheels with tyres is presented. Considered rotating system is the essential part of a large machine for the dynamic investigation of pneumatic aeroplane tyres, which simulate dynamic load of the examined wheel with tyre. The presented method uses differential equations of motion and regression method. The paper includes the results of the computation for the chosen rotating system.*

1. INTRODUCTION

The multimass rotating in its own bearing is considered. This system is rotating with the initial angular velocity ω_0. Part of the kinetic energy of this system is used for overcoming of the unknown resistance to motion (i.e. rolling resistance, air resistance) and the rest of energy of the system is supplied to the wheel with the examined tyre. This second part of energy is most important to dynamical investigation of tyres in view of international standards ISO.

The considered rotating system is depicted diagrammatically in Fig. 1.

Dynamic investigation of aeroplane tyres consists of:

– the choice of rotating mass set, simulating dynamic load for the given type of tyres and for the kind of simulation research (starting, landing),
– defining of angular speed ω_0 of the race corresponding to linear speed of the aeroplane,
– after heaving reached by the rotating system angular speed ω_0 the drive is switched

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off, the wheel with the examined tyre is pressed to the determined standard force and standard time,
– on the base of recorded diagram of the race angular speed $\omega(t)$, the value of energy provided by the rotating system to the wheel with the tyre is computed.

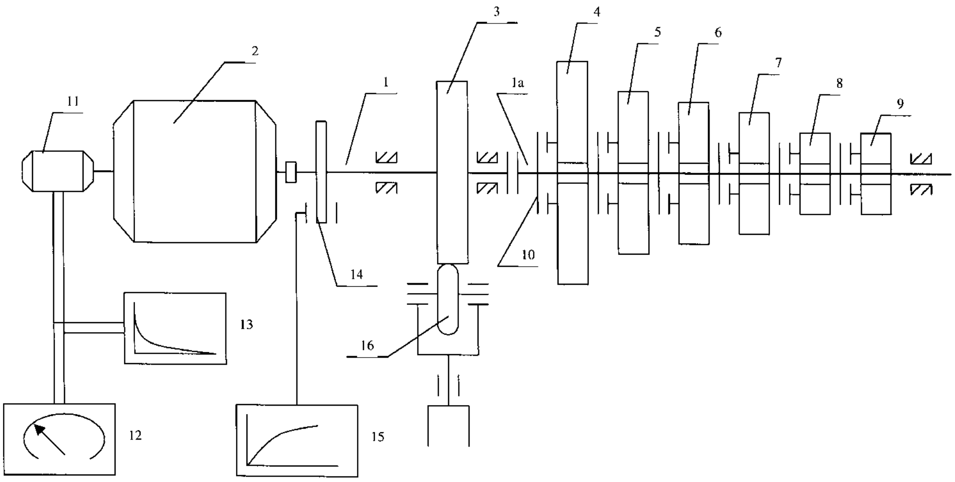

Fig. 1. The investigated rotating system: 1-main shaft, 1a-additional shaft, 2-electric drive, direct-current motor, 3-a wheel mounted on the main shaft, simulating the race for the tyre, 4-9-additional masses connected with the shaft 1a by means of the claw couplings, 10-claw couplings, 11-rate generator, tachometer, 12,13-angular speed meter and recorder, 14-brake, 15-braking torque recorder, 16-examined wheel with tyre.

2. THE METHOD

Now we describe the determining dynamic parameters of rotating system by means of measurement of breaking torque and angular speed.

For simplicity of consideration the method is presented on the example of the rotating single mass shown in Fig. 2, where:

$\omega(t)$, $\dot{\omega}(t)$ – the angular speed of the rotating system registered during the experiment and angular acceleration computed on the basis of $\omega(t)$,

$M_h(t)$ – the known braking torque, registered during the experiment,

$M_r(\omega,t)$ – the unknown moment of motion resistance which has to be computed.

Dynamics of the rotating system shown in Fig. 2 is governed by the equation:

$$B\dot{\omega}(t) + M_r(\omega, t) = -M_h(t) . \tag{1}$$

Moment of motion resistance can be expressed as power series:

$$M_r(\omega, t) = c_0 + c_1\omega(t) + c_2\omega^2(t) + ... + c_k\omega^k(t) , \tag{2}$$

where: c_i – the unknown coefficients of motion resistance, which have to be computed, $i = 1...k$.

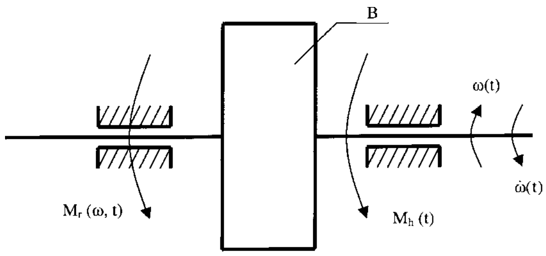

Fig. 2. One-mass system.

Usually it is sufficient to limit the number of the terms of the series (2) to four, i.e. k = 3.
Introducing (2) into (1), apart from t, we obtain:

$$B\dot{\omega} + c_0 + c_1\omega + c_2\omega^2 + ... + c_k\omega^k = -M_h \, . \tag{3}$$

The real laboratory experiment is carried out in the following way. First, the rotating part of the experimental rig is brought up to required rotational speed ω_0, and next the brakes are switched with a simultaneous registration of a time history of both the rotational speed $\omega(t)$ and the braking torque $M_h(t)$.

In Figure 3 the exemplary responses of the measured quantities are shown. The angular acceleration $\dot{\omega}(t)$ which occurred in equation (3) can be found by differentiating the known time history of $\omega(t)$. Dividing the time corresponding to measurement of $\omega(t)$ and $M_h(t)$ into n equal intervals of length Δt (see Fig. 3) one can formulate the following system of conditional algebraic equations (the values of $\omega_i(t_i)$, $\dot{\omega}_i(t_i)$, M_{hi} are known):

$$B\dot{\omega}_i + c_0 + c_1\omega_i + c_2\omega_i^2 + ... + c_k\omega_i^k + M_{hi} = 0 \, , \quad i = 1...n \tag{4}$$

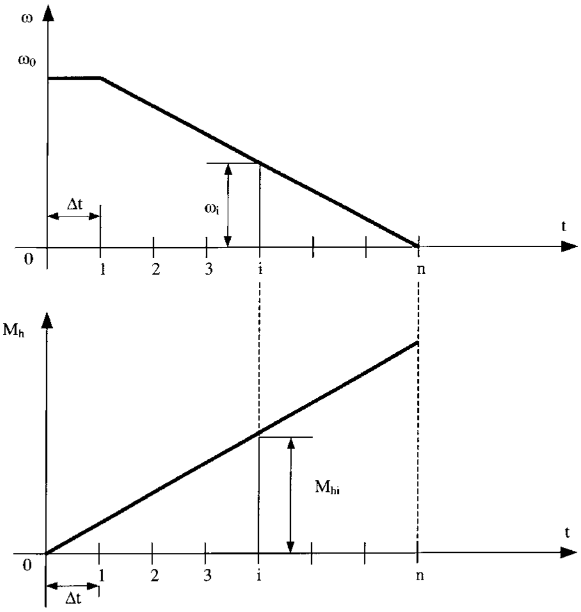

Fig. 3. Angular speed and braking torque versus time.

This set of equations (4) is in general contradictory (at n>k+2), therefore we seek for the unknown parameters B and c_i the most probable values governed by equations

$$B\dot{\omega}_i + c_0 + c_1\omega_i + c_2\omega_i^2 + ... + c_k\omega_i^k + M_{hi} = \varepsilon_i \,. \tag{5}$$

Assuming the normal distribution of deviations ε_i, for the most probable values of unknown parameters B and c_i, one obtains the minimal sum of squares of deviations ε_i.

It results from the principles of regression method [1, 2]. Denoting this sum by

$$S = \sum_{i=1}^{n} \varepsilon_i^2 \tag{6}$$

and minimalizing it with respect to the parameters B and c_i one obtains

$$\frac{\partial S}{\partial B} = 0;\ \frac{\partial S}{\partial c_0} = 0;\ \frac{\partial S}{\partial c_1} = 0;\ ....\frac{\partial S}{\partial c_k} = 0 \,. \tag{7}$$

The obtained set of normal equations can be written in a matrix form (see the next item).

3. Influence of pressing force of the wheel on the race

On the basis of the literature concerning rolling bearings theory one can settle that the total bearing frictions is the sum of rolling and sliding friction [2, 3].

The moment of bearing frictions consists of the following members:

M_0 – moment of hydraulic resistance independent of a load and dependent of angular speed,

M_1 – moment of rolling resistance resulting from elastic bearing deformation, dependent on a bearing load due to the formulae

$$M_0 = 10^{-7} f_0 (\nu n)^{2/3} d_m^3 \quad [\text{Nmm}], \tag{8}$$

and for $\nu n < 2000$ by the relation

$$M_0 = 160 \cdot 10^{-7} f_0 d_m^3 \quad [\text{Nmm}], \tag{9}$$

where:

ν – oil kinematic (lubrication) viscosity coefficient ([cSt] or [mm^2/s]),

n – angular speed of the bearing [rev/min],

f_0 – coefficient depending on bearing construction and on lubrication (from a bearing catalogue),

d_m = (d+D)/2 - mean diameter of the bearing [mm],

d – diameter of the bearing hole [mm],

D – outer diameter of the bearing [mm].

The moment M_1 of rolling resistance is calculated from the formula [2, 3]

$$M_1 = \mu_1 f_1 P \frac{d_m}{2} \quad [\text{Nmm}], \tag{10}$$

where:

μ_1 – friction coefficient depending on the type of the bearing and on the load value (from a catalogue),

f_l – coefficient depending on bearing construction and the load (from a catalogue),
P – bearing load [N],
d_m – mean bearing diameter [mm].

Taking into account the influence of the pressing force on the rolling resistance in the bearing, the correction coefficient has been introduced

$$e = \frac{M_0 + M_1(Z)}{M_0 + M_1(G)}, \tag{11}$$

where:

$M_1(Z)$ – moment of rolling resistance of the bearing at the resultant load

$$Z = \sqrt{G^2 + F^2}, \tag{12}$$

where:

$$\left.\begin{array}{lll} a_i = 10^{-7} f_0 (\nu n_i)^{2/3} d_m^3 & \text{for} & \nu n_i \geq 2000 \\ a_i = 160 \cdot 10^{-7} f_0 d_m^3 & \text{for} & \nu n_i < 2000 \end{array}\right\} \text{ - depending on angular speed,}$$

$b = \mu_1 f_1 \frac{d_m}{4}$ - constant depending on the kind of bearing.

Substituting (12) to (2) we obtain (for i-th period of time Dt)

$$M_r(\omega, t) = e_i[c_0 + c_1\omega(t_i) + c_2\omega^2(t_i) + ... + c_k\omega^k(t_i)]. \tag{13}$$

Substituting (13) to (4) we get

$$\begin{bmatrix} \sum \dot{\omega}_i^2 & \sum e_i\dot{\omega}_i & \sum e_i\omega_i\dot{\omega}_i & \sum e_i\omega_i^2\dot{\omega}_i & .... & \sum e_i\omega_i^k\dot{\omega}_i \\ \sum e_i\dot{\omega}_i & \sum e_i^2 & \sum e_i^2\omega_i & \sum e_i^2\omega_i^2 & .... & \sum e_i^2\omega_i^k \\ \sum e_i\omega_i\dot{\omega}_i & \sum e_i^2\omega_i & \sum e_i^2\omega_i^2 & \sum e_i^2\omega_i^3 & .... & \sum e_i^2\omega_i^{k+1} \\ \sum e_i\omega_i^2\dot{\omega}_i & \sum e_i^2\omega_i^2 & \sum e_i^2\omega_i^3 & \sum e_i^2\omega_i^4 & .... & \sum e_i^2\omega_i^{k+2} \\ \sum e_i\omega_i^3\dot{\omega}_i & \sum e_i^2\omega_i^3 & \sum e_i^2\omega_i^4 & \sum e_i^2\omega_i^5 & .... & \sum e_i^2\omega_i^{k+3} \\ .... & .... & .... & .... & .... & .... \\ \sum e_i\omega_i^k\dot{\omega}_i & \sum e_i^2\omega_i^k & \sum e_i^2\omega_i^{k+1} & \sum e_i^2\omega_i^{k+2} & .... & \sum e_i^2\omega_i^{k+k} \end{bmatrix} \times \begin{bmatrix} B \\ c_0 \\ c_1 \\ c_2 \\ c_3 \\ ...... \\ c_k \end{bmatrix} = \begin{bmatrix} -\sum M_{hi}\dot{\omega}_i \\ -\sum M_{hi}e_i \\ -\sum M_{hi}\omega_i e_i \\ -\sum M_{hi}\omega_i^2 e_i \\ -\sum M_{hi}\omega_i^3 e_i \\ .... \\ -\sum M_{hi}\omega_i^k e_i \end{bmatrix} \tag{14}$$

where c_i – are the corrected coefficients of motion resistance with allowances for influence of pressing force of the wheel to the race.

4. ENERGY DETERMINATION PROVIDED TO THE EXAMINED WHEEL

The exemplary diagram of an angular speed of the rotating system after pressing the examined wheel to the race is shown in Fig. 4.

In this diagram two ranges are chosen, in which the relation is approximately linear. The first range (t_0 - t_1) corresponds to the mating unbraked wheel, the second one (t_1 - t_2) corresponds to the mating braked wheel. The equation of the motion of the rotating system with the wheel pressed to the race has the form

$$B_s\dot{\omega}(t) + M(\omega, t) = 0, \tag{15}$$

where:

- B_s – summary moment of inertia of the chosen combination of the rotating masses computed with (15),
- $\dot{\omega}(t)$ – negative acceleration of the system,
- $M(\omega,t)$ – the moment of motion resistance reduced on the shaft axis of the rotating system with the wheel.

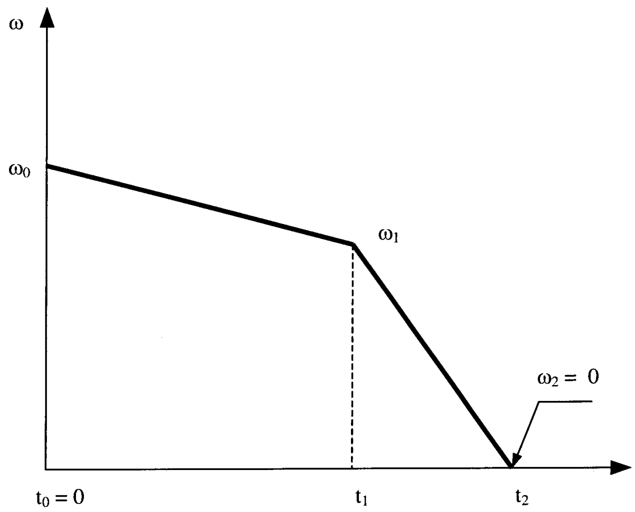

Fig. 4. Angular speed of the system versus time with marked ω_i values.

The moment of the motion resistance of the system includes two terms

$$M(\omega,t) = M_r(\omega,t) + M_k(\omega,t)\,, \tag{16}$$

where:

- $M_r(\omega,t)$ – moment of motion resistance of the rotating system,
- $M_k(\omega,t)$ – moment of motion resistance of the examined wheel, reduced to the shaft of the rotating system.

The energy lost due to overcoming the motion resistance of the rotating system with allowance for the force pressing the wheel to the race, for determined time period (t_{i-1} - t_i) has the following form

$$E_{ri} = \int_{t_{i-1}}^{t_i} M_r(\omega,t)\cdot\omega(t)dt\,. \tag{17}$$

In the same period the rotating system loses the kinetic energy, due to decrease of the angular speed from ω_{i-1} to ω_i:

$$E_i = \frac{B_s}{2}(\omega_{i-1}^2 - \omega_i^2)\,. \tag{18}$$

Finally, the energy provided to the wheel in the i-th period is presented in the form

$$E_{ki} = E_i - E_{ri}\,. \tag{19}$$

5. Numerical computation

The equations (14), and (17)–(19) have been numerically solved. The obtained results for the chosen rotating system are presented below:
without allowance for the pressing force (F = 0)

$B = 2007{,}67$ [kgm^2], $c_0 = -398{,}016$ [Nm],
$c_1 = 51{,}4248$ [Nms], $c_2 = -1{,}00815$ [Nm],
$c_3 = 5{,}8925 \cdot 10^{-3}$ [Nms^3], $E_k = 1909290$ [J];

with allowance for the pressing force (F = 32690 [N])

$B = 2007{,}51$ [kgm^2], $c_0 = -430{,}5$ [Nm],
$c_1 = 60{,}2354$ [Nms], $c_2 = -1{,}22586$ [Nm],
$c_3 = 7{,}4484 \cdot 10^{-3}$ [Nms^3], $E_k = 1824220$ [J].

The calculations clearly show the influence of pressing force on the decrease of energy provided to the examined tyre.

6. Conclusions

The proposed method is devoted to dynamical analysis of the air tyres investigation.

The machine consists of rotating running track with the tyre pressed to it (with required force) and with the attached additional rotating masses, which simulate the different dynamical loads during landing of an aeroplane.

In the first part of this work the identification method of the rotating system is proposed using the measurements of the braking torque and the rotating velocity during time of braking process.

In result one gets the following parameters of the investigated rotating system: an inertial mass moment B, and the resistance c_i coefficients.

In addition, in the constructed experimental rig one can measure a pressing force of the investigated tyre.

In the second part of this work the method of a value of energy E_k transfer from the rotating system to the investigated tyre is proposed. It corresponds to energy which is absorbed by an aeroplane tyre during the aeroplane landing. In result a knowledge of an absorbed energy by an aeroplane tyre can be applied to an appropriate construction of a tyre.

References

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DINAMIKA I IDENTIFIKACIJA SISTEMA ROTACIONE MAŠINE ZA ISPITIVANJE AVIONSKIH GUMA

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U ovom radu prikazan je eksperimentalno-numerički metod određivanja dinamičkih parametara tj. masenog momenta inercije i koeficijenta otpora kretanja sistema sačinjenog od više rotirajućih masa i energije obezbeđenih za ispitivanje avionskih točkova sa gumama. Razmatrani rotacioni sistem je ključni element velike mašine za ispitivanje avionskih pneumatika, koja simulira dinamičko opterećenje ispitivanog točka sa pneumatikom.

Prikazani metod koristi diferencijalne jednačine kretanja i metod regresije. Rad obuhvata i rezultate proračuna za izabrani rotacioni sistem.