FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol.1, N° 7, 2000, pp. 799 - 807

CALCULATION OF LOAD CAPACITY OF GEARS IN RANDOM VARYING EXPLOITATION CONDITIONS

UDC 621.83:621.86.016 62.233.1/.9:621.86.016

Vojislav Miltenović, Dragan Milčić

The Faculty of Mechanical Engineering, Niš, Beogradska 14, 18000 Niš

Abstract. *A high degree of accuracy in dimensioning and controlling the safety from destruction can be reached by measuring exploitation load of the vital elements of the construction. The choice of the adequate load spectrum is performed by introducing a number of representative spectrums for particular operating conditions, each of these having its probability of occurrence. The easiest way to solve the problem is to introduce the standardised load spectrums with the aim of forming the catalogues of the standardised load spectrums for representative mechanical systems.*

INTRODUCTION

The requirements of the market are constantly growing more complex with regard to productivity, quality and speed of mastering new products. Rapid development is the cause of the increase of designing and constructional tasks with even greater degree of complexity. The conventional "traditional" designing, based on empirical results and intuition, does not suffice for successful following of progress in other areas of human activity. In the engineering practice today, automatised designing is an imperative, which is caused by the use of computers in the process of developing a product.

New tools for overcoming the flaws of the classical approach can be classified under the term **CIM**- Computer Integrated Manufacturing- which defines the integrated flow of all information both within and outside the company. **CIM** consists of modules such as **CAD** (Computer Aided Design), **CAM** (Computer Aided Manufacturing), **CAPP** (Computer Aided Process Planning), **CAQ** (Computer Aided Quality) etc.

The utilities and tools of **CAD** technology can be most widely applied in automatization of the designing process and they appear as the support mechanisms in the object modelling processes, in engineering analysis and documentation. The applicability of the **CAE** (Computer Aided Engineering) is limited to the process of engineering analysis (**FEM** analysis and simulation) while **CAPP** and **CAM** can be used in modelling

Received August 20, 2000
Presented at 5th YUSNM Niš 2000, Nonlinear Sciences at the Threshold of the Third Millenium, October 2-5, 2000, Faculty of Mechanical Engineering University of Niš

and designing (elaboration) operating procedures. The architecture of the intelligent programme for designing toothed gearing, being developed at the Faculty of Mechanical Engineering in Nis, is presented in Figure 1.

The structure of the system for designing toothed gearing is very complex and heterogeneous. The system has been developed from a modular principle enabling to perform, with the aid of the computer, certain activities and tasks of the designers. The basic task of the system is to enable an integrated application of different programme modules and systems developed by different authors and companies, which should be used in automatization of certain activities in designing toothed gearing. This is the reason why the software basis of a developed system rests on the maximal application of all available standards in the area of data flow, communication and computers.

The programme for designing toothed gearing consists of 6 basic programme modules integrated in a single totality:

1. the programme module for calculating gearing pairs,
2. the programme module for calculating roller and antifriction bearings (plain bearing, sleeve bearing, slide bearing),
3. the programme module for choosing and calculating the shaft-hub joint,
4. the programme module for calculating and shaping the shaft,
5. the programme module for calculating intermediate belt gearing,
6. the programme modules for modelling and generating constructional documents for integral parts of gearing (Auto CAD, GENIUS, Pro/ENGINEER)

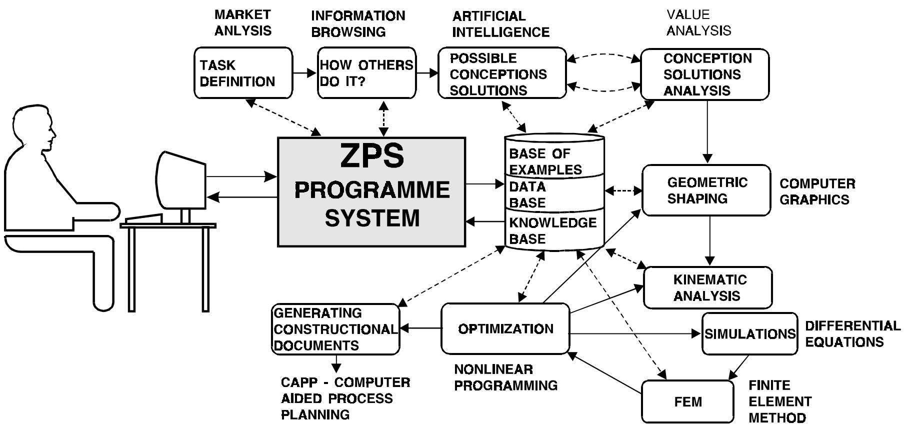


Fig. 1. The activities and computer methods applied in the process of toothed gearing design

The programme module for calculating the gearing pairs consists of:

ZPS1 - the programme module for calculating and optimisation of gearing geometrical-constructional characteristics,

ZPS2 - the programme module for final calculation of cylindrical and cone gearing,

ZPS3 - the programme module for initial designing of worm gearing,

ZPS4 - the programme module for calculating the geometry and strength of worm gearing pairs.

The programme module **ZPS2** is intended for the final calculation of gearing and offers the following possibilities:

- the calculation of precise geometry of cylindrical involute gear with outer and inner serration for the defined parameters of the tool and the calculation of cone gearing,
- automatic choice of the centre distance for the previously defined standard sequence,
- automatic determination of the addendum air gap and shortening of tooth head for an integral number of addendum circle radius,
- the calculation of factors of working conditions by means of load spectrum,
- the calculation of load capacity with respect to DIN-3990, AGMA- standard and ISO-recommendations,
- the calculation of gearing elastic deformation,
- the calculation of load capacity with respect to seizing and pitting, violent and wear fracture,
- the calculation of load capacity by simulating the exploitation conditions,
- the calculation of load capacity of cylindrical and conical involent gears with straight and helical gear.

The paper deals with calculating the gear load capacity in random varying exploitation conditions by simulating the exploitation conditions.

1. The fundamentals of calculation

The quality of design results is considerably higher with regard to the fact that the application of information technology eliminates in a great degree the subjective sources of error.

The most important constituent with regard to safety and reliability of MS is the load capacity of its constructional elements. The load capacity can be as well be used to measure the quality of the constituents. The calculation of load capacity can be performed by comparing the operating and critical stress; therefore, the evaluation of quality can be considered in terms of evaluation of exactness of the size of working and critical stress.

The accuracy of the operating stress calculation depends on the appropriateness of the methods applied and the manner of determining the load. Whether a method is appropriate is determined by its possibilities for identifying the state, position and degree of stress. The quality of the methods applied is tested experimentally. The degree of the operating stress depends on the load; it is therefore necessary to know the degree, course of change and frequency, as well as the probability of occurrence of the highest stress appearing during life for the observed part or element of the construction.

The current testing of the safety of mechanical parts against destruction is performed by determining the nominal load, i.e. the nominal operating load, which corresponds to the most frequent stress in the course of operation. The exploitation conditions, i.e. the occurrence of load greater than the nominal is taken into consideration by way of the factor of operating conditions chosen from the corresponding tables. A conclusion can be drawn that this way of determining the operating load offers approximate values, which leads to inadequate dimensions of the construction elements.

A high degree of accuracy in dimensioning and testing the safety from destruction can be accomplished by measuring the exploitation load and identifying the load spectrum for the vital elements of the structure. The load spectrums can be obtained on the basis of experimental MS in the process of operating for certain conditions; therefore each load spectrum has its probability of occurrence. The problem of choice of the appropriate spectrum is resolved by introducing a greater number of representative spectrums for certain operating conditions, which allows for an adequate evaluation of all the inter conditions. The load spectrums can be expressed in terms of a relative frequency of certain amplitudes $f(x)$ where

$$f(x) = \frac{\sigma_{ai}}{\sigma_{a\max}} = \frac{size\ of\ any\ stress\ amplitude}{max.stress\ amplitude} \tag{1}$$

The load spectrum represents a basis for determining the critical stress as well. With current methods of calculating the critical stress of dynamically strained pieces, dynamical endurance, obtained in the continual relation of the lowest and highest stress for the probability of destruction of about 0,5, is used. For a more precise calculation of the safety of pieces against destruction it is necessary to introduce endurance corresponding to the manner and number of changes in operating stress in the operating life of the piece observed - operating endurance. Data about operating endurance are obtained in experimental research of the test pieces or elements for particular spectrums of operating stress in laboratories. The basic problem is a wide range and duration of the research conditioned by the need to conduct the research on a greater number of load spectrums and with a greater number of load spectrums in order to obtain a more realistic picture about the size and laws of scattering of the research results. Therefore, this problem is solved by finding the relation between the operating and basic endurance i.e. by applying the hypothesis on the accumulation of damage of material. The basis of this hypothesis is the supposition that every stress σ_i in a spectrum adds to damage proportional to the number of circles to destruction N_i, where $n_i<N_i$. Destruction occurs when the sum of the damages reaches the value

$$D = \sum \frac{n_i}{N_i} = \sum D_i \tag{2}$$

According to some hypothesis $D = 1$.

The experimental research of this hypothesis for different stress states has revealed significant deviations. Therefore, the hypothesis has been altered in different ways primarily with regard to the sum of damages etc.

The size of the load of the MS elements can be obtained by a computer simulation of the exploitation conditions. The course of the life calculation is presented in Figure 2.

The starting point is the time function of the load change, discretization of which to one or two parameters provides the load spectrum. By introducing the correctional dependence between endurance and load spectrum (2) graphic dependence of the life for a standardised load spectrum is obtained, i.e. the obtained output represents the available piece resource in the form of interdependence of reliability and life. The basic problem is how to obtain the time function of the load change. The best way is to measure load in exploitation conditions. Taking into consideration that this is a long-lasting and expensive procedure, such a function is usually obtained in the accelerated laboratory or polygon research; it serves as a basis to obtain the corresponding load spectrums. For load spectrums defined in this manner it is possible to obtain the corresponding load spectrums for similar systems.

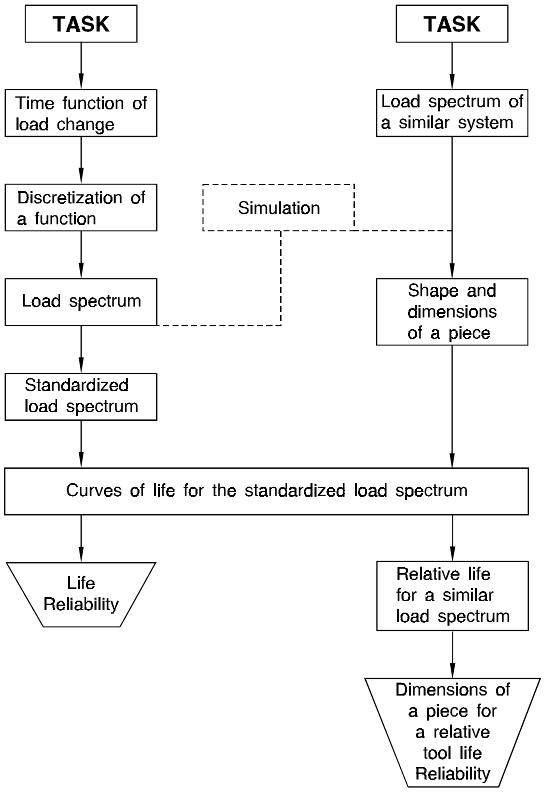


Fig. 2. The relative life for the standardized load spectrum

2. THE CALCULATION OF THE GEAR LOAD CAPACITY FOR THE CRITERIA OF ENDURANCE OF THE TOOTH FLANK AND ROOT

The calculation rests on the fact that each gear revolution has a corresponding damaging effect. The effect of damage depends on the size of stress; it can be disregarded with small stress. The life obtained is a measure denoting the available resource of the material. For such a calculation to be possible, it is necessary to have a load spectrum, exact characteristics of durability of material and applicability of appropriate hypothesis on the accumulation of damage of material. Our calculation is based on Palmgren-Miner hypothesis where $D \approx 1$.

The load spectrum is aproximated with corresponding blocks whereas each torque has a corresponding number of cycles of load change n_i. In Figure. 3, besides the load spectrum, the Veler curve of the tooth load is represented.

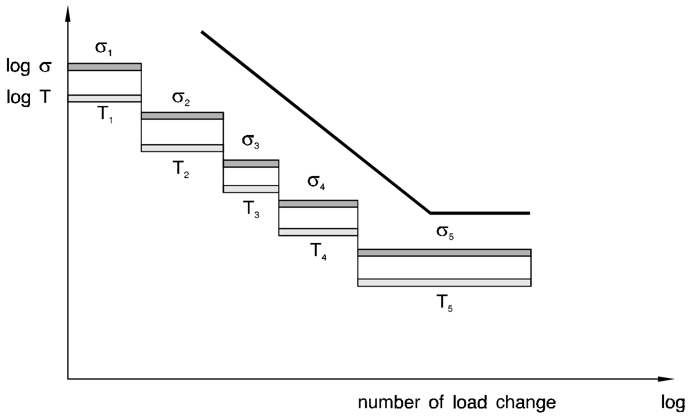


Fig. 3. Load spectrum

For each of the load spectrum levels T_i the corresponding contact stress is determined according to:

$$\sigma_{Hi} = Z_H \cdot Z_E \cdot Z_\varepsilon \cdot Z_\beta \cdot Z_{B,D} \sqrt{\frac{2000 \cdot T_i}{d_1{}^2 \cdot b} \cdot \frac{u+1}{u} K_{vi} \cdot K_{H\beta i} \cdot K_{H\alpha i}} \tag{3}$$

where:
Z_H – zone factor, Z_β - helix angle factor,
Z_E – elasticity factor, Z_ε - contact ratio factor,
$Z_{B,D}$ – single pair tooth contact factor for the pinion, for the wheel,
K_{vi} – dynamic factor, $K_{H\beta i}$ – face load factor, $K_{H\alpha i}$ – transverse load factor.

Aplication factor K_A has not been used in the expression (3), since it is taken into consideration via the load spectrum.

The characteristic of the endurance of material is determined by the Veler curve i.e. the damage line.

For example, for C5426 endurance characteristics have been presented in figures 4.a and 4.b.

The inclination of the Veler curve (Fig. 4a) is:

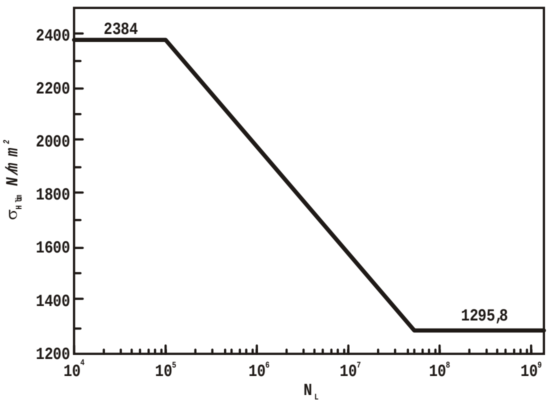
$$p = \frac{\log \frac{N_{BD}}{N_{BS}}}{\log \frac{[\sigma_H]_s}{[\sigma_H]_D}} = \frac{\log \frac{5 \cdot 10^7}{10^5}}{\log \frac{2384}{1295.8}} = 10,19 \tag{4}$$

The number of load change cycles to the fracture for any of the levels N_i equals:

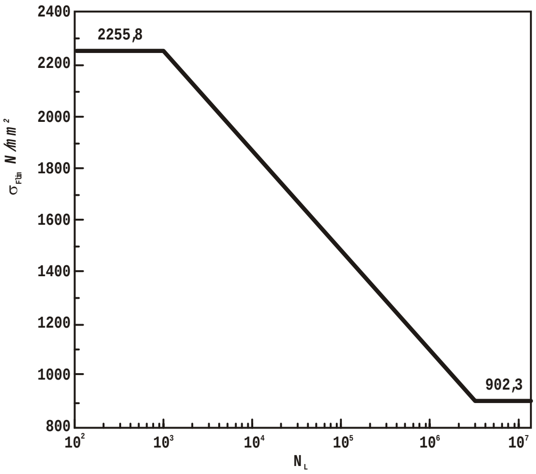
$$\log N_i = p \cdot \{\log[\sigma_H]_s - \log(S_H \cdot \sigma_{Hi})\} + \log N_{BS} \tag{5}$$

The tooth root stress for each of the load spectrum levels T_i is determined by

$$\sigma_{Fi} = Y_{Fa} \cdot Y_{sa} \cdot Y_\beta \cdot Y_\varepsilon \cdot \frac{2000 \cdot T_i}{d_1 \cdot b \cdot m_n} K_{vi} \cdot K_{F\beta i} \cdot K_{F\alpha i} \tag{6}$$



a)



b)

Fig. 4. Damage lines for damage statistical probability of 10%

The characteristics of the endurance of material are determined by damage curve presented in Fig.4, whence

$$p = \frac{\log \dfrac{N_{BD}}{N_{BS}}}{\log \dfrac{[\sigma_F]_s}{[\sigma_F]_D}} = \frac{\log \dfrac{3 \cdot 10^6}{10^3}}{\log \dfrac{2255.8}{902.3}} = 8{,}7378 \qquad (7)$$

The number of load change cycles to fracture for any of the levels N_i is

$$\log N_i = p \cdot \{\log[\sigma_F]_s - \log(S_F \cdot \sigma_{Fi})\} + \log N_{BS} \qquad (8)$$

The algorithm of the calculation of the blocks safety degree S_H and root of teeth S_F is presented in Figure 5. The calculation is iterated until the damage is in the scope of $0{,}95 < D < 1{,}05$.

Example: Cylindrical gear pair $z_1 = 20$; $z_2 = 73$; $b = 106$ mm; $a = 580$ mm; $m_n = 12$ mm; $\beta = 6^0$; gear material C5426 cemented/C5431 induction hardened; life $L_h = 50000$ hours; nominal power $P = 900$ kW; input number of revolutions $n = 1080$ min^{-1} ; load spectrum defined in the table 1:

Table 1.

Load block	Relative time participation	T_i/T_n
1	0.15	0.9
2	0.4	1
3	0.23	1.2
4	0.12	1.6
5	0.1	1.8

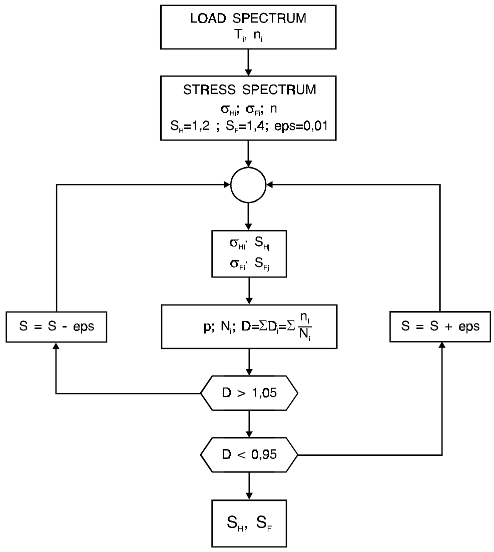


Fig. 5. The algorithm of the calculation factor of safety from tooth breakage and factor of safety from pitting

The results of the calculation have been presented in Table 2.

Table 2.

i	n_i	T_i [Nm]	σ_{Hi} [N/mm^2]	σ_{Fi} [N/mm^2]
1	8100000	7161.75	735.61	190.53
2	21600000	7957.50	770.82	209.84
3	12420000	9549.00	836.80	247.90
4	6480000	12732.00	957.34	325.14
5	5400000	14323.50	1013.84	364.66
		$S_H = 1,458$		
		$S_F = 2,1899$		

3. Conclusion

The following conclusions can be drawn:

1. In order to obtain the exact results of the calculation of operating load it is necessary to know the size, course of change, frequency as well as the probability of occurrence of the greatest loads appearing during life for the part of the construction being observed.
2. The size of load of MS elements is obtained by experimental research or a computer simulation of the exploitation conditions.
3. The calculation of gears is based on the fact that each gear revolution has the corresponding damaging effect, i.e. the calculation has been made for Palmgren-Miner hypothesis.
4. An integrated, intelligent system for simultaneous designing of gearing has been developed at the Faculty of Mechanical Engineering in Niš. This programme enables to perform calculations for gears by simulating the exploitation conditions.

References

1. Miltenović V., Ognjanović M.: *Mašinski elementi-II - elementi za prenos snage*, Mašinski fakultet Niš - Beograd, 1995.
2. Niemann G., Winter H.: *Maschinenelemente, Band II - Getriebe allgemein*, Zahnradgetriebe- Grundlagen, Stirnradgetriebe, Springer-Verlag, 1985.
3. *DIN 3990* Teil 41
4. Miltenović,V., Milčić, D.: *Proračun nosivosti zupčastih prenosnika simulacijom eksploatacionih uslova*. Naučno-stručni skup "Istraživanje i razvoj mašinskih elemenata i sistema" - IRMES '98, 10. - 11. septembar 1998., Beograd, s.235-240.

PRORAČUN NOSIVOSTI ZUPČANIKA U NESTACIONARNO PROMENLJIVIM EKSPLOATACIONIM USLOVIMA

Vojislav Miltenović, Dragan Milčić

Visok stepen tačnosti pri dimenzionisanju i provera sigurnosti protiv razaranja može se postići merenjem eksploatacionih opterećenja vitalnih elemenata konstrukcije. Izbor merodavnog spektra opterećenja vrši se uvođenjem više spektara reprezenata za određene radne uslove pri čemu svaki od njih ima svoju verovatnoću pojave. Problem se najlakše rešava uvođenjem normiranih spektara opterećenja sa težnjom da se formiraju katalozi normiranih spektara opterećenja za reprezentativne mašinske sisteme.