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AN ANALYSIS OF THE MATHEMATICAL MODELS IN THE DEFORMATION PROCESS OF ECCENTRICALLY LOADED BOLTS

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Abstract. *Two mathematical models for determining bolt elastic lines, being eccentrically loaded by a working force, have been considered in the paper. Based on the elastic lines defined, it is very easy to obtain bolt bending stress, which at calculating such bolted connections might not be neglected. An example of the calculation for a concrete bolted connection has been given in the paper.*

Key words: *bolted connection, model, elastic line, force, bending stress*

1. INTRODUCTION

Within real working conditions, as the most widely used structural element connection, the bolted connection is most frequently exposed to an action of an eccentric working force. Such a working force disturbs a linearity of the forces and deformations in the connection, so the forces, namely stress are much higher if compared to the bolted connections, at which working force is of centric character. The elements of connection are exposed to a complex tension, originating out of stretching and bending. Thus, an additional bolt tension becomes more expressive, even at very low working force values, whereas the deformation in the connection are multiple increased.

The working force effect leads to a separation of the connected parts, this is beginning at one end of a contact surface, thus, as the working stress increases, it gradually spreads toward an opposite edge, where the bolt axis is.

At connections where jointing is to be provided, by widening an interspace, a jointing zone is decreased, and in a moment, when the separation of the connected parts reaches the bolt's axis, the bolted connection looses its jointing properties.

At the connections serving to transmit the force from one onto another connected part, the appearance of inter-distance causes an unfovourable tension distribution, enabling a

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relative moving of the jointed parts, the dirt penetration as well, this resulting in a mutual wear and tear of the connected parts. These negative effects are especially evident at the changeable force action.

The bolted connection calculations based on linear relations of the forces and deformations, corresponding to centrically stressed connections, in this case , do not give valid results. That is why an analysis of the relation among the forces and bolt deformations as well as the jointed parts of an eccentrically loaded connection has been done, and the corresponding supplements of the existing methods of calculation of the previosly fastened bolted connections have been made.

2. THE MODEL OF AN ECCENTRICALLY LOADED BOLTED CONNECTION

The bolted connection working force, being of assymetrical character, causes a complex connection element stress in the joint.

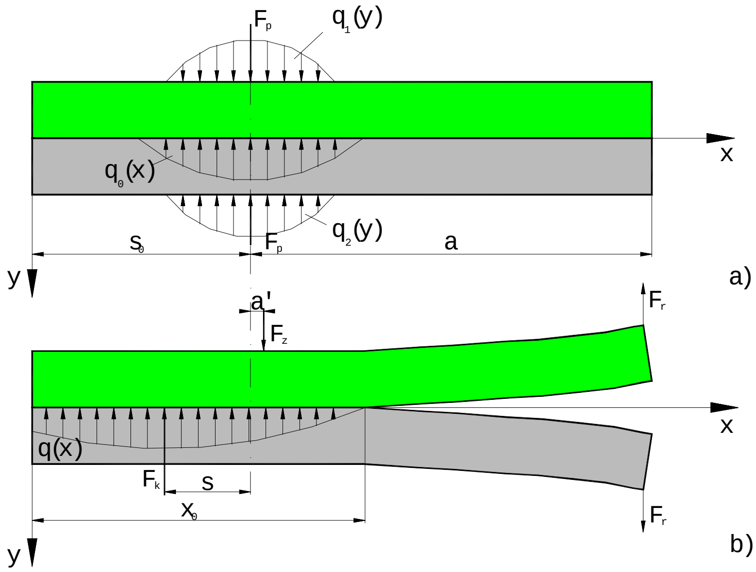

Fig. 1. Contact pressures (forces) arrangement
a) due to the F_p force of the previous seizing b) due to the effect of F_r working force

After the nut's seizing (Fig. 1a), there appears the force of the previous seizing F_p, being balanced with a contact pressure between the connected parts $q_0(x)$, namely with the reaction of the connected parts F_k.

By the working force action (Fig. 1b), a balanced state in the connection is disturbed, thus the force in the bolt increases to a quantity of F_z, whereas the connected parts $F_k = \int_0^{x_0} q(x)$, is displaced for an s values in relation to the bolt axis.

From the balanced connection state condition, it follows:

$$F_r = F_z - F_k \tag{1}$$

$$F_r \cdot a = F_k \cdot s + F_z \cdot a' \tag{2}$$

Due to the working force action, the force in the bolt is displaced in relation to the bolt axis for an a' quantity, but such a displacement is limited and depends on the bolt dimensions, its rigidity and the connected parts stifness.

The s quantity, defining the position of an F_k connected parts reacton, plays a very important role at calculating bolt connections loaded in this way. It may be, at the most, equal the size of the s_0 edge distance, when, there comes a critical case, namely the separation of the connected parts, and the total working force is further on accepted by the bolt.

There are several ways of s quantity determinantion, and one of the possible methods is given in the papers [1,2]. This procedure is based on superposing the results obtained by the separate observation of deformation resulted from single kinds of tension, to which the connection has been exposed (Fig. 2).

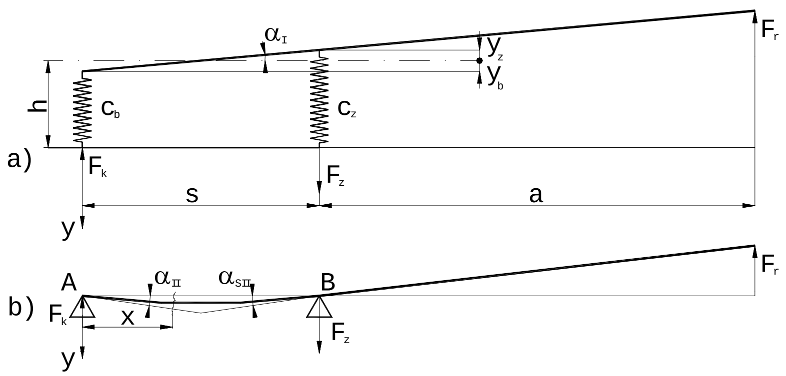

Fig. 2. The model of the eccentrically bolted connection
a) solid beam on movable supports b) elastic beam on immovable supports

If the slope angles in two separately observed cases are marked by α_I and α_{II} in sequence, their sum is:

$$\alpha_I + \alpha_{II} = 0 \tag{3}$$

$$\alpha_I = tg\alpha_I = -\frac{y_k + y_z}{s} = -\frac{1}{s}\left(\frac{F_k - F_p}{c_b} + \frac{F_z - F_p}{c_z}\right) \tag{4}$$

The angle α_I is determined out of the equation of the elastic line of the beam on immovable supports (Fig. 2b):

$$E \cdot I_p \cdot y_x'' = -M_f(x) \Rightarrow y_x'' = -\frac{1}{E \cdot I_p}(F_k \cdot x) \tag{5}$$

By integrating this equation, it follows:

$$y(x) = -\frac{1}{E \cdot I_p}(F_k \frac{x^3}{6} + C_1 \cdot x + C_2) \quad (6)$$

The integrating constants are determined out of a condition that the beam deflection in the support poins equals zero, thus:

$$y(x) = \frac{1}{E \cdot I_p}\frac{F_k}{6}(s^2 x - x^3) \quad (7)$$

$$y'_x = \frac{1}{E \cdot I_p}\frac{F_k}{6}(s^2 - 3x^2) \quad (8)$$

Then, it follows:

$$\alpha_{II} = y'_{x=0} = \frac{1}{E \cdot I_p}\frac{F_k \cdot s^2}{6} \quad (9)$$

Out of condition (3), taking care of force balance, acting upon bolted connection, a characteristic equation of bolted connection is obtained:

$$\frac{a \cdot c}{6 \cdot E \cdot I_p} \cdot s^3 + \left(\frac{F_p}{F_r} - \frac{c}{c_z}\right) \cdot s - a = 0 \quad (10)$$

Solving the equation (10), for the given force of the previous F_p seizing and F_r working force, an s quantity is obtained, based on which F_z and F_k force quantities may be determined.

3. Console model for the bolt elastic line determination

To determine the elastic line of the bolt whose shape is given in Fig. 3a, the bolt deformation analogy has been used, at connecting of the plates, whose surfaces are not parallel (Fig. 3b).

The bending angle of α_S plates (Fig. 3b) is determined out of the model shown in Fig. 2a, Fig. 2b, and that angle is:

$$\alpha_s = \alpha_I + \alpha_{sII} \quad (11)$$

The α_I angle is determined by the equation (4), whereas α_{SII} is obtained from the equation (8), if $x = s$:

$$\alpha_{sII} = y'_{x=s} = -\frac{F_k \cdot s^2}{3E \cdot I_p} \quad (12)$$

Now the α_S angle, according to the absolute size is:

$$\alpha_s = \frac{1}{s}\left(\frac{F_k}{c_b} - \frac{F_p}{c} + \frac{F_z}{c_z}\right) + \frac{F_k \cdot s^2}{3E \cdot I_p} \quad (13)$$

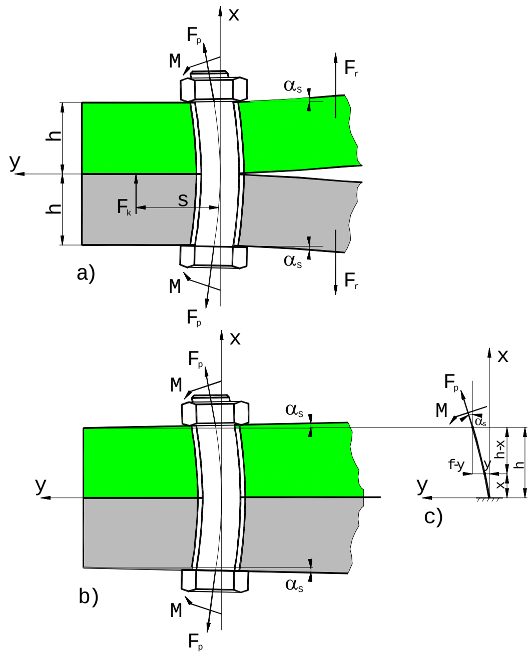

Fig. 3. Console model for obtaining bolt elastic line a) real shape of a deformed bolt b) shape of a deformed bolt in the connection of two oblique plates c) model of the console loaded by M moment and F_p force

3.1. Equation of the bolt elastic line

If, at F_r working force action, deformations are small, then in the connection of two oblique plates (Fig. 3b) the bolt deformation may approximate the deformation of the console being squeezed in cross section $y = 0$ (Fig. 3c), whereas the force F_p of the previous seize and M moment act at its end.

This is certainly not a classical example of console, but it may be taken as an adequate model, as in the cross section $x = 0$, slope is $y_x' = 0$, and here are maximum bolt bending stresses, this resulting from the connection symmetry.

Also, not a classical seizure at the place $x = 0$ is observed, but the loads at the end of the console. For the console stressed by M moment at its end, the following is valid:

$$\alpha_{s(x=h)} = \frac{M \cdot h}{E \cdot I_z} \Rightarrow M = \frac{\alpha_s \cdot M \cdot I_z}{h} \qquad (14)$$

Differential equation of the bolt elastic line is:

$$E \cdot I_z \cdot y'' = M + F_p \cdot \sin\alpha_s (h - x) - F_p \cdot \cos\alpha_s \cdot (f - y) \qquad (15)$$

The solution of this equation is:

$$y = y_h + y_p = A \cdot ch\omega x + B \cdot sh\omega x + C \cdot x + D$$

Here is: $\omega = \sqrt{\dfrac{F_p \cdot \cos\alpha_s}{E \cdot I_z}}$

Integrating constants are determined out of limit conditions, where in the cross section $x = 0$, deflection and slope equal to zero, thus the bolt elastic line equation is of the following form:

$$y = \left(\frac{M}{F_p \cdot \cos\alpha_s} + h \cdot tg\alpha_s - f \right) \cdot ch\omega x - \frac{tg\alpha_s}{\omega} \cdot sh\omega x - \frac{M}{F_p \cdot \cos\alpha_s} - tg\alpha_s (h - x) + f \tag{16}$$

Deflection at the end of console is obtained out of following condition f=y if x=h, then:

$$f = \left[\frac{M}{F_p \cdot \cos\alpha_s} \cdot \left(\frac{ch\omega h - 1}{ch\omega h} \right) + tg\alpha_s \cdot \left(h - \frac{th\omega h}{\omega} \right) \right] \tag{17}$$

3.2. Bolt bending stress

The aim of this analysis was to determine the stresses appearing in the bolt due to F_r working force action. Thus, the bolt bending stress is: $\sigma_f = M_f / W_x$, where:

- bolt bending moment is: $M_f = M + F_p \cdot \sin\alpha_s \cdot (h - x) - F_p \cdot \cos\alpha_s \cdot (f - y)$
- moment of resistance of a cross section of the bolt is: $W_x = \pi \cdot d_s^3 / 32$

Maximal bolt bending stress is in the cross section $x = 0$:

$$\sigma_{f\max} = \frac{32}{\pi \cdot d_s^3} (M + F_p \cdot \sin\alpha_s \cdot h - F_p \cdot \cos\alpha_s \cdot f) \tag{18}$$

4. Polynomial model for determining bolt elastic line

As the bolt connection observed is symmetrical, polynomial model given in Fig. 4 may be used for determining bolt elastic line.

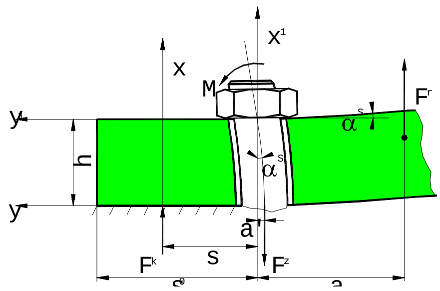

Fig. 4. Polynomial model for determining bolt elastic line

When F_r working force acts, it results in deformation of the connected parts, thus, at the supporting place, the nut is turned for α_S angle, and moment appears in that place is:

$$M = \frac{E \cdot I_z \cdot \alpha_s}{h} \tag{19}$$

The nut turning angle is determined in the same way as in the section 3, thus the equation (13) is valid:

$$\alpha_s = \frac{1}{s}\left(\frac{F_k}{c_b} - \frac{F_p}{c} + \frac{F_z}{c_z}\right) + \frac{F_k \cdot s^2}{3E \cdot I_p} \tag{20}$$

4.1. Bolt elastic line equation

The bolt elastic line equation may be supposed in the polynomial form. The coefficients A_i (i = 1,2,3 and 4) are determined out of the limit conditions:

1. for $x = 0$, it follows $y = -(s + a')$
2. for $x = 0$, it follows $y_x' = 0$
3. for $x = h$, it follows $y_x' = \mathrm{tg}\alpha_S$
4. for $x = h$, it follows $y_x'' = \dfrac{M}{E \cdot I_z}$

The quantity a’ may be determined out of the force balance condition (2):

$$a' = \frac{F_r \cdot a - F_k \cdot s}{F_z} \tag{21}$$

By changing the limit conditions, the bolt elastic line equation is obtained:

$$y = -(s + a') + \left(\frac{tg\alpha_s}{h} - \frac{M}{2 \cdot E \cdot I_z}\right) \cdot x^2 + \left(-\frac{tg\alpha_s}{3 \cdot h^2} + \frac{M}{3 \cdot E \cdot I_z \cdot h}\right) \cdot x^3 \tag{22}$$

4.2. Bolt bending stress

Bending stress is $\sigma_f = M_f / W_x$, whereas bending moment is:

$$M_f = E \cdot I_z \cdot y'' \tag{23}$$

From the bolt elastic line equation (22), it follows:

$$y_x'' = 2 \cdot \left(\frac{tg\alpha_s}{h} - \frac{M}{2E \cdot I_z}\right) + 6 \cdot \left(-\frac{tg\alpha_s}{3h^2} + \frac{M}{3E \cdot I_z \cdot h}\right) x \tag{24}$$

Now, the bolt bending moment from equation (23) is:

$$M_f = E \cdot I_z \cdot \left[2 \cdot \left(\frac{tg\alpha_s}{h} - \frac{M}{2 \cdot E \cdot I_z}\right) + 6 \cdot \left(-\frac{tg\alpha_s}{3h^2} + \frac{M}{3 \cdot E \cdot I_z \cdot h}\right) x\right] \tag{25}$$

The maximum bolt bending in the cross section $x = 0$ is:

$$M_{f\,\max} = 2 \cdot E \cdot I_z \cdot \left(\frac{tg\alpha_s}{h} - \frac{M}{2 \cdot E \cdot I_z}\right) \tag{26}$$

The maximum bolt bending stress is:

$$\sigma_{f\max} = \frac{64 \cdot E \cdot I_z}{\pi \cdot d_s^3}\left(\frac{tg\alpha_s}{h} - \frac{M}{2 \cdot E \cdot I_z}\right) \tag{27}$$

5. CALCULATION RESULTS

The calculation of a bolted connection that is loaded by working force, can not be done without using computer. Based on algorhytm, there have been made computer programs, where, on the base of input quantities, all the necessary quantities may be computed in a qiuck and easy way. Also, by varying input quantities, the dependences of single quantities on the working force growth may be obtained.

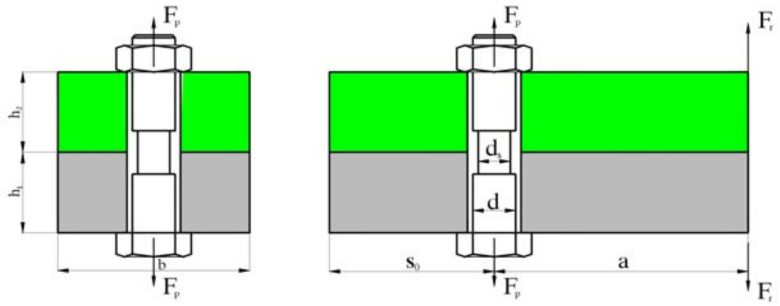

Fig. 5. Bolted connection computation model

The computation model, for which experimental investigations have been done, is shown in Fig. 5, and it has the following characteristics:

$s_0 = 60$ mm, $h_1 = h_2 = h = 30$ mm, $a = 100$ mm, $b = 60$mm, $d_s = 9$ mm

$E = 2.1 \cdot 10^5$ N/mm^2, $c_z = 0.1198 \cdot 10^6$ N/mm, $c_b = 0.5292 \cdot 10^6$ N/mm

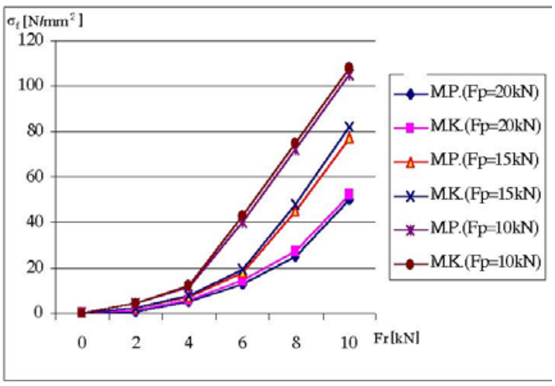

Fig. 6. Bending stress dependence on working force growth
(M.P.- Polynimial model, M.K.-Console model)

The bending stress σ_f computation results depending on F_r working force growth, for different forces of previous seize will be given in this paper.

Both models, as it may be seen in the diagram, give good results coinciding considerably with experimental investigations.

6. Conclusion

As it has been pointed out in the paper, the action of eccentric working force disturbs linear relations of the force and deformation in a connection. For these reasons, even at very small working force acting, additional force in bolt increases, at which there appear not only bolt stretching stresses but also bolt bending stresses, and they should be taken into consideration at computing such connections.

Both mathematical models for determining bolt elastic line give good results which, to a great extent, agree with experimental results, though it may be said that the parabole model is more simple to be used and less approximations are made if compared to the console model.

References

1. Agatonović, P. *Zusammengesetzte Betriebsgeanspruchung von Schraubenverbindungen*, Konstruktion, No.26, (1974).
2. Agatonović, P. *Beitrag zur Berechnung von Schraubenverbindungen*, Drahtwelt, No. 58, (1972).
3. Birger, I.A., Iosilevič, G.B. *Rezbovie soedinenija*, Mašinostroenie, Moskow, (1973).
4. Bulatović, R. *Investigation of causes of self-unscrewing of longitudinaly, transversaly, static and dynamic loaded bolted connection*, Doctoral thesis, Faculty of Mechanical Engineering, Belgrade, (1989).
5. Bulatović, R., Jovanović, J. *Theoretical and experimental investigation of eccentrically loaded bolted connection*, IRMES '95, Niš, (1995).
6. Iosilevič, G.B., Šarpovskij, Yu.V. *Zatjažka i stoporenije rezbovih soedinenij*, Mašinostroenie, Moskow, (1971).
7. Kowalske, D. *Die Schraubenverbindung als ein Problem elastisch gebetteter Platten*, Konstruktion, No.30, (1978).

ANALIZA MATEMATIČKIH MODELA U PROCESU DEFORMACIJA EKCENTRIČNO OPTEREĆENIH ZAVRTNJEVA

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U radu su razmatrana dva matematička modela, za određivanje elastične linije zavrtnja, koji je opterećen ekcentričnom radnom silom. Na osnovu definisanih elastičnih linija, veoma lako i brzo se dolazi do napona savijanja zavrtnja, koji se pri proračunu ovakvih veza ne bi smjeli zanemarivati. U radu je dat primjer proračuna za konkretnu zavrtanjsku vezu.

Ključne reči: *zavrtanjska veza, model, elastična linija, sila, napon savijanja.*