FACTA UNIVERSITATIS
Series: **Mechanical Engineering** Vol.1, N° 7, 2000, pp. 769 - 773

THE ENERGY BALANCE FOR DAMPED WIND-EXCITED VIBRATIONS

UDC 625.5:551.311.3

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Abstract. *In the calculation of wind-excited overhead transmission line with Stockbridge dampers the damper behavior is usually represented by its impedance corresponding to a vertical translatory damper clamp motion. The moments introduced by the damper into the cable are normally disregarded. In this paper the dampers are characterized by means of a* 2×2 *complex impedance matrix which can be experimentally determinated in the laboratory and which includes the effects of the rotatory motion of the clamp. The energy balance method is then adapted to this case and the bending strains in the cable are calculated at the dangerous points. It turns out that the moments introduced by the damper into the cable are of little or no importance with regard to the energy absorbed. They may however affect strongly the local bending strains in the cable at the damper clamp.*

1. INTRODUCTION

Different types of mechanical vibrations occur in overhead lines and frequently lead to severe damage. In the present paper the aeolian vibrations are studied, which are usually observed in the frequency range 10-100 Hz, and may cause failure due to material fatigue. In this paper the energy balance for damped wind excited vibrations, and the power dissipated by the damper and the cable are considered.

2. POWER DISSIPATED BY THE DAMPER

The energy balance for steady state vibrations is shown through the equation:

$$P_w = P_D + P_C \tag{1}$$

Pw being the power of the aerodynamic forces, P_D the power corresponding to the

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Received August 15, 2000
Presented at 5th YUSNM Niš 2000, Nonlinear Sciences at the Threshold of the Third Millenium, October 2-5, 2000, Faculty of Mechanical Engineering University of Niš

dissipation of mechanical energy in the damper and P_C the power of the material damping in the cable.

The damper extracts mechanical energy from the cable at the isolated point $x = l_1$, the wind forces act over the whole span, introducing power over the complete length of the cable. Mechanical energy has therefore to travel along the cable towards the point $x = l_1$, where it is dissipated in the damper. The damping power is computed for the case of a simple harmonic wave travelling from the middle of the span towards the damper. To this incident wave corresponds a reflected wave travelling from the damper towards the middle of the span, as well as oscillations in the cable between the damper and the clamped end at $x = 0$. These oscillations are calculated by requiring the proper boundary conditions to be fulfilled, with account taken of the dynamic behavior of the damper. Of course the power dissipation by the damper depends strongly on the damper location l_1.

The wind excited vibrations of small amplitude are described by:

$$EIw^{IV} - Tw^{II} + \rho\ddot{w} = q(x,t) + \bar{d}(w,\dot{w},t) \qquad x \neq l_1 \tag{2}$$

where EI is the bending stiffness of the cable, T is its tension, ρ is the mass per unit length, $w(x,t)$ is the transverse displacement of the cable, q(x,t) is the wind force due to vortex shedding and $\bar{d}$ is the material damping. The distance l_1 from the damper to the fixed point is usually small with respect to the span l. Including the new dimensionless co-ordinate $z = 2\pi/\lambda = kx$, normalized with respect to the wavelength and writing the solutions of equation (2) with $q \equiv 0$, $d \equiv 0$ in the form

$$w(x,t) = U(z)\sin\omega t + V(z)\cos\omega t \qquad \omega = 2\pi / f \tag{3}$$

one obtains, from equation (2) the ordinary differential equations:

$$\begin{aligned} \mu^2 d^4U / dz^4 - d^2U / dz - a^2U = 0 \\ \mu^2 d^4V / dz^4 - d^2V / dz - a^2V = 0 \end{aligned} \tag{4}$$

with the abbreviations

$$\mu^2 = (2\pi)^2 EI / T\lambda^2 \qquad a^2 = f^2\lambda^2\rho / T \tag{5}$$

Since λ is supposed to be the wavelength corresponding to the frequency f, one of the roots of the characteristic equation corresponding to equations (4) has to be equal to unity, so that

$$a^2 = 1 + \mu^2 \tag{6}$$

is the relation between frequency f and the wavelength λ. The general solution of equations (4) is

$$\begin{aligned} U(z) = B_U \sin z + C_U \cos z + D_U e^{pz} + F_U e^{-pz} \\ V(z) = B_V \sin z + C_V \cos z + D_V e^{pz} + F_V e^{-pz} \end{aligned} \tag{7}$$

with B_U, C_U,...F_U as integration constants and $p = \sqrt{1 + 1/\mu^2}$. The solution for the domain $x \geq l_1$ is written as

$$w(x,t) = U_1(z)\sin\omega t + V_1(z)\cos\omega t \quad (8)$$

and for $0 \le x \le l_1$

$$w(x,t) = U_2(z)\sin\omega t + V_2(z)\cos\omega t \quad (9)$$

The solution for $x \to \infty$ is given by the superposition of simple harmonic waves traveling in opposite directions,

$$\begin{gathered} w(x,t) = [(u-g)\sin z + h\cos z]\sin\omega t + [(u+g)\cos z + h\sin z]\cos\omega t \\ k = 2\pi/\lambda, \quad c = \omega/k \end{gathered} \quad (10)$$

At the point $x = l_1$ the displacement $w(l_1,t)$ and the slope $w'(l_1,t)$ must be continuous:

$$U_1(\bar{l}) = U_2(\bar{l}) \quad V_1(\bar{l}) = V_2(\bar{l})$$

$$\left(\frac{dU_1}{dz}\right)_{\bar{l}} = \left(\frac{dU_2}{dz}\right)_r \left(\frac{dV_1}{dz}\right)_{\bar{l}} = \left(\frac{dV_2}{dz}\right)_{\bar{l}} \quad \bar{l} = \frac{2\pi l_1}{\lambda} \quad (11)$$

The damper is a passive system, which can only dissipate and not generate mechanical energy, the mean power

$$P_D = \frac{1}{T}\int_0^T F_{(t)}\dot{y}(t)dt + \frac{1}{T}\int_0^T M_{(t)}\dot{\varphi}(t)dt \quad (12)$$

must always be positive. In equation (11), $F(t)$ and $M(t)$ are the vertical force and a moment that damper clamp exerts on the cable, and $\dot{y}(t)$ and $\dot{\varphi}(t)$ are the velocity and angular velocity of the damper clamp:

$$\begin{aligned} F(t) &= Z_{11}\dot{y}_0\sin(\omega t + \alpha_{11}) + Z_{12}\dot{\varphi}_0\sin(\omega t - \beta + \alpha_{12}) \\ M(t) &= Z_{21}\dot{y}_0\sin(\omega t + \alpha_{21} + Z_{22}\dot{\varphi}_0\sin(\omega t - \beta + \alpha_{22}) \end{aligned} \quad (13)$$

$$\begin{aligned} \dot{y}(t) &= \dot{y}_0\sin\omega t \\ \dot{\varphi}(t) &= \dot{\varphi}_0\sin(\omega t - \beta) \end{aligned}$$

The coefficients Z_{ij} and α_{ij}, i,j = 1,2 define the impedance of the damper and can be experimentally determined. The dynamic characteristics of the damper could be completely described by the complex 2 × 2 impendance matrix:

$$\begin{pmatrix} Z_{11}e^{i\alpha_{11}} & Z_{12}e^{i\alpha_{12}} \\ Z_{21}e^{i\alpha_{21}} & Z_{22}e^{i\alpha_{22}} \end{pmatrix} = \begin{pmatrix} R_{11} + iI_{11} & R_{12} + iI_{12} \\ R_{21} + iI_{21} & R_{22} + iI_{22} \end{pmatrix} \quad (14)$$

with $R_{jk} = Z_{jk}\cos\alpha_{jk} \quad I_{jk} = Z_{jk}\sin\alpha_{jk} \quad j,k = 1,2.$

3. Power of the Aerodynamic Forces

The power of the aerodynamic forces is calculated for the case of standing waves in an infinite cable, i.e. the distortion at the ends of the cable and near the damper were due to small bending stiffness disregarded in this calculation. Typically the length of the span is about 500 m and the bending stiffness is so weak that these perturbations are not perceptible except at a very short distance from the end or from the damper respectively, so that these distortions do not introduce a measurable error in the calculated wind power P_w. Using experimental results due to reference [2] the following expression for the power of the aerodynamic forces could be given:

$$P_W = lf^3 D^4 \frac{1}{\pi} \sum_{i=1}^{10} b_i \left(\frac{A}{D} \right)^i \tag{15}$$

D being the diameter of the cable. The coefficients b can be computed from reference 2, and

$$f = c_s v / D$$

$c_s = 0.19$, v being the wind velocity.

4. Material and structural damping in the cable

The material damping and structural damping in the cable has been investigated experimentally by several authors. Usually an expression of type:

$$P_C = C_2 f^n \beta^m \left(\frac{P}{T} \right) l$$

is employed, where C_2, m and n are constants which may vary from one author to another. It is assumed that m and n are constant for all cables and that C_2 characterizes the damping properties of the particular cable being considered. The variable $\beta = 2\pi A/\lambda$ is the angular amplitude, and P is the limit load of the cable.

The material and structural damping in the cable is unimportant in transmission lines if dampers are used,

$$P_D \succ\succ P_C$$

5. Conclusion

In the present paper the energy balance method in which account is taken of the location of the damper on the cable is generalized in such a way that not only the forces but also the moment transmitted by the damper clamp to the cable are included. To do this, one requires knowledge of a complex 2 × 2 impedance matrix for the damper instead of a single scalar complex impedance as used so far. The energy balance for steady state vibrations shown through the equation (1) can be adapted to the case of conductor equipped with Stockbridge dampers and the vibration amplitudes and the bending strains in the cable could be calculated at the dangerous points.

References

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ENERGIJSKI BILANS ZA AMORTIZOVANE VETROM POBUĐENE VIBRACIJE

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Različiti tipovi mehaničkih vibracija nastaju kod žičara i često dovode do ozbiljnih oštećenja. U radu su izučavane eolske vibracije, koje imaju uobičajenu učestanost 10-100 Hz i koje mogu da dovedu do otkaza zbog zamora materijala. U ovom radu su razmatrani i energijski bilansi za amortizovane vetrom pobuđene vibracije kao i energija koju disipiraju amortizer i kabl.