SIMULATION MODELING OF AN ESSENTIALLY NON-LINEAR DYNAMIC SYSTEM

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Abstract. The forced vibrations of an essentially non-linear dynamic system are examined. The physical nature of the essential non-linearity is due to the existence of dry friction forces with a changing magnitude. A comparative research of the forced vibrations of the two-mass essentially non-linear system by the presence of dry friction of “harmonics” type is done. A frequency analysis of the vibrations is given. A simulation model of a system of differential equations with different parameters is completed from a view of building up an effective vibroprotection system. A broad spectrum of a disturbing frequency is searched so that the system completes its function as a protector against vibrations. Conclusions of the decisions characterizing the influence of the model parameters on the movement of the mechanical system are drawn.

DYNAMIC MODELS

The dynamic dampening of vibrations is a method where additional devices called dynamic dampers are introduced into a vibrating system. They realize the dynamic dampening of vibrations using the principle of redistributing the vibration energy and directing it from the object protected to the damper and the principle of increasing the quantity of scattered energy in the system.

The paper examines the vibrations of the simplest inertia dynamic damper (the so called Fram's damper) in the presence of dry friction of "harmonic" type. Five dynamic models given below are studied. The differential equations describing the oscillations of the corresponding models have the kind of:

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Model I
\[ \begin{align*} 
\ddot{x}_1 + k_1^2 x_1 - k_2^2 v(x_2 - x_1) &= h \sin pt, \\
\ddot{x}_2 + k_2^2 (x_2 - x_1) &= 0. 
\end{align*} \]

Model II
\[ \begin{align*} 
\ddot{x}_1 + k_1^2 x_1 - k_2^2 v(x_2 - x_1) + f \cos pt | \text{sign } \dot{x}_1 &= h \sin pt, \\
\ddot{x}_2 + k_2^2 (x_2 - x_1) &= 0. 
\end{align*} \]

Model III
\[ \begin{align*} 
\ddot{x}_1 + k_1^2 x_1 - k_2^2 v(x_2 - x_1) + f \cos pt | \text{sign } \dot{x}_1 &= h \sin pt, \\
\ddot{x}_2 + k_2^2 (x_2 - x_1) + f \cos pt | \text{sign } \dot{x}_2 &= h \sin pt. 
\end{align*} \]

Model IV
\[ \begin{align*} 
\ddot{x}_1 + k_1^2 x_1 - k_2^2 v(x_2 - x_1) + f \cos pt | \text{sign } \dot{x}_1 &= h \sin pt, \\
\ddot{x}_2 + k_2^2 (x_2 - x_1) + f \cos pt | \text{sign } \dot{x}_2 &= -h \sin pt. 
\end{align*} \]

Model V
\[ \begin{align*} 
\ddot{x}_1 + k_1^2 x_1 - k_2^2 v(x_2 - x_1) &= h \sin pt, \\
\ddot{x}_2 + k_2^2 (x_2 - x_1) + f \cos pt | \text{sign } \dot{x}_2 &= 0. 
\end{align*} \]
Here $x_1$ and $x_2$ are the generalized co-ordinates corresponding to the dynamic models; $k_1$ and $k_2$ are the corresponding natural frequencies; $p$ is the frequency of the disturbing force; $\nu = m_1/m_2$ is the ratio between the inertia features of the two masses. Coefficient $h$ characterizes the amplitude of the harmonic disturbing force and coefficient $f$ characterizes the amplitude of the dry friction force, which is of "harmonic" type as it can be seen.

RESULTS OF SIMULATIONS

Using MATLAB software package, simulation modelling of the differential equations written above has been done. The integration is based on an explicit Runge-Kutta (4,5) formula, the Dormand-Prince pair. The following parameters of the system have been accepted: $p = 1; h = 1; f = 0.1; \nu = 0.5$.

Natural frequencies $k_1$ and $k_2$ change while carrying out the simulation modelling. A frequency analysis of the oscillations along the two generalized co-ordinates is made for corresponding combination. The distribution of the maximal values of the amplitude spectral density with different values of $k_1$ and $k_2$ is given in next figures.

Amplitude $a_1$                                      Amplitude $a_2$

Model-I

Model-II

Model-III
The general analysis of the results shows the following peculiarities:

1. The amplitude of the basic mass $a_1$ decreases insignificantly in comparison to the basic model-I with switching to the damper with dry friction of "harmonics" type (model-V). Here it is characteristic that the system "locking" has not been watched of the both models as it is of systems with dry friction and a constant magnitude.

2. Zones of system "locking" have been watched in the presence of disturbing and resistance forces acting together on both of the masses. These zones are in the area of high natural frequencies with anti-phase disturbances (model-IV) and mainly around the resonance areas with synchronic disturbances (model-III).

3. Without looking for the coefficient of vibroprotection efficiency, we can estimate that a wide area around the resonance is characteristic for model-II: low values of basic mass $a_1$ amplitude have been watched in that area.

REFERENCES
