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FORCING DETOUR MONOPHONIC NUMBER OF A GRAPH*

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Abstract. For a connected graph $G = (V, E)$ of order at least two, a *chord* of a path P is an edge joining two non-adjacent vertices of P. A path P is called a *monophonic path* if it is a chordless path. A longest $x - y$ monophonic path is called an $x - y$ *detour monophonic path*. A set S of vertices of G is a *detour monophonic set* of G if each vertex v of G lies on an $x - y$ detour monophonic path for some elements x and y in S. The minimum cardinality of a detour monophonic set of G is the *detour monophonic number* of G and is denoted by $dm(G)$. A subset T of a minimum detour monophonic set S of G is a *forcing detour monophonic subset* for S if S is the unique minimum detour monophonic set containing T. A forcing detour monophonic subset for S of minimum cardinality is a *minimum forcing detour monophonic subset* of S. The *forcing detour monophonic number* $f_{dm}(S)$ in G is the cardinality of a minimum forcing detour monophonic subset of S. The *forcing detour monophonic number* of G is $f_{dm}(G) = min\{f_{dm}(S)\}$, where the minimum is taken over all minimum detour monophonic sets S in G. We determine bounds for it and find the forcing detour monophonic number of certain classes of graphs. It is shown that for every pair a, b of positive integers with $0 \le a < b$ and $b \ge 2$, there exists a connected graph G such that $f_{dm}(G) = a$ and $dm(G) = b$.

1. Introduction

By a graph $G = (V, E)$ we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q respectively. For basic graph theoretic terminology we refer to Harary [5]. The *distance* $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest $u - v$ path in G. A $u - v$ path of length $d(u, v)$ is called a $u - v$ *geodesic*. For a vertex v of G, the *eccentricity* $e(v)$ is the distance between v and a vertex farthest from v. The minimum eccentricity among the vertices of G is the *radius*, *rad* G and the maximum eccentricity is its *diameter*, *diam* G of G. Two vertices u and v of G are called *antipodal* if $d(u, v) = diam\,G$. The *neighborhood* of a vertex v is the set $N(v)$ consisting of all vertices u which are adjacent with v. The *closed neighborhood* of a

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vertex v is the set $N[v] = N(v) \bigcup \{v\}$. A vertex v is an *extreme vertex* if the subgraph induced by its neighbors is complete.

The *detour distance* $D(u, v)$ between two vertices u and v in G is the length of a longest $u - v$ path in G. A $u - v$ path of length $D(u, v)$ is called a $u - v$ *detour*. It is known that D is a metric on the vertex set V of G. The concept of detour distance was introduced in [1] and further studied in [2]. The closed detour interval $I_D[x, y]$ consists of x, y, and all the vertices in some $x - y$ detour of G. For $S \subseteq V$, $I_D[S]$ is the union of the sets $I_D[x, y]$ for all $x, y \in S$. A set S of vertices of a graph G is a *detour set* if $I_D[S] = V$, and the minimum cardinality of a detour set is the *detour number* $dn(G)$. The concept of detour number of a graph was introduced in [3] and further studied in [4].

A *chord* of a path P is an edge joining two non-adjacent vertices of P. A path P is called a *monophonic path* if it is a chordless path. A longest $x - y$ monophonic path is called an $x - y$ *detour monophonic path*. A set S of vertices of G is a *detour monophonic set* if each vertex v of G lies on an $x - y$ detour monophonic path for some $x, y \in S$. The minimum cardinality of a detour monophonic set of G is the *detour monophonic number* of G and is denoted by $dm(G)$. The detour monophonic set of cardinality $dm(G)$ is called *dm*-set. The detour monophonic number of a graph was introduced in [7] and further studied in [6]. There are interesting applications of these concepts to the problem of designing the route for a shuttle and communication network design.

For the graph G given in Figure 1.1, $S_1 = \{z, w, v\}$, $S_2 = \{z, w, u\}$ and $S_3 = \{z, w, x\}$ are the minimum detour monophonic sets of G and so $dm(G)$=3.

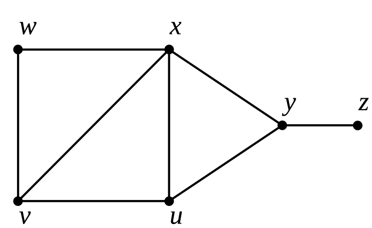

Figure 1.1: Graph G

A connected graph G may contain more than one minimum detour monophonic sets. For example, the graph G given in Figure.1.1 contains three minimum detour monophonic sets. For each minimum detour monophonic set S in G there is always some subset T of S that uniquely determines S as the minimum detour monophonic set containing T. Such sets are called "forcing detour monophonic subsets "and we discuss these sets in this paper.

The following theorems will be used in the sequel.

Theorem 1.1. [7] Each extreme vertex of a connected graph G belongs to every detour monophonic set of G.

Theorem 1.2. [7] For the complete graph $K_p (p \geq 2)$, $dm(K_p) = p$.

Theorem 1.3. [7] No cutvertex of a connected graph G belongs to any minimum detour monophonic set of G.

Theorem 1.4. [7] For the cycle $C_n (n \geq 3)$,

$$dm(C_n) = \begin{cases} 2 & \text{if } n \text{ is even} \\ 3 & \text{if } n \text{ is odd} \end{cases}$$

Throughout the paper G denotes a connected graph with at least two vertices.

2. Forcing Detour Monophonic Number

Definition 2.1. *Let G be a connected graph and let S be a minimum detour monophonic set of G. A subset T of a minimum detour monophonic set S of G is a forcing detour monophonic subset for S if S is the unique minimum detour monophonic set containing T. A forcing detour monophonic subset for S of minimum cardinality is a minimum forcing detour monophonic subset of S. The forcing detour monophonic number $f_{dm}(S)$ in G is the cardinality of a minimum forcing detour monophonic subset of S. The forcing detour monophonic number of G is $f_{dm}(G) = min\{f_{dm}(S)\}$, where the minimum is taken over all minimum detour monophonic sets S in G.*

Example 2.1. *For the graph G given in Figure 1.1, $S_1 = \{z, w, v\}$, $S_2 = \{z, w, u\}$ and $S_3 = \{z, w, x\}$ are the minimum detour monophonic sets of G. It is clear that $f_{dm}(S_1) = 1$, $f_{dm}(S_2) = 1$ and $f_{dm}(S_3) = 1$ so that $f_{dm}(G) = 1$. For the graph G given in Figure 2.1, $S = \{y, v\}$ is the unique minimum detour monophonic set of G and so $f_{dm}(G) = 0$.*

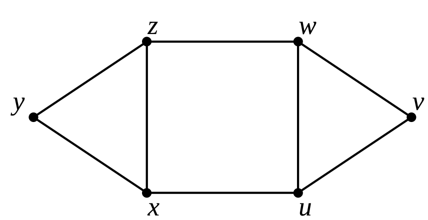

Figure 2.1: Second graph G

The next theorem follows immediately from the definition of the detour monophonic number and forcing detour monophonic number of a graph G.

Theorem 2.2. *For a connected graph G, $0 \leq f_{dm}(G) \leq dm(G) \leq p$.*

Remark 2.1. *The bounds in Theorem 2.2 are sharp. For the graph G given in Figure 2.1, $f_{dm}(G) = 0$. By Theorem 1.2, for the complete graph $K_p (p \geq 2)$, $dm(K_p) = p$. The inequalities in Theorem 2.2 are strict. For the graph G given in Figure 1.1, $dm(G) = 3$ and $f_{dm}(G) = 1$. Thus $0 < f_{dm}(G) < dm(G) < p$.*

The following theorem is an easy consequence of the definitions of the detour monophonic number and forcing detour monophonic number. In fact, the theorem characterizes graphs G for which the lower bound in Theorem 2.2 is attained and also graphs G for which $f_{dm}(G) = 1$ and $f_{dm}(G) = dm(G)$.

Theorem 2.3. *Let G be a connected graph . Then*

(i) $f_{dm}(G) = 0$ *if and only if G has a unique minimum detour monophonic set.*

(ii) $f_{dm}(G) = 1$ *if and only if G has at least two minimum detour monophonic sets, one of which is a unique minimum detour monophonic set containing one of its elements, and*

(iii) $f_{dm}(G) = dm(G)$ *if and only if no minimum detour monophonic set of G is the unique minimum detour monophonic set containing any of its proper subsets.*

Definition 2.4. *A vertex v of a connected graph G is said to be a detour monophonic vertex of G if v belongs to every minimum detour monophonic set of G.*

We observe that if G has a unique minimum detour monophonic set S, then every vertex in S is a detour monophonic vertex of G. Also, if x is an extreme vertex of G, then x is a detour monophonic vertex of G. For the graph G given in Figure 1.1, w and z are the detour monophonic vertices of G.

The following theorem and corollary follows immediately from the definitions of detour monophonic vertex and forcing detour monophonic subset of G.

Theorem 2.5. *Let G be a connected graph and let $\mho_{dm}$ be the set of relative complements of the minimum forcing detour monophonic subsets in their respective minimum detour monophonic sets in G. Then $\bigcap_{F \in \mho_{dm}} F$ is the set of detour monophonic vertices of G.*

Corollary 2.6. *Let G be a connected graph and let S be a minimum detour monophonic set of G. Then no detour monophonic vertex of G belongs to any minimum forcing detour monophonic subset of S.*

Theorem 2.7. *Let G be a connected graph and let M be the set of all detour monophonic vertices of G. Then $f_{dm}(G) \leq dm(G) - |M|$.*

Proof. Let S be any minimum detour monophonic set of G. Then $dm(G) = |S|$, $M \subseteq S$ and S is the unique minimum detour monophonic set containing $S - M$. Thus $f_{dm}(G) \leq |S - M| = |S| - |M| = dm(G) - |M|$. $\square$

Corollary 2.8. *If G is a connected graph with l extreme vertices, then $f_{dm}(G) \leq dm(G) - l$.*

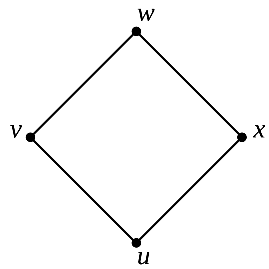

Figure 2.2: Third graph G

Remark 2.2. *The bound in Theorem 2.7 is sharp. For the graph G given in Figure 1.1,* $dm(G) = 3$ *and* $f_{dm}(G) = 1$*. Also,* $M = \{w, z\}$ *is the set of all detour monophonic vertices of G and so* $f_{dm}(G) = dm(G) - |M|$*. Also the inequality in Theorem 2.7 can be strict. For the graph G given in Figure 2.2,* $S_1 = \{x, v\}$ *and* $S_2 = \{u, w\}$ *are the minimum detour monophonic sets so that* $dm(G) = 2$ *and* $f_{dm}(G) = 1$*. Also, no vertex of G is a detour monophonic vertex of G, we have* $f_{dm}(G) < dm(G) - |M|$*.*

Theorem 2.9. *Let G be a connected graph and let S be a minimum detour monophonic set of G. Then no cutvertex of G belongs to any minimum forcing detour monophonic subset of S.*

Proof. Let v be a cutvertex of G. By Theorem 1.3, v does not belong to any minimum detour monophonic set of G. Since any minimum forcing detour monophonic subset of S is a subset of S, the result follows from Theorem 2.5. □

Theorem 2.10. *If G is a connected graph with* $dm(G) = 2$*, then* $f_{dm}(G) \leq 1$*.*

Proof. Let $dm(G) = 2$. Then by Theorem 2.2, $f_{dm}(G) \leq 2$. Suppose that $f_{dm}(G) = 2$. Then by Theorems 2.3(i) and 2.3(iii), G has at least two minimum detour monophonic sets and no minimum detour monophonic set of G is the unique minimum detour monophonic set containing any of its proper subsets. Since $dm(G) = 2$, there exists a unique element, say x, is common for any two minimum detour monophonic sets, say S_1 and S_2. Let $S_1 = \{x, u\}$ and $S_2 = \{x, v\}$. Since S_1 is a minimum detour monophonic set, v lies on an $x - u$ detour monophonic path. Similarly, since S_2 is a minimum detour monophonic set, u lies on an $x - v$ detour monophonic path. Then the vertices x, u and v lie on a cycle. Let C be a longest cycle containing the vertices x, u and v. Then the length of C is more than 4. If C is an even cycle, then either S_1 or S_2 is not a detour monophonic set of G, which is a contradiction. If C is an odd cycle, then any internal vertex of an $x - u$ geodesic does not lie on an $x - u$ detour monophonic path and so S_1 is not a detour monophonic set of G, which is a contradiction. Hence $f_{dm}(G) \leq 1$. □

Now, we proceed to determine the forcing detour monophonic number of certain classes of graphs.

Theorem 2.11. *For any cycle $C_n (n \geq 4)$,*

$$f_{dm}(C_n) = \begin{cases} 1 & \text{if } n \text{ is even} \\ 3 & \text{if } n \text{ is odd} \end{cases}$$

Proof. Let $C_n : v_1, v_2, \ldots, v_m, v_{m+1}, \ldots, v_n, v_1$ be a cycle of order n.

Case (i) n is even. Let $n = 2m$. Then every minimum detour monophonic set of C_n consists of a pair of antipodal vertices and C_n has exactly m minimum detour monophonic sets. Clearly every minimum detour monophonic set containing one of its elements. Then by Theorem 2.3(ii), $f_{dm}(C_n) = 1$.

Case (ii) n is odd. Let $n = 2m+1$. It is clear that no two point set will form a detour monophonic set of C_n. Now, $\{v_1, v_2, v_3\}$ is a minimum detour monophonic set of C_n and so $dm(C_n) = 3$. We observe that any minimum detour monophonic set of C_n is any one of the following.

(i) any three consecutive vertices

(ii) a vertex and its antipodal vertices

(iii) any three non-adjacent vertices

Then clearly no minimum detour monophonic set of C_n is the unique detour monophonic set containing any of its proper subsets. Hence by Theorem 2.3(iii), $f_{dm}(C_n) = dm(G) = 3$. □

Theorem 2.12. *For any complete graph $G = K_p (p \geq 2)$ or any non-trivial tree $G = T$, $f_{dm}(G) = 0$.*

Proof. For $G = K_p$, it follows from Theorem 1.2 that the set of all vertices of G is the unique minimum detour monophonic set of G. Now, it follows from Theorem 2.3 (i) that $f_{dm}(G) = 0$. If G is a non-trivial tree, then by Theorems 1.1 and 1.3, the set of all endvertices of G is the unique minimum detour monophonic set of G and so by Theorem 2.3 (i), $f_{dm}(G) = 0$. □

Theorem 2.13. *For the complete bipartite graph $G = K_{m,n} (m, n \geq 2)$,*

$$f_{dm}(G) = \begin{cases} 0 & \text{if } 2 = m < n \text{ or } 3 = m < n \\ 1 & \text{if } 2 = m = n \text{ or } 3 = m = n \\ 3 & \text{if } 4 = m \leq n \\ 4 & \text{if } 5 \leq m \leq n \end{cases}$$

Proof. We prove this theorem by considering four cases. Let $U = \{u_1, u_2, \ldots, u_m\}$ and $W = \{w_1, w_2, \ldots, w_n\}$ be the bipartition of G, where $m \leq n$.

Case 1. $2 = m = n$ or $3 = m = n$. Then U and W are the only minimum detour monophonic set of G and so by Theorem 2.3(ii), $f_{dm}(G) = 1$.

Case 2. $2 = m < n$ or $3 = m < n$. Then U is the unique minimum detour monophonic set of G and so by Theorem 2.3(i), $f_{dm}(G) = 0$.

Case 3. $4 = m \le n$. If $4 = m = n$, then the minimum detour monophonic sets of G are U, W and any set got by choosing any two elements from each of U and W. Clearly, neither an 1-element or nor a 2-element subset of any minimum detour monophonic set is a forcing subset and any 3-element subset of U is a forcing subset for U. Hence $f_{dm}(G) = 3$. If $4 = m < n$, then the minimum detour monophonic sets of G are U and any set got by choosing any two elements from each of U and W. Then similar to the above argument, we have $f_{dm}(G) = 3$.

Case 4. $5 \le m \le n$. Then any minimum detour monophonic set is got by choosing any two elements from each of U and W, and G has at least two minimum detour monophonic sets. Hence $dm(G) = 4$. Clearly, no minimum detour monophonic set of G is the unique minimum detour monophonic set containing any of its proper subsets. Then by Theorem 2.3(iii), we have $f_{dm}(G) = dm(G) = 4$. □

Theorem 2.14. *For every pair a, b of positive integers with $0 \le a < b$ and $b \ge 2$, there exists a connected graph G such that $f_{dm}(G) = a$ and $dm(G) = b$.*

Proof. If $a = 0$, let $G = K_b$. Then by Theorem 2.12, $f_{dm}(G) = 0$ and by Theorem 1.2, $dm(G) = b$. Thus we assume that $0 < a < b$. We consider four cases.

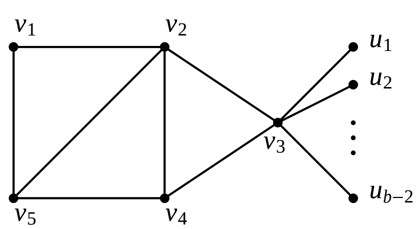

Figure 2.3: G

Case 1. $a = 1$. If $b = 2$, then for any even cycle G, $f_{dm}(G) = a$ by Theorem 2.11 and $dm(G) = b$ by Theorem 1.4. So, we assume that $b \ge 3$. Let G be the graph obtained from the cycle $C_5 : v_1, v_2, v_3, v_4, v_5, v_1$ of order 5 by adding $b - 2$ new vertices $u_1, u_2, \ldots, u_{b-2}$ and joining each $u_i (1 \le i \le b - 2)$ to v_3; and joining two more edges v_2v_5 and v_2v_4. The graph G is shown in Figure 2.3. Let $S = \{u_1, u_2, \ldots, u_{b-2}, v_1\}$ be the set of all extreme vertices of G. By Theorem 1.1, every detour monophonic set of G contains S. It is clear that S is not a detour monophonic set of G. It is easily verified that $S_1 = S \cup \{v_5\}$, $S_2 = S \cup \{v_2\}$ and $S_3 = S \cup \{v_4\}$ are the minimum detour monophonic sets of G. Hence $dm(G) = b$. Moreover, since S_1 is the unique minimum detour monophonic set containing $\{v_5\}$, it follows that $f_{dm}(S_1) = 1$ and so $f_{dm}(G) = 1$.

Case 2. $a = 2$. Then $b \ge 3$. Let G be the graph obtained from the cycle $C_5 : v_1, v_2, v_3, v_4, v_5, v_1$ of order 5 by adding $b - 2$ new vertices $u_1, u_2, \ldots, u_{b-2}$ and joining each $u_i (1 \le i \le b - 2)$ to v_3. The graph G is shown in Figure 2.4. Let $S = \{u_1, u_2, \ldots, u_{b-2}\}$ be the set of all extreme vertices of G. By Theorem 1.1, every detour monophonic set of G contains S. Clearly S is not a detour monophonic set of G. Also $S \cup \{x\}$, where $x \in V(G) - S$, is not a detour monophonic set of G. It is easily verified that $S_1 = S \cup \{v_1, v_5\}$, $S_2 = S \cup \{v_1, v_4\}$, $S_3 = S \cup \{v_2, v_4\}$, $S_4 = S \cup \{v_1, v_2\}$, $S_5 = S \cup \{v_4, v_5\}$ and $S_6 = S \cup \{v_2, v_5\}$ are the minimum detour monophonic sets of G. Hence $dm(G) = b$. If x is an element of $S_i (1 \leq i \leq 6)$, then $\{x\}$ is a subset of at least two minimum detour monophonic sets of G. Hence it follows from Theorem 2.3(i) and (ii) that $f_{dm}(G) \geq 2$. Since S_1 is the unique minimum detour monophonic set containing $\{v_1, v_5\}$, we have $f_{dm}(G) = 2$.

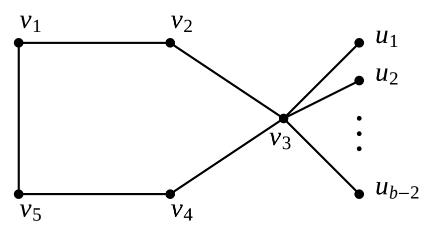

Figure 2.4: G

Case 3. $a \geq 3$ and $b = a + 1$. For each integer i with $0 \leq i \leq b$, let $F_i : u_i, v_i$ be a path of order 2. Let G be the graph obtained from $F_i (0 \leq i \leq b)$ by adding $2b$ edges $u_0 u_j$, $v_0 v_j$ for all j with $1 \leq j \leq b$. The graph G is shown in Figure 2.5. First we show that $dm(G) = b$. Let $U = \{u_1, u_2, \ldots u_b\}$ and $W = \{v_1, v_2, \ldots v_b\}$. We observe that a set S of vertices of G is a minimum detour monophonic set only if S has the following two properties: (1) S contains exactly one vertex from each set $\{u_j, v_j\} (1 \leq j \leq b)$, and (2) $S \cap U \neq \emptyset$ and $S \cap W \neq \emptyset$. Then (1) implies that $dm(G) \geq b$. Since $S' = \{u_2, u_3, \ldots, u_b, v_1\}$ is a detour monophonic set of G with $|S'| = b$, it follows that $dm(G) = b = a + 1$.

Now, we prove that $f_{dm}(G) = a$. First assume that a minimum detour monophonic set contains at least one vertex from U and W. Without loss of generality, let $S_1 = \{u_2, u_3, \ldots, u_b, v_1\}$ be a minimum detour monophonic set of G. We claim that $f_{dm}(G) = b - 1$. Let T be a subset of S_1 such that $|T| \leq b - 2$. Then there exist at least two vertices, say $x, y \in S_1$, such that $x, y \notin T$. Suppose that $x = v_1$ and $y = u_j$ for some $j (2 \leq j \leq b)$. Now, $S_2 = (S_1 - \{v_1, u_j\}) \cup \{u_1, v_j\}$ satisfies (1) and (2) and so S_2 is a minimum detour monophonic set such that $T \subseteq S_2$. Therefore S_1 is not the unique minimum detour monophonic set containing T and so T is not a forcing subset of S_1. Suppose that $x = u_i$ for some $i (2 \leq i \leq b)$ and $y = u_j$ for some $j (2 \leq j \leq b)$ and $i \neq j$. Now, $S_3 = (S_1 - \{u_i, u_j\}) \cup \{v_i, v_j\}$ satisfies (1) and (2) and so S_3 is a minimum detour monophonic set containing T. Hence T is not a forcing subset of S_1 and so $f_{dm}(S_1) \geq b - 1$. Now, it is clear that S_1 is the unique minimum detour monophonic set containing $\{u_2, u_3, \ldots, u_b\}$ so that $f_{dm}(S_1) = b - 1$. Hence it follows that $f_{dm}(G) = b - 1 = a$.

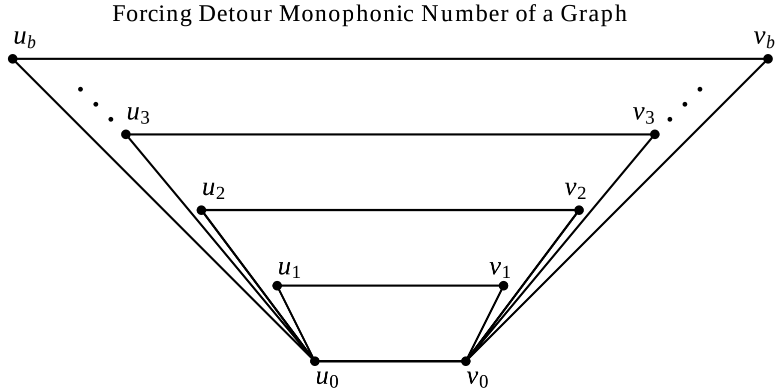

Figure 2.5: G

Case 4. $a \geq 3$ and $b \geq a+2$. Let $F_i : l_i, m_i, n_i, o_i, p_i, q_i, r_i, l_i (1 \leq i \leq a)$ be "a" number of copies of C_7. Let G be the graph obtained from $F_i (1 \leq i \leq a)$ by identifying the vertices o_{i-1} of F_{i-1} and l_i of $F_i (2 \leq i \leq a)$; and adding $b-a$ new vertices $z_1, z_2, \ldots z_{b-a-1}, z$ and joining each $z_i (1 \leq i \leq b-a-1)$ to l_1; and joining each $m_i, n_i (1 \leq i \leq a)$ to the vertices $r_i, q_i, p_i (1 \leq i \leq a)$; and joining the vertex z to o_a. The graph G is shown in Figure 2.6. Let $S = \{z_1, z_2, \ldots z_{b-a-1}, z\}$ be the set of all extreme vertices of G. Then by Theorem 1.1, every detour monophonic set of G contains S. Clearly, S is not a detour monophonic set of G. We observe that every minimum detour monophonic set contains exactly one vertex from $\{m_i, n_i\}$ for every $i (1 \leq i \leq a)$. Thus $dm(G) \geq b$. Since $S_1 = S \cup \{m_1, m_2, \ldots, m_a\}$ is a detour monophonic set of G, it follows that $dm(G) = b$.

Next we show that $f_{dm}(G) = a$. Since every minimum detour monophonic set of G contains S, it follows from Theorem 2.7 that $f_{dm}(G) \leq dm(G) - |S| = b - (b-a) = a$. Now, since $dm(G) = b$ and every minimum detour monophonic set of G contains S, it is easily seen that every minimum detour monophonic set S' of G is of the form $S \cup \{x_1, x_2, \ldots, x_a\}$, where $x_i \in \{m_i, n_i\}$ for every $i (1 \leq i \leq a)$. Let T be any proper subset of S' with $|T| < a$. Then there is a vertex $x \in S' - S$ such that $x \notin T$. If $x = m_i (1 \leq i \leq a)$, then $S'' = (S' - \{m_i\}) \cup \{n_i\}$ is a minimum detour monophonic set containing T. Similarly, if $x = n_j (1 \leq j \leq a)$, then $S''' = (S' - \{n_j\}) \cup \{m_j\}$ is a minimum detour monophonic set containing T. Thus S' is not the unique minimum detour monophonic set containing T and so T is not a forcing subset of S'. This is true for all minimum detour monophonic sets of G and so $f_{dm}(G) = a$. $\square$

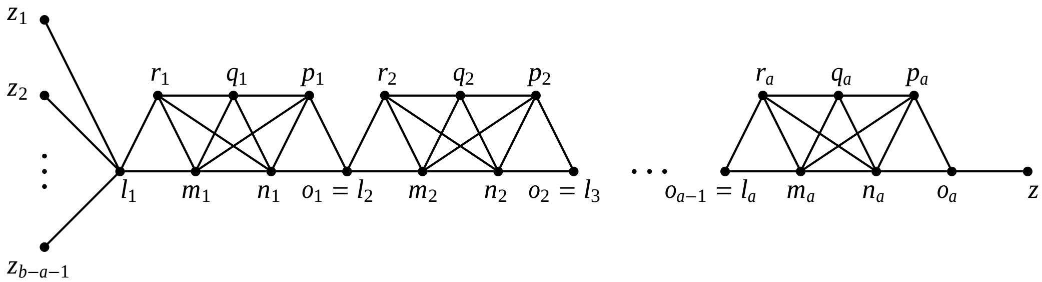

Figure 2.6: G

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