FACTA UNIVERSITATIS (NIŠ)
Ser. Math. Inform. Vol. 28, No 2 (2013), 151–159

SIMPSON TYPE INTEGRAL INEQUALITIES IN WHICH THE POWER OF THE ABSOLUTE VALUE OF THE FIRST DERIVATIVE OF THE INTEGRAND IS s-PREINVEX*

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Abstract. In the article, the authors introduce a new notion "s-preinvex function", establish an identity for such a kind of functions, and find some Simpson type integral inequalities in which the power of the absolute value of the first derivative of the integrand is s-preinvex.

1. Introduction

Let us recall some definitions of various convex functions.

Definition 1.1. A function $f : I \subseteq \mathbb{R} = (-\infty, \infty) \to \mathbb{R}$ is said to be convex if

$$f(\lambda x + (1-\lambda)y) \leqslant \lambda f(x) + (1-\lambda) f(y) \tag{1.1}$$

holds for all $x, y \in I$ and $\lambda \in [0, 1]$.

Definition 1.2. [1, 5, 28] A set $S \subseteq \mathbb{R}^n$ is said to be invex with respect to the map $\eta : S \times S \to \mathbb{R}^n$ if for every $x, y \in S$ and $t \in [0, 1]$

$$y + t\eta(x, y) \in S. \tag{1.2}$$

Definition 1.3. Let $S \subseteq \mathbb{R}^n$ be an invex set with respect to $\eta : S \times S \to \mathbb{R}^n$. For every $x, y \in S$, the η-path P_{xv} joining the points x and $v = x + \eta(y, x)$ is defined by

$$P_{xv} = \{z \mid z = x + t\eta(y, x), t \in [0, 1]\}. \tag{1.3}$$

Received February 19, 2013; Accepted May 15, 2013.
2010 *Mathematics Subject Classification.* Primary 26D15; Secondary 26A51, 26B12, 41A55, 49J52
*This work was partially supported by the Foundation of Research Program of Science and Technology at Universities of Inner Mongolia Autonomous Region under Grant No. NJZY13159 and by the Science Research Funding of Inner Mongolia University for Nationalities under Grant No. NMD1225, China.

Definition 1.4. [5] Let $S \subseteq \mathbb{R}^n$ be an invex set with respect to $\eta : S \times S \to \mathbb{R}^n$. A function $f : S \to \mathbb{R}$ is said to be preinvex with respect to η, if for every $x, y \in S$ and $t \in [0,1]$,

$$f(y + t\eta(x,y)) \leqslant tf(x) + (1-t)f(y). \tag{1.4}$$

Let us reformulate some inequalities of Hermite-Hadamard type for the above mentioned convex functions.

Theorem 1.1. [9, Theorem 2.2] *Let $f : I^\circ \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping and $a, b \in I^\circ$ with $a < b$. If $|f'(x)|$ is convex on $[a,b]$, then*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x\right| \leqslant \frac{b-a}{8}\bigl(|f'(a)| + |f'(b)|\bigr). \tag{1.5}$$

Theorem 1.2. [14, Theorems 2.3 and 2.4] *Let $f : I \subseteq \mathbb{R}_0 \to \mathbb{R}$ be differentiable on I° and $a, b \in I$ with $a < b$. If $|f'(x)|^p$ is s-convex on $[a,b]$ for some $s \in (0,1]$ and $p > 1$, then*

$$\left|f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x\right| \leqslant \frac{b-a}{16}\left(\frac{4}{p+1}\right)^{1/p}\bigl(|f'(a)| + |f'(b)|\bigr) \tag{1.6}$$

and

$$\begin{aligned}\left|f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)\,\mathrm{d}x\right| \leqslant \frac{b-a}{4}\left(\frac{4}{p+1}\right)^{1/p}\Bigl\{\Bigl[|f'(a)|^{p/(p-1)} \\ + 3|f'(b)|^{p/(p-1)}\Bigr]^{1-1/p} + \Bigl[3|f'(a)|^{p/(p-1)} + |f'(b)|^{p/(p-1)}\Bigr]^{1-1/p}\Bigr\}.\end{aligned} \tag{1.7}$$

Theorem 1.3. [5, Theorem 2.1] *Let $A \subseteq \mathbb{R}$ be an open invex subset with respect to $\theta : A \times A \to \mathbb{R}$ and let $f : A \to \mathbb{R}$ be a differentiable function. If $|f'|$ is preinvex on A, then for every $a, b \in A$ with $\theta(a,b) \neq 0$ we have*

$$\begin{aligned}\left|\frac{f(b) + f(b+\theta(a,b))}{2} - \frac{1}{\theta(a,b)}\int_b^{b+\theta(a,b)} f(x)\,\mathrm{d}x\right| \\ \leqslant \frac{|\theta(a,b)|}{8}\bigl[|f'(a)| + \bigl|f'(b)\bigr|\bigr].\end{aligned} \tag{1.8}$$

For more information on Hermite-Hadamard type inequalities for various convex functions, please refer to recently published articles [2, 3, 4, 6, 7, 8, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30] and closely related references therein.

In this article, we will introduce a new notion "s-preinvex function", establish an identity for such a kind of functions, and find some Simpson type integral inequalities in which the power of the absolute value of the first derivative of the integrand is s-preinvex.

2. A definition and a lemma

The so-called “s-preinvex function” may be defined as follows.

Definition 2.1. Let $S \subseteq \mathbb{R}^n$ be an invex set with respect to $\eta : S \times S \to \mathbb{R}^n$. A function $f : S \to \mathbb{R}_0 = [0, \infty)$ is said to be s-preinvex with respect to η and $s \in (0, 1]$ if for every $x, y \in S$ and $t \in [0, 1]$

$$f(y + t\eta(x, y)) \leqslant t^s f(x) + (1 - t)^s f(y). \tag{2.1}$$

For establishing our new integral inequalities of Simpson type for s-preinvex functions, we need the following integral identity.

Lemma 2.1. *Let $A \subseteq \mathbb{R}$ be an open invex subset with respect to $\theta : A \times A \to \mathbb{R}$ and let $a, b \in A$ with $\theta(a, b) \neq 0$. If $f : A \to \mathbb{R}$ is a differentiable function and f' is integrable on the θ-path P_{bc}, then*

$$\begin{aligned}
&\frac{1}{6}\left[f(b) + 4f\left(\frac{2b + \theta(a, b)}{2}\right) + f(b + \theta(a, b))\right] - \frac{1}{\theta(a, b)} \int_b^{b+\theta(a,b)} f(x)\,\mathrm{d}x \\
&\quad= \frac{\theta(a, b)}{4} \int_0^1 \left[\left(t - \frac{2}{3}\right) f'\left(b + \frac{1 + t}{2}\theta(a, b)\right) + \left(\frac{2}{3} - t\right) f'\left(b + \frac{1 - t}{2}\theta(a, b)\right)\right] \mathrm{d}t.
\end{aligned}$$

Proof. Since $a, b \in A$ and A is an invex set with respect to θ, for every $t \in [0, 1]$, we have $b + t\theta(a, b) \in A$. Integrating by part gives

$$\begin{aligned}
&\int_0^1 \left[\left(t - \frac{2}{3}\right) f'\left(b + \frac{1 + t}{2}\theta(a, b)\right) + \left(\frac{2}{3} - t\right) f'\left(b + \frac{1 - t}{2}\theta(a, b)\right)\right] \mathrm{d}t \\
&= \frac{2}{\theta(a, b)}\left[f\left(b + \frac{1 + t}{2}\theta(a, b)\right)\left(t - \frac{2}{3}\right)\Big|_0^1 - \int_0^1 f\left(b + \frac{1 + t}{2}\theta(a, b)\right) \mathrm{d}t\right] \\
&\quad - \frac{2}{\theta(a, b)}\left[f\left(b + \frac{1 - t}{2}\theta(a, b)\right)\left(\frac{2}{3} - t\right)\Big|_0^1 + \int_0^1 f\left(b + \frac{1 - t}{2}\theta(a, b)\right) \mathrm{d}t\right] \\
&= \frac{2}{\theta(a, b)}\left[\frac{1}{3} f(b + \theta(a, b)) + \frac{2}{3} f\left(\frac{2b + \theta(a, b)}{2}\right) - \int_0^1 f\left(b + \frac{1 + t}{2}\theta(a, b)\right) \mathrm{d}t\right] \\
&\quad + \frac{2}{\theta(a, b)}\left[\frac{1}{3} f(b) + \frac{2}{3} f\left(\frac{2b + \theta(a, b)}{2}\right) - \int_0^1 f\left(b + \frac{1 - t}{2}\theta(a, b)\right) \mathrm{d}t\right] \\
&= \frac{2}{3\theta(a, b)}\left[f(b) + 4f\left(\frac{2b + \theta(a, b)}{2}\right) + f(b + \theta(a, b))\right] \\
&\quad - \frac{2}{\theta(a, b)}\left[\int_0^1 f\left(b + \frac{1 + t}{2}\theta(a, b)\right) dt + \int_0^1 f\left(b + \frac{1 - t}{2}\theta(a, b)\right) \mathrm{d}t\right] \\
&= \frac{2}{3\theta(a, b)}\left[f(b) + 4f\left(\frac{2b + \theta(a, b)}{2}\right) + f(b + \theta(a, b))\right] - \frac{4}{\theta^2(a, b)} \int_b^{b+\theta(a,b)} f(x)\,\mathrm{d}x.
\end{aligned}$$

The proof of Lemma 2.1 is completed. □

3. Some new integral inequalities of Simpson type

We are now in a position to establish some Simpson type integral inequalities in which the power of the absolute value of the first derivative of the integrand is s-preinvex.

Theorem 3.1. *Let $A \subseteq \mathbb{R}$ be an open invex subset with respect to $\theta : A \times A \to \mathbb{R}$ and let $f : A \to \mathbb{R}$ be a differentiable function. If $|f'|^q$ is s-preinvex on A for $q \geqslant 1$, then for every $a, b \in A$ with $\theta(a, b) \neq 0$*

$$
\begin{aligned}
&\left|\frac{1}{6}\left[f(b) + 4f\left(\frac{2b + \theta(a, b)}{2}\right) + f(b + \theta(a, b))\right] - \frac{1}{\theta(a, b)} \int_b^{b+\theta(a,b)} f(x)\, \mathrm{d}x\right| \\
&\quad \leqslant \frac{5|\theta(a, b)|}{72}\left[\frac{2}{5 \times (s + 1)(s + 2)6^s}\right]^{1/q} \\
&\quad \times \left\{\left[\left(2 \times 5^{s+2} - (4 - s)6^{s+1} - (2s + 7)3^{s+1}\right)|f'(a)|^q \right.\right.\\
&\quad \left. + \left((2s + 1)3^{s+1} + 2\right)|f'(b)|^q\right]^{1/q} + \left[\left((2s + 1)3^{s+1} + 2\right)|f'(a)|^q \right.\\
&\quad \left.\left. + \left(2 \times 5^{s+2} - (4 - s)6^{s+1} - (2s + 7)3^{s+1}\right)|f'(b)|^q\right]^{1/q}\right\}.
\end{aligned} \tag{3.1}
$$

Proof. Since A is an invex set with respect to θ, for every $t \in [0, 1]$, we obtain $b + t\theta(a, b) \in A$. Using Lemma 2.1 and Hölder's inequality, we have

$$
\begin{aligned}
&\left|\frac{1}{6}\left[f(b) + 4f\left(\frac{2b + \theta(a, b)}{2}\right) + f(b + \theta(a, b))\right] - \frac{1}{\theta(a, b)} \int_b^{b+\theta(a,b)} f(x)\, \mathrm{d}x\right| \\
&\leqslant \frac{|\theta(a, b)|}{4} \int_0^1 \left|t - \frac{2}{3}\right|\left[\left|f'\left(b + \frac{1 + t}{2}\theta(a, b)\right)\right| + \left|f'\left(b + \frac{1 - t}{2}\theta(a, b)\right)\right|\right] \mathrm{d}t \\
&\leqslant \frac{|\theta(a, b)|}{4}\left(\int_0^1 \left|t - \frac{2}{3}\right| \mathrm{d}t\right)^{1-1/q} \\
&\quad \times \left\{\left[\int_0^1 \left|t - \frac{2}{3}\right|\left(\left(\frac{1 + t}{2}\right)^s |f'(a)|^q + \left(\frac{1 - t}{2}\right)^s |f'(b)|^q\right) \mathrm{d}t\right]^{1/q}\right. \\
&\quad \left. + \left[\int_0^1 \left|t - \frac{2}{3}\right|\left(\left(\frac{1 - t}{2}\right)^s |f'(a)|^q + \left(\frac{1 + t}{2}\right)^s |f'(b)|^q\right) \mathrm{d}t\right]^{1/q}\right\} \\
&= \frac{5|\theta(a, b)|}{72}\left[\frac{2}{5 \times (s + 1)(s + 2)6^s}\right]^{1/q} \\
&\quad \times \left\{\left[\left(2 \times 5^{s+2} - (4 - s)6^{s+1} - (2s + 7)3^{s+1}\right)|f'(a)|^q \right.\right.\\
&\quad \left. + \left((2s + 1)3^{s+1} + 2\right)|f'(b)|^q\right]^{1/q} + \left[\left((2s + 1)3^{s+1} + 2\right)|f'(a)|^q \right.\\
&\quad \left.\left. + \left(2 \times 5^{s+2} - (4 - s)6^{s+1} - (2s + 7)3^{s+1}\right)|f'(b)|^q\right]^{1/q}\right\}.
\end{aligned}
$$

The proof of Theorem 3.1 is completed. □

Corollary 3.1. *Under the conditions of Theorem 3.1, if $q = 1$, we have*

$$\left|\frac{1}{6}\left[f(b) + 4f\left(\frac{2b+\theta(a,b)}{2}\right) + f(b+\theta(a,b))\right] - \frac{1}{\theta(a,b)}\int_b^{b+\theta(a,b)} f(x)\,\mathrm{d}x\right| \leqslant \frac{|\theta(a,b)|}{(s+1)(s+2)6^{s+2}}\left[2\times 5^{s+2} - (4-s)6^{s+1} + 2 - 6\times 3^{s+1}\right][|f'(a)| + |f'(b)|].$$

Theorem 3.2. *Let $A \subseteq \mathbb{R}$ be an open invex subset with respect to $\theta : A\times A \to \mathbb{R}$ and let $f : A \to \mathbb{R}$ be a differentiable function. If $|f'|^q$ is s-preinvex on A for $q > 1$, then for every $a, b \in A$ with $\theta(a,b) \neq 0$,*

$$\left|\frac{1}{6}\left[f(b) + 4f\left(\frac{2b+\theta(a,b)}{2}\right) + f(b+\theta(a,b))\right] - \frac{1}{\theta(a,b)}\int_b^{b+\theta(a,b)} f(x)\,\mathrm{d}x\right| \leqslant \frac{|\theta(a,b)|}{36}\left(\frac{q-1}{2q-1}\right)^{1-1/q}\left[\frac{3}{2^s(s+1)}\right]^{1/q}\left[2^{(2q-1)/(q-1)}+1\right]^{1-1/q} \times\left\{\left[\left(2^{s+1}-1\right)|f'(a)|^q + |f'(b)|^q\right]^{1/q} + \left[|f'(a)|^q + \left(2^{s+1}-1\right)|f'(b)|^q\right]^{1/q}\right\}. \tag{3.2}$$

Proof. For every $t \in [0,1]$, we have $b + t\theta(a,b) \in A$. So using Lemma 2.1 and Hölder's inequality, we obtain

$$\begin{aligned}
&\left|\frac{1}{6}\left[f(b) + 4f\left(\frac{2b+\theta(a,b)}{2}\right) + f(b+\theta(a,b))\right] - \frac{1}{\theta(a,b)}\int_b^{b+\theta(a,b)} f(x)\,\mathrm{d}x\right| \\
&\leqslant \frac{|\theta(a,b)|}{4}\int_0^1\left|t-\frac{2}{3}\right|\left[\left|f'\left(b+\frac{1+t}{2}\theta(a,b)\right)\right| + \int_0^1\left|f'\left(b+\frac{1-t}{2}\theta(a,b)\right)\right|\right]\mathrm{d}t \\
&\leqslant \frac{|\theta(a,b)|}{4}\left[\int_0^1\left|t-\frac{2}{3}\right|^{q/(q-1)}\mathrm{d}t\right]^{1-1/q}\left\{\left[\int_0^1\left|f'\left(b+\frac{1+t}{2}\theta(a,b)\right)\right|^q\mathrm{d}t\right]^{1/q}\right. \\
&\quad\left. + \left[\int_0^1\left|f'\left(b+\frac{1-t}{2}\theta(a,b)\right)\right|^q\mathrm{d}t\right]^{1/q}\right\} \\
&\leqslant \frac{|\theta(a,b)|}{4}\left[\int_0^1\left|t-\frac{2}{3}\right|^{q/(q-1)}\mathrm{d}t\right]^{1-1/q} \\
&\quad\times\left\{\left[\int_0^1\left(\left(\frac{1+t}{2}\right)^s|f'(a)|^q + \left(\frac{1-t}{2}\right)^s|f'(b)|^q\right)\mathrm{d}t\right]^{1/q}\right. \\
&\quad\left. + \left[\int_0^1\left(\left(\frac{1-t}{2}\right)^s|f'(a)|^q + \left(\frac{1+t}{2}\right)^s|f'(b)|^q\right)\mathrm{d}t\right]^{1/q}\right\} \\
&= \frac{|\theta(a,b)|}{36}\left(\frac{q-1}{2q-1}\right)^{1-1/q}\left[\frac{3}{2^s(s+1)}\right]^{1/q}\left[2^{(2q-1)/(q-1)}+1\right]^{1-1/q} \\
&\times\left\{\left[\left(2^{s+1}-1\right)|f'(a)|^q + |f'(b)|^q\right]^{1/q} + \left[|f'(a)|^q + \left(2^{s+1}-1\right)|f'(b)|^q\right]^{1/q}\right\}.
\end{aligned}$$

The proof of Theorem 3.2 is complete. □

Theorem 3.3. *Let $A \subseteq \mathbb{R}$ be an open invex subset with respect to $\theta : A \times A \to \mathbb{R}$ and let $f : A \to \mathbb{R}$ be a differentiable function. If $|f'|^q$ is s-preinvex on A for $q > \frac{3}{2}$, then for every $a, b \in A$ with $\theta(a,b) \neq 0$,*

$$
\begin{aligned}
&\left|\frac{1}{6}\left[f(b)+4f\left(\frac{2b+\theta(a,b)}{2}\right)+f(b+\theta(a,b))\right]-\frac{1}{\theta(a,b)}\int_b^{b+\theta(a,b)}f(x)\,\mathrm{d}x\right| \\
&\quad \leqslant \frac{|\theta(a,b)|}{36}\left(\frac{q-1}{2q-3}\right)^{1-1/q}\left[\frac{3}{2^s(s+1)(s+2)(s+3)}\right]^{1/q}\left[2^{(2q-3)/(q-1)}+1\right]^{1-1/q} \\
&\quad \times\Big\{\Big[\Big(\big(s^2-7s+42\big)2^{s+1}-\big(4s^2+32s+78\big)\Big)|f'(a)|^q \\
&\quad +\big(4s^2+8s+6\big)|f'(b)|^q\Big]^{1/q}+\Big[\big(4s^2+8s+6\big)|f'(a)|^q \\
&\quad +\Big(\big(s^2-7s+42\big)2^{s+1}-\big(4s^2+32s+78\big)\Big)|f'(b)|^q\Big]^{1/q}\Big\}.
\end{aligned}
$$

Proof. Since $b + t\theta(a,b) \in A$ for $t \in [0,1]$, form Lemma 2.1 and Hölder's inequality, we have

$$
\begin{aligned}
&\left|\frac{1}{6}\left[f(b)+4f\left(\frac{2b+\theta(a,b)}{2}\right)+f(b+\theta(a,b))\right]-\frac{1}{\theta(a,b)}\int_b^{b+\theta(a,b)}f(x)\,\mathrm{d}x\right| \\
&\leqslant \frac{|\theta(a,b)|}{4}\int_0^1\left|t-\frac{2}{3}\right|\left[\left|f'\left(b+\frac{1+t}{2}\theta(a,b)\right)\right|\mathrm{d}t+\left|f'\left(b+\frac{1-t}{2}\theta(a,b)\right)\right|\right]\mathrm{d}t \\
&\leqslant \frac{|\theta(a,b)|}{4}\left[\int_0^1\left|t-\frac{2}{3}\right|^{(q-2)/(q-1)}\mathrm{d}t\right]^{1-1/q} \\
&\quad \times\left\{\left[\int_0^1\left|t-\frac{2}{3}\right|^2\left|f'\left(b+\frac{1+t}{2}\theta(a,b)\right)\right|^q\mathrm{d}t\right]^{1/q}\right. \\
&\quad \left.+\left[\int_0^1\left|t-\frac{2}{3}\right|^2\left|f'\left(b+\frac{1-t}{2}\theta(a,b)\right)\right|^q\mathrm{d}t\right]^{1/q}\right\} \\
&\leqslant \frac{|\theta(a,b)|}{4}\left[\int_0^1\left|t-\frac{2}{3}\right|^{(q-2)/(q-1)}\mathrm{d}t\right]^{1-1/q} \\
&\quad \times\left\{\left[\int_0^1\left|t-\frac{2}{3}\right|^2\left(\left(\frac{1+t}{2}\right)^s|f'(a)|^q+\left(\frac{1-t}{2}\right)^s|f'(b)|^q\right)\mathrm{d}t\right]^{1/q}\right. \\
&\quad \left.+\left[\int_0^1\left|t-\frac{2}{3}\right|^2\left(\left(\frac{1-t}{2}\right)^s|f'(a)|^q+\left(\frac{1+t}{2}\right)^s|f'(b)|^q\right)\mathrm{d}t\right]^{1/q}\right\} \\
&= \frac{|\theta(a,b)|}{36}\left(\frac{q-1}{2q-3}\right)^{1-1/q}\left[\frac{3}{2^s(s+1)(s+2)(s+3)}\right]^{1/q}\left[2^{(2q-3)/(q-1)}+1\right]^{1-1/q} \\
&\quad \times\Big\{\Big[\Big(\big(s^2-7s+42\big)2^{s+1}-\big(4s^2+32s+78\big)\Big)|f'(a)|^q \\
&\quad +\big(4s^2+8s+6\big)|f'(b)|^q\Big]^{1/q}+\Big[\big(4s^2+8s+6\big)|f'(a)|^q \\
&\quad +\Big(\big(s^2-7s+42\big)2^{s+1}-\big(4s^2+32s+78\big)\Big)|f'(b)|^q\Big]^{1/q}\Big\}.
\end{aligned}
$$

The proof of Theorem 3.3 is complete. □

REFERENCES

1. T. Antczak: *Mean value in invexity analysis*. Nonlinear Anal. **60** (2005), no. 8, 1473–1484; Available online at http://dx.doi.org/10.1016/j.na.2004.11.005.
2. R.-F. Bai, F. Qi, and B.-Y. Xi: *Hermite-Hadamard type inequalities for the m- and (α, m)-logarithmically convex functions*. Filomat **26** (2013), no. 1, 1–7.
3. S.-P. Bai and F. Qi: *Some inequalities for (s_1, m_1)-(s_2, m_2)-convex functions on the co-ordinates*. Glob. J. Math. Anal. **1** (2013), no. 1, 22–28.
4. S.-P. Bai, S.-H. Wang, and F. Qi: *Some Hermite-Hadamard type inequalities for n-time differentiable (α, m)-convex functions*. J. Inequal. Appl. 2012, **2012**:267, 11 pages; Available online at http://dx.doi.org/10.1186/1029-242X-2012-267.
5. A. Barani, A. G. Ghazanfari, and S. S. Dragomir: *Hermite-Hadamard inequality for functions whose derivatives absolute values are preinvex*. J. Inequal. Appl. 2012, **2012**:247, 9 pages; Available online at http://dx.doi.org/10.1186/1029-242X-2012-247.
6. L. Chun and F. Qi: *Integral inequalities of Hermite-Hadamard type for functions whose* 3rd *derivatives are s-convex*. Appl. Math. **3** (2012), no. 11, 1680–1685; Available online at http://dx.doi.org/10.4236/am.2012.311232.
7. S. S. Dragomir: *On Hadamard's inequalities for convex functions*. Math. Balkanica (N.S.) **6** (1992), no. 3, 215–222.
8. S. S. Dragomir: *Two mappings on connection to Hadamard's inequality*. J. Math. Anal. Appl. **167** (1992), no. 1, 49–56; Available online at http://dx.doi.org/10.1016/0022-247X(92)90233-4.
9. S. S. Dragomir and R. P. Agarwal: *Two inequalities for differentiable mappings and applications to special means of real numbers and to trapezoidal formula*. Appl. Math. Lett. **11** (1998), no. 5, 91–95; Available online at http://dx.doi.org/10.1016/S0893-9659(98)00086-X.
10. S. S. Dragomir, J. Pečarić, and L. E. Persson: *Some inequalities of Hadamard type*. Soochow J. Math. **21** (1995), no. 3, 335–341.
11. S. S. Dragomir, J. E. Pečarić, and J. Sándor: *A note on the Jensen-Hadamard inequality*. Anal. Numér. Théor. Approx. **19** (1990), no. 1, 29–34.
12. D. A. Ion: *Some estimates on the Hermite-Hadamard inequality through quasi-convex functions*. An. Univ. Craiova Ser. Mat. Inform. **34** (2007), 83–88.
13. W.-D. Jiang, D.-W. Niu, Y. Hua, and F. Qi: *Generalizations of Hermite-Hadamard inequality to n-time differentiable functions which are s-convex in the second sense*. Analysis (Munich) **32** (2012), no. 3, 209–220; Available online at http://dx.doi.org/10.1524/anly.2012.1161.
14. U. S. Kirmaci: *Inequalities for differentiable mappings and applications to special means of real numbers to midpoint formula*. Appl. Math. Comp. **147** (2004), no. 1, 137–146; Available online at http://dx.doi.org/10.1016/S0096-3003(02)00657-4.
15. F. Qi, Z.-L. Wei, and Q. Yang: *Generalizations and refinements of Hermite-Hadamard's inequality*. Rocky Mountain J. Math. **35** (2005), no. 1, 235–251; Available online at http://dx.doi.org/10.1216/rmjm/1181069779.

16. Y. Shuang, H.-P. Yin, and F. Qi: *Hermite-Hadamard type integral inequalities for geometric-arithmetically s-convex functions*. Analysis (Munich) **33** (2013), in press; Available online at http://dx.doi.org/10.1524/anly.2013.1192.

17. S.-H. Wang, B.-Y. Xi, and F. Qi: *On Hermite-Hadamard type inequalities for* (α, m)*-convex functions*. Int. J. Open Probl. Comput. Sci. Math. **5** (2012), no. 4, 47–56.

18. S.-H. Wang, B.-Y. Xi, and F. Qi: *Some integral inequalities in terms of supremum norms of n-time differentiable functions*. Math. Sci. Lett. **2** (2013), no. 2, in press; Available online at http://dx.doi.org/10.12785/msl/02????.

19. S.-H. Wang, B.-Y. Xi, and F. Qi: *Some new inequalities of Hermite-Hadamard type for n-time differentiable functions which are m-convex*. Analysis (Munich) **32** (2012), no. 3, 247–262; Available online at http://dx.doi.org/10.1524/anly.2012.1167.

20. B.-Y. Xi, R.-F. Bai, and F. Qi: *Hermite-Hadamard type inequalities for the m- and* (α, m)*-geometrically convex functions*. Aequationes Math. **184** (2012), no. 3, 261–269; Available online at http://dx.doi.org/10.1007/s00010-011-0114-x.

21. B.-Y. Xi, J. Hua, and F. Qi: *Hermite-Hadamard type inequalities for extended s-convex functions on the co-ordinates in a rectangle*. J. Appl. Anal. **20** (2014), no. 1, in press.

22. B.-Y. Xi and F. Qi: *Hermite-Hadamard type inequalities for functions whose derivatives are of convexities*. Nonlinear Funct. Anal. Appl. **18** (2013), in press.

23. B.-Y. Xi and F. Qi: *Some Hermite-Hadamard type inequalities for differentiable convex functions and applications*. Hacet. J. Math. Stat. **42** (2013), in press.

24. B.-Y. Xi and F. Qi: *Some inequalities of Hermite-Hadamard type for h-convex functions*. Adv. Inequal. Appl. **2** (2013), no. 1, 1–15.

25. B.-Y. Xi and F. Qi: *Some integral inequalities of Hermite-Hadamard type for convex functions with applications to means*. J. Funct. Spaces Appl. **2012** (2012), Article ID 980438, 14 pages; Available online at http://dx.doi.org/10.1155/2012/980438.

26. B.-Y. Xi, S.-H. Wang, and F. Qi: *Some inequalities of Hermite-Hadamard type for functions whose 3rd derivatives are P-convex*. Appl. Math. **3** (2012), no. 12, 1898–1902; Available online at http://dx.doi.org/10.4236/am.2012.312260.

27. B.-Y. Xi, Y. Wang, and F. Qi: *Some integral inequalities of Hermite-Hadamard type for* (s, m)*-convex functions*. Transylv. J. Math. Mechanics **5** (2013), no. 1, in press.

28. X. M. Yang and D. Li: *On properties of preinvex functions*. J. Math. Anal. Appl. **256** (2001), no. 1, 229–241; Available online at http://dx.doi.org/10.1006/jmaa.2000.7310.

29. T.-Y. Zhang, A.-P. Ji, and F. Qi: *On integral inequalities of Hermite-Hadamard type for s-geometrically convex functions*. Abstr. Appl. Anal. **2013** (2013), Article ID 560586, 14 pages; Available online at http://dx.doi.org/10.1155/2012/560586.

30. T.-Y. Zhang, A.-P. Ji, and F. Qi: *Some inequalities of Hermite-Hadamard type for GA-convex functions with applications to means*. Matematiche (Catania) **68** (2013), no. 1, 229–239; Available online at http://dx.doi.org/10.4418/2013.68.1.17.

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