UNIVERSITY OF NIŠ
The scientific journal **FACTA UNIVERSITATIS**
Series: **Mechanics, Automatic Control and Robotics** Vol.2, No 9, 1999 pp. 983 - 994
Editor of series: Katica (Stevanovi}) Hedrih, e-mail: katica@masfak.masfak.ni.ac.yu
Address: Univerzitetski trg 2, 18000 Niš, YU, Tel: +381 18 547-095, Fax: +381 18 547-950
http:// ni.ac.yu/Facta

DISCRETE-TIME VARIABLE STRUCTURE CONTROL SYSTEMS - MULTIVARIABLE LINEAR PLANT CASE -

UDC 62-52

Zoran M. Bučevac

Mechanical Engineering Department, University of Belgrade

Abstract. *Linear time-invariant discrete digital plant, with no restrictions on the form of the state equation and the number of controls, is considered. The stabilizing state feedback control algorithm is developed by Lyapunov's second method leading to the variable structure system with sliding modes. Essentially the control algorithm drives the system from an arbitrary initial state to a prescribed so-called sliding subspace S, in finite time. Inside the sliding subspace S the system is switched to the sliding mode regime and stay in it for ever.*

Key words: *Discrete-time digital system, variable structure systems, Lyapunov's second method, sliding mode*

1. INTRODUCTION

In the sixties, S.V. Emel'yanov originated the variable structure control systems (VSCS) theory, although this type of ideas could have been recognized in the early works of discontinuous systems in the West in the fifties; see, for example, Flüge-Lotz (1953). Enormous number of papers appeared as a result of early research in the area of VSCS. The results were mostly gathered in two monographs (Emel'yanov, 1967; Emel'yanov et. al., 1970).

In Utkin's research in this area we meet more modern and general approach. He oriented himself especially to the VSCS with sliding modes (Utkin, 1974; Utkin, 1981) where did generalization observing no restrictions on the form of the state equations and the number of controls. Research was continued and many papers on the subject have appeared (see, for example, Young, 1977 and 1978; Itkis, 1983; Ryan, 1983; Slotine and

Received November 20, 1996; in revised form October 20, 1998
This work was supported in part by the National Science Foundation under project No.11E08PT1

Sastry, 1983; Ambrosino et al., 1984; Gough et al., 1984; Khurana et al., 1986; Geering and Guzzella, 1986; Bartolini and Zolezzi, 1986; Slotine et al., 1986; Xing-Huo and Chun-Bo, 1987; Li et al., 1994; El-Khazali et al., 1995; Karakasoglu et al., 1995; Zheung et al., 1995 and their bibliographies).

Almost all published in the area is related to the VSCS with sliding modes where the main feature of the systems is existence of their so - called sliding mode regime inside the so - called sliding subspace. The popularity of the systems stems from their well known advantages compared to the other types of VSCS or other types of control systems at all:

1. In sliding mode regime, the systems' behavior is independent of plant parameters' variations, in some cases,
2. Realizability of desired dinamical properties of the systems in their sliding mode regime,
3. Realization simplicity of compensator of the systems,
4. Sliding mode regime invariance to disturbances, in some cases.

The first motivation for the paper to be related to VSCS with sliding modes goes from the all previously mentioned advantages of the systems.

The second and the most important motivation for this work stems from the fact that results to date mostly concern continuous time type of these systems. There were small number of papers related to the discrete time version of the problem (see, Itkis, 1976; Milosavljevic, 1985; Sarpturk, 1987; Kotta, 1989; Furuta, 1990; Sira-Ramirez, 1991; Bartolini et al., 1995; Gao et al., 1995). The author of this paper, treated discrete digital type of the problem (see, Bucevac, 1985, 1988), clearly to make place of microprocessor compensator application in the systems. By this paper the author continues the series of his discrete digital type of the problem papers.

In Itkis (1976), the author considers amplitude as well as width modulation of a scalar control applied to a continuous linear plant, in companion canonical form, of a VSS. Since the resulting control is piecewise constant, discrete nature of the special case system is evident. Also, in the work of Milosavljevic (1985), the linear plants in companion canonical form with scalar control were considered. He observed all combinations of discretization of compensator input variables. Sarpturk considered discrete equivalent of linear time invariant continuous system and gave necessary and sufficient convergence and sliding conditions. Kotta claimed that Sarpturk's conditions led to the upper and lower bounded control. Furuta treated the scalar control case and used Lyapunov function which was analogous to the one used for continuous type of the problem. Sira-Ramirez treated single input single output nonlinear time discrete system. His approach led to chattering around sliding manifold. Bartolini et al. gave control algorithm guaranteeing finite reaching time to the sliding manifold as well as taking in account boundness in value. Also, the "equivalent control" was introduced providing motion in sliding manifold. Gao treated scalar control case of time discrete VSCS. Bucevac discovered phenomenon of discrete digital VSCS: during sliding mode regime inside so-called sliding subspace actual so-called sliding control must be applied to the system. Also, one step control was discovered.

In the work linear time invariant discrete digital system is under consideration. A state feedback discrete-time variable structure control algorithm is designed. The control algorithm is stabilizing but through the art of VS with sliding modes. Lyapunov's second method, as an elegant technique, is used. This alines the work among the small number of papers where the method was primarily used for analysis or design of VSCS, see, Gough et al. (1984), Xing-Huo and Chun-Bo (1987). Essentially, the objective is to push the system state from an arbitrary initial position to the so-called sliding subspace S, in finite time. Once the system reaches the sliding subspace S it stays there for ever working in so-called sliding mode regime. The main feature of the regime is the system state assymptotical approaching to the zero equilibrium state. The solution of the problem makes place for making of commercial microprocessor compensator of the type. So far the problem related to the most general case of multivariable linear time invariant plant was not solved by Lyapunov's approach as proposed in the work.

2. PROBLEM STATEMENT, NOTATION AND SOME DEFINITIONS

Linear time - invariant discrete digital system is considered, which is described by its state equation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \tag{2.1}$$

where $k \in N_0$, $N_0 = \{0,1,2,...,\}$, $x(k) \in \Re^n$ is state vector at time k, $\mathbf{u}(k) \in \Re^m$ is control vector at time k, $m < n$; $\mathbf{A} \in \Re^{n \times n}$ and $\mathbf{B} \in \Re^{n \times m}$ are real constant matrices and $(\mathbf{A},\mathbf{B})$ is assumed to be controllable pair.

A set of hyperplanes described by

$$\mathbf{c}^i x = 0\,; \quad i = 1,2,...,m\,,$$

where $\mathbf{c}^i \in \Re^n$ is a constant non zero row vector, is taken and jointed to the system (2.1). The intersection of these hyperplanes is the so-called sliding subspace S (in further text only S). Clearly, S is described by

$$S = \{\mathbf{x}\colon \mathbf{C}\mathbf{x} = 0\}$$

where $\mathbf{C} \in \Re^{m \times n}$ has i-th row equal to $\mathbf{c}^j$. Also, invertibility of matrix $\mathbf{CB}$ is assumed.

The objective of this paper is to develop variable structure type of state feedback control law

$$\mathbf{u} = \mathbf{u}(\mathbf{x}) \tag{2.2}$$

which guarantees that the state $\chi(k;\mathbf{x}(0);\mathbf{u}(\cdot))$ of the system (2.1) reaches S in finite time. Once S has been reached the controller is required to keep the state within it thereafter which means positive invariance of S relative to the system motion and what is denoted as **sliding mode regime**. During this regime, inside S, convergence to zero equilibrium with appropriately prescribed modes λ_i, can be guaranteed if the $\mathbf{c}^j$ has been appropriately chosen. For $\mathbf{c}^j$ choice see Utkin (1981).

More rigorously , S is positive invariant relative to the system (2.1) motion iff (if and only if) $x(0) \in S$ implies $\chi(k;\mathbf{x}(0);\mathbf{u}(\cdot)) \in S$, $\forall k \in N_0$.

Further, some other notations and definitions are given for the reason of their usage in theorems which are main results and by means of which formulated problem is solved.

Real n-dimensional state space $\Re^n$ is with Euclidean norm denoted by $\|\cdot\|$.
For S,

$$d(\mathbf{x}, S) = \inf\,(\|\mathbf{x} - \mathbf{y}\|\colon \mathbf{y} \in S)$$

is the distance between S and the point $\mathbf{x} \in \Re^n$. Also for S

$$\mathrm{d}_{N_0} = (\chi(k;\mathbf{x}(0);\mathbf{u}(\cdot)),S) = \sup[\mathrm{d}\,(\mathbf{x}(k),S)\colon k\in N_0]$$
$$= \sup\{\inf\,[\|\chi(k;\mathbf{x}(0);\mathbf{u}(\cdot)) - \mathbf{y}\|\colon \mathbf{y}\in S]\colon k\in N_0\}$$

is distance between solution $\chi(k;\mathbf{x}(0);\mathbf{u}(\cdot))$ of system (2.1) and S on N_0.

Definition 2.1: The state $\mathbf{x} = 0$ of the system (2.1), (2.2) is stable in S (with respect to S) iff $\forall\varepsilon\in\Re_+(\Re_+ = [0,+\infty[)$, $\exists\delta = \delta(\varepsilon)\in\Re_+$ such that $x(0)\in S$ and $\|x(0)\| < \delta(\varepsilon)$ implies that $\chi(k;\mathbf{x}(0);u(\cdot))$ exists, $\|\chi(k;\mathbf{x}(0);\mathbf{u}(\cdot))\| < \varepsilon$ and $\chi(k;\mathbf{x}(0);u(\cdot))\in S$, $\forall k \geq 0$.

Definition 2.2: The state $x = 0$ of the system (2.1), (2.2) is attractive (globally) in S (with respect to S) iff $\exists\Delta > 0$ $(\Delta = +\infty)$ such that $x(0) \in S$ and $\|x(0)\| < \Delta$ implies that $\chi(k;x(0);u(\cdot))$ exists,

$$\lim\,\{\|\chi(k;x(0);u(\cdot))\|\colon k\to+\infty\} = 0 \text{ and } \chi(k;x(0);u(\cdot))\in S,\ \forall k\in N_0.$$

Definition 2.3: The state $\mathbf{x} = 0$ of the system (2.1), (2.2) is (globally) asymptotically stable in S (with respect to S) iff it is both stable and (globally) attractive in S.

Definition 2.4: The system (2.1), (2.2) is stable in S (with respect to S) iff its state $\mathbf{x} = 0$ is globally asymptotically stable in S.

Definition 2.5: S is stable, relative to the system (2.1), (2.2), iff $\forall\varepsilon \in \Re_+$, $\exists\delta = \delta(\varepsilon)\in\Re_+$ such that distance $\mathrm{d}[\mathbf{x}(0),S] < \delta$ implies that $\chi(k;\mathbf{x}(0);\mathbf{u}(\cdot))$ exists and $\mathrm{d}_{N_0}[\chi(k;\mathbf{x}(0);\mathbf{u}(\cdot)),S] < \varepsilon$, $\forall k \geq 0$.

Definition 2.6: S is attractive (globally) relative to the system (2.1), (2.2) iff $\exists\Delta\in\,]0,+\infty[$ $(\Delta = +\infty)$ such that $\mathrm{d}[\mathbf{x}(0), S] < \Delta$ implies $\lim\{\mathrm{d}[\chi(k;\mathbf{x}(0);\mathbf{u}(\cdot)),S]\colon k \to +\infty\} = \lim\{\mathrm{d}[\mathbf{x}(k), S]\colon k \to +\infty\} = 0$.

Definition 2.7: S is (globally) asymptotically stable relative to the system (2.1), (2.2) iff S is stable and (globally) attractive at the same time.
The problem stated in the objective of the paper can be reformulated as following: the objective of the paper is to develop the control law (2.2) such that S is **globally asymptotically stable** relative to the system (2.1), (2.2), and the system (2.1), (2.2) is stable in the sliding mode regime with appropriately prescribed modes.

3. Main Results

In this section solution of the problem stated in section 2 is provided by the following theorems.

Theorem 3.1: Application of actual so-called sliding control $\mathbf{u}^{sl}(k) = -(\mathbf{CB})^{-1}\mathbf{CAx}(k)$ to the system (2.1) inside S is necessary and sufficient for S to be positive invariant relative to the system (2.1) solution $\chi(k;\,\mathbf{x}(0);\,\mathbf{u}^{sl}(k))$.

The application of actual sliding control $\mathbf{u}^{sl}(k)$ (in further text only $\mathbf{u}^{sl}(k)$) to the system (2.1) inside S is phenomenon of discrete digital VSCS with sliding modes. Using this fact and intention for developing of variable structure type of controller for the system (2.1), control (2.2) components could be more precisely specified as

$$\mathbf{u}_i[\mathbf{x}(k)] = \begin{cases} \mathbf{u}_i^+[\mathbf{x}(k)], \mathbf{c}^i\mathbf{x}(k)>0 \\ \mathbf{u}_i^{sl}[\mathbf{x}(k)], \mathbf{c}^i\mathbf{x}(k) = 0 \\ \mathbf{u}_i^-[\mathbf{x}(k)], \mathbf{c}^i\mathbf{x}(k)<0 \end{cases} \tag{3.1}$$

Evidently, the control is discontinuous which makes the right hand side of the system (2.1) state equation also discontinuous. Opposite to the $\mathbf{u}^{sl}(k)$, which is applied to the system (2.1) inside S, let us call control, which is applied to the system (2.1) outside of S, as outer control, $\mathbf{u}^{ot}(k)$. The following theorems are basic for solving of the stated problem by Lyapunov's second method.

Theorem 3.2: Let the system (2.1), (3.1) be considered. S is globally asymptotically stable relative to the system (2.1), (3.1) if there exists scalar function v such that:

a) $v(\mathbf{x})\in C(\Re^n)$
b) $v(\mathbf{x}) > 0, \forall\mathbf{x}\in \Re^n \setminus S$
c) $v(\mathbf{x}) = 0, \forall\mathbf{x}\in S$
d) $v(\mathbf{x}) \to \infty$ as $\mathrm{d}(\mathbf{x}, S) \to +\infty$
e) $\Delta v(\mathbf{x}) < 0, \forall\mathbf{x}\in \Re^n \setminus S$
f) $\Delta v(\mathbf{x}) = 0, \forall\mathbf{x}\in S$

where $\Delta v(\mathrm{x})$ is the first forward finite difference of $v(\mathbf{x})$.

Scalar function v defined by

$$v(x)=(\mathrm{sign}\mathbf{Cx})^{\mathrm{T}}\cdot\mathbf{Cx}$$

as Lyapunov candidate and outer control law

$$\mathbf{u}^{ot}(k) = -(\mathbf{CB})^{-1}[\mathbf{CAx}(k) - \Psi(k)\mathrm{sign}\mathbf{Cx}(k)],\ \Psi = \mathrm{diag}[\psi_{11},\psi_{22},\ldots, \psi_{mm},],\ \psi_{ii}(k) \geq 0, \forall\mathbf{x}\in N_0$$

are chosen relative to the system (2.1). Evidently, such sort of $\mathbf{u}^{ot}$ together with $\mathbf{u}^{sl}$, which is unique and already stated, represents control of (3.1) type. Using all these facts the following theorem was obtained as corollary to the previous theorem.

Theorem 3.3: S is globally asymptotically stable relative to the system (2.1) if the following control is applied:

$$\mathbf{u}(k) = \begin{cases} \mathbf{u}^{\mathrm{ot}}(k) = -(\mathbf{CB})^{-1}[\mathbf{CAx}(k) - \Psi(k)\mathrm{sign}\mathbf{Cx}(k)], \\ \psi_{\mathrm{ii}}(k) = \alpha\,|\,c^i\mathbf{x}(k)\,|,\ \alpha\in[0,1[\\ \\ \mathbf{u}^{sl}(k) = -(\mathbf{CB})^{-1}\mathbf{CAx}(k) \end{cases}.$$

To be guaranteed for $\mathbf{x}(k)$ to reach S in finite time $\Delta v(\mathbf{x}(k)) = -v(\mathbf{x}(0))/P$ is taken with "P" as desired number of reaching steps. Taking the fact into account the following theorem is come to.

Theorem 3.4: S is globally asymptotically stable relative to the system (2.1) if the following control is applied:

$$\mathbf{u}(k)=\begin{cases}\mathbf{u}^{ot}(k)=-(\mathbf{CB})^{-1}[\mathbf{CAx}(k)-\Psi(k)\mathrm{sign}\mathbf{Cx}(k)]\\ \psi_{ii}(k)=\rho_{ii}(k)\sum_{j=1}^{m}[|\mathbf{c}^j\mathbf{x}(k)|-\frac{1}{P}|\mathbf{c}^j\mathbf{x}(0)|],\forall i=1,\cdots,m;\ \forall k=0,\cdots,P-1\\ \rho_{ii}(k)\in[0,1[\wedge\sum_{i=1}^{m}\rho_{ii}(k)|\mathrm{sign}c^i\mathbf{x}(k)|=1\\ \mathbf{u}^{sl}(k)=-(\mathbf{CB})^{-1}\mathbf{CAx}(k)\end{cases}$$

4. Proofs of theorems

In this section proofs of theorems from the preceding section are provided.

Proof of theorem 3.1: (*Necessity*) Positive invariance of S relative to the system (2.1) solution $\chi(k;\, \mathbf{x}(0);\, \mathbf{u}^{sl}(k))$ is assumed. So, every time $\mathbf{x}(k)\in S$, $\mathbf{x}(k+1)\in S$ also. Last fact, description of S and state equation of the system (2.1), give equation

$$\mathbf{CAx}(k)+\mathbf{CBu}^{sl}(k)=0$$

and further

$$\mathbf{u}^{sl}(k)=-(\mathbf{CB})^{-1}\mathbf{CAx}(k)\ .$$

(*Sufficiency*) Let assume, relative to system (2.1), $\mathbf{x}(k)\in S$ and control

$$\mathbf{u}^{sl}(k)=-(\mathbf{CB})^{-1}\mathbf{CAx}(k)$$

is applied. Then, state equation of system (2.1) and expression for S give

$$\mathbf{Cx}(k+1)=\mathbf{CAx}(k)-\mathbf{CB}(\mathbf{CB})^{-1}\mathbf{CAx}(k)=0$$

which implies that $\mathbf{x}(k+1)\in S$.

Proof of theorem 3.2: The proof starts with assumption that sufficient conditions are fulfilled while stability and attraction of S should be shown.

The following notation is introduced, too. $O(S)\subseteq\Re^n$ is neighbourhood of S in meaning $O(S)$ is superset of an open set containing S. $O_{(.)}(S)=\{\mathbf{x}\colon \mathrm{d}(\mathbf{x},S)<(\cdot)\}$ is $(\cdot)$ − neighbourhood of S, $(\cdot)>0$. $V_{(.)}=\{\mathbf{x}\colon 0\le\nu(\mathbf{x})<(\cdot)\}$ is the largest connected neighbourhood of S with indicated property. $\varphi(.)\in K$ is comparison function from class K, with property $\varphi(\nu_1)<\varphi(\nu_2)$, $\forall\nu_1,\nu_2\in[0,+\infty[$, $\nu_1<\nu_2$.

Stability: As function v is positive definite on $\Re^n$ with respect to S, there exists open and connected set $A=O(S)$ such that $V_{\xi_1}\subset V_{\xi_2}\wedge\partial V_{\xi_1}\cap\partial V_{\xi_2}=\varnothing$ when $\xi_1<\xi_2\wedge V_\xi\subseteq A$, $\forall i=1,2$. Let $\xi_m=\max\{\xi\colon V_\xi\subseteq A\}$ and $\varepsilon^*=\max\{\varepsilon\colon O_\varepsilon(S)\subseteq V_{\xi_m}\}$. Let $\varepsilon\in\,]0,\varepsilon^*]$. As $\nu(x)$ is positive definite on $\Re^n$ relative to S, there exists $\varphi\in K$ such that $\nu(\mathbf{x})\ge\varphi[\mathrm{d}(\mathbf{x},S)]$ $\forall x\in\Re^n$. Last statement and $\mathrm{d}(\mathbf{x},S)=\varepsilon$, $\forall\mathbf{x}\in\partial O_\varepsilon(S)$ imply $\nu(\mathbf{x})\big|_{\partial O_\varepsilon(S)}\ge\varphi(\varepsilon)$. Now $\delta\in\,]0,+\infty[$ is chosen in the way $O_\delta(S)$ is the largest δ-neighbourhood inside $V_{\varphi(\varepsilon)}$ which finally implies

$O_\delta(S)\subseteq V_{\varphi(\varepsilon)}\subseteq O_\varepsilon(S)\subseteq O_{\varepsilon^*}(S)\subseteq V_{\xi_m}\subseteq A$. For initial state $\mathbf{x}(0)\in O_\delta(S)$, $\nu[\mathbf{x}(0)] < \varphi(\varepsilon)$ which means that hypersurface defined by $\nu[\mathbf{x}(0)] = \text{constans}$ is inside $V_{\varphi(\varepsilon)}$. This fact, conditions d) and e) of the theorem and expression $\Delta\nu[\mathbf{x}(k)] = \nu[\chi(k+1;k;\mathbf{x}(k))]-\nu[\chi(k;k;\mathbf{x}(k))]$, $\mathbf{x}(k) = \chi(k;0;\mathbf{x}(0))$ imply $v[\chi(k;0;\mathbf{x}(0))] < \nu[\mathbf{x}(0)] < \varphi(\varepsilon)$ $\forall k\in N_0$, $\forall\mathbf{x}(0)\in O_\delta(S)\backslash S \,\underline{\vee}\, v[\chi(k;0;\mathbf{x}(0))] = \nu[\mathbf{x}(0)] < \varphi(\varepsilon)$ $\forall k\in N_0$, $\forall\mathbf{x}(0)\in S$. On the other hand $v[\chi(k;0;\mathbf{x}(0))] \geq \varphi\{d[\chi(k;0;\mathbf{x}(0)),S]\}$ $\forall k\in N_0$. Last three inequalities give $\varphi(\varepsilon) > \varphi\{d[\chi(k;0;\mathbf{x}(0)),S]\}$ $\forall k\in N_0$, $\forall\mathbf{x}(0)\in O_\delta(S)$ and further $\varepsilon > d[\chi(k;0;\mathbf{x}(0)),S]$ $\forall k\in N_0$, $\forall\mathbf{x}(0)\in O_\delta(S)$, $\forall\varepsilon\in]0,\varepsilon^*]$. For $\varepsilon > \varepsilon^*$, $\delta(\varepsilon) = \delta(\varepsilon^*)$ is chosen. Previous analysis gives $d[\chi(k;0;\mathbf{x}(0)),S] < \varepsilon^*$ $\forall k\in N_0$ and hence $d[\chi(k;0;\mathbf{x}(0)),S] < \varepsilon$ $\forall k\in N_0$. In that way it is shown that all what is stated by definition 2.5 is fulfilled, that is, that stability of S is proved.

Attraction: S is assumed not to be globally attractive while the conditions of the theorem are fulfilled. The consequence is that $\lim_{k\to\infty} d[\chi(k;0;\mathbf{x}(0)),S] = v$, $v\in]0,\infty[$ for some $\mathbf{x}(0)$, $\mathbf{x}(0)$ is such that $d[\mathbf{x}(0),S] < \infty$. In that case there exists $\alpha\in]0,\ v]$ such that $\alpha = \inf\{d[\chi(k;0;\mathbf{x}(0)),S]:\ k\in N_0\}$. Let $\xi_1 = \inf[\nu(\mathbf{x}):\ x\in\partial O_\alpha(S)]$. Previous assumptions imply that $\inf\{v[\chi(1;0;\mathbf{x}(0))]:\ k\in N_0\} \geq \xi_1$ for $\mathbf{x}(0)\in\Re^n$, that is, working point of (2.1), (3.1) system never comes into V_{ξ_1}, $\chi(k;0;\mathbf{x}(0))\in\Re^n\backslash V_{\xi_1}$, $\forall k\in N_0$. Condition e) of the theorem implies that $\Delta\nu(\mathbf{x}) < 0$ $\forall\mathbf{x}\in\Re^n\backslash V_{\xi_1}$ so that there exists supremum of function $\Delta\nu$ on set $\Re^n\backslash V_{\xi_1}$, $\sup[\Delta\nu(\mathbf{x}):\ \mathbf{x}\in\Re^n\backslash V_{\xi_1}] = \eta < 0$. This further gives

$$
\begin{gathered}
v[\chi(1;0;\mathbf{x}(0))] - \nu[\mathbf{x}(0)] \leq \eta \\
v[\chi(2;1;\mathbf{x}(1))] - \nu[\mathbf{x}(1)] \leq \eta \\
\vdots \\
v[\chi(k;k-1;\mathbf{x}(k-1))] - \nu[\mathbf{x}(k-1)] \leq \eta .
\end{gathered}
$$

By summing all these relations, the following expression is got: $\nu[\mathbf{x}(k)] - \nu[\mathbf{x}(0)] \leq k\eta$, that is, $\nu[\mathbf{x}(k)] \leq \nu[\mathbf{x}(0)] + k\eta$. It implies that $\nu[\mathbf{x}(k)] < 0$ $\forall k \geq [0,\nu[\mathbf{x}(0)]/(-\eta)]_N + 1$ where $[0,\ \nu[\mathbf{x}(0)]/(-\eta)]_N$ denotes the biggest integer of the indicated segment. The last statement is contradictory to the b) condition of the theorem. This is caused by failed assumption that S is not globally attractive while conditions of the theorem are fulfilled, that is, S is globally attractive.

Proof of theorem 3.3: Previously introduced Lyapunov's function candidate fulfills evidently conditions a) – d) of theorem 3.2. Let be shown that conditions e) and f) are fulfilled. First, state $\mathbf{x}(k)$ of the system (2.1) is adopted to be out of S. Then

$$
\begin{aligned}
\Delta\nu[\mathbf{x}(k)] &= \nu[\mathbf{x}(k+1)]-\nu[\mathbf{x}(k)] = \\
&= \{\mathrm{sign}[\boldsymbol{\Psi}(k)\mathrm{sign}\mathbf{Cx}(k)]\}^T\boldsymbol{\Psi}(k)\mathrm{sign}\mathbf{Cx}(k) - [\mathrm{sign}\mathbf{Cx}(k)]^T\mathbf{Cx}(k) = \\
&= \sum_{i=1}^{m}|\psi_{ii}(k)|\cdot|\mathrm{sign}\mathbf{c}^i\mathbf{x}(k)| - 1^T|\mathbf{Cx}(k)| \leq 1^T\Psi(k)\mathbf{1} - 1^T|\mathbf{Cx}(k)| = (\alpha-1)\,1^T|\mathbf{Cx}(k)| < 0
\end{aligned}
$$

where $|\mathbf{Cx}(k)| = [|\mathbf{c}^1\mathbf{x}(k)|,\ |\mathbf{c}^2\mathbf{x}(k)|,\ \ldots,\ |\mathbf{c}^m\mathbf{x}(k)|]^T$, $\mathbf{1} = (1\ 1\ \ldots 1)^T$, $\mathbf{1}\in\Re^m$. Second, state $\mathbf{x}(k)$ of the system (2.1) is adopted to be inside of S. Then

$$
\Delta\nu[\mathbf{x}(k)] = \{\mathrm{sign}[\mathbf{CAx}(k) - \mathbf{CB}(\mathbf{CB})^{-1}\mathbf{CAx}(k)]\}^T[\mathbf{CAx}(k) - \mathbf{B}(\mathbf{CB})^{-1}\mathbf{CAx}(k)] - 1^T|\mathbf{Cx}(k)| = 0 .
$$

Proof of theorem 3.4: Differently from previous theorem, concerning condition e) of theorem 3.2, in this theorem decreasing of $\nu[\mathbf{x}(k)]$ along motion of (2.1) with prescribed "velocity" in section 3 should be shown. State of system (2.1) at time k is adopted to be out of S. Then

$$\Delta\nu[\mathbf{x}(k)] = 1^{\mathrm{T}}\,|\,\Psi(k)\mathrm{sign}\mathbf{Cx}(k)\,| \;-\; 1^{\mathrm{T}}|\mathbf{Cx}(k)| = -\frac{1^{\mathrm{T}}|\mathbf{Cx}(k)|}{P},\; k = 0, 1, \ldots, P-1,$$

that is,

$$\sum_{i=1}^{m}\psi_{\mathrm{ii}}(k)\,|\,\mathrm{sign}\, c^{i}\mathbf{x}(k)\,| = \sum_{j=1}^{m}\left[|\,\mathbf{c}^{j}\mathbf{x}(k)\,| - \frac{|\,c^{j}\mathbf{x}(0)\,|}{P}\right].$$

Further, including expression for $\psi_{ii}(k)$ left-hand side of the last relation becomes:

$$\sum_{i=1}^{m}\left\{\rho_{ii}(k)\sum_{j=1}^{m}\left[|\,c^{j}\mathbf{x}(k)\,| - \frac{1}{P}|\,\mathbf{c}^{j}\mathbf{x}(0)\,|\right]\right\}\cdot|\,\mathrm{sign}\mathbf{c}^{i}\mathbf{x}(k)\,| = \sum_{j=1}^{m}\left[|\,c^{j}\mathbf{x}(k)\,| - \frac{1}{P}|\,\mathbf{c}^{j}\mathbf{x}(0)\,|\right]\cdot$$

$$\sum_{i=1}^{m}\rho_{ii}(k)\,|\,\mathrm{sign}\mathbf{c}^{i}\mathbf{x}(k)\,| = \sum_{\mathrm{j}=1}^{m}\left[|\,\mathbf{c}^{j}\mathbf{x}(k)\,| - \frac{1}{P}|\,\mathbf{c}^{j}\mathbf{x}(0)\,|\right],\;\; \forall k = 0, 1, 2, \ldots, P-1$$

in which way the proof is finished.

5. EXAMPLE

In this section an example is provided to illustrate presented results.

Example 4.1: Let us consider unstable system

$$\mathbf{x}(k+1) = \begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & -1 \\ -1 & 1 & -2 \end{bmatrix}\mathbf{x}(k) + \begin{bmatrix} 1 & -1 \\ 2 & -1 \\ -2 & 1 \end{bmatrix}\mathbf{u}(k) \tag{4.1}$$

with controllable pair (A,B). S, which guarantees prescribed mode $\lambda = 0.9$ in sliding mode regime, is designed by analogical procedure to the known one in literature; see [16]. This sliding subspace S is defined by

$$\mathbf{C} = \begin{bmatrix} 1 & 1.9 & -2.1 \\ -1 & -1.1 & 0.9 \end{bmatrix}.$$

Let be chosen, in presented control algorithm

$$\rho_{ii}(k) = \rho(k) = \begin{cases} \dfrac{1}{\mathrm{m} - l(k)}, & l(k) \neq \mathrm{m} \\ 0, & l(k) = \mathrm{m} \end{cases}$$

where $l(k)$ is number of zero components in vector $\mathbf{Cx}(k)$. Further, "P" is adopted to be 10. System (4.1) with developed control algorithm included becomes:

$$\mathbf{x}(k+1) = \begin{bmatrix} 0.9 & -0.6 & 0.9 \\ -1.8 & 12 & -1.8 \\ -1.2 & 0.8 & -1.2 \end{bmatrix} \mathbf{x}(k) + \rho(k)\{\Omega(k) - 0.1\Omega(0)\} \begin{bmatrix} -1 & -2 \\ 0.5 & 0.5 \\ -0.5 & -0.5 \end{bmatrix} \cdot \begin{Bmatrix} \mathrm{sign}[\sigma_1(k)] \\ \mathrm{sign}[\sigma_2(k)] \end{Bmatrix}$$

where,

$$\sigma_1(\cdot) = x_1(\cdot) + 1.9x_2(\cdot) - 2.1x_3(\cdot),$$
$$\sigma_2(\cdot) = -x_1(\cdot) - 1.1x_2(k) + 0.9x_3(\cdot),$$
$$\Omega(\cdot) = |\sigma_1(\cdot)| + |\sigma_2(\cdot)|.$$

Solving of the discrete digital system was done by digital computer. As illustration, simulation results from the initial state $\mathbf{x}(0) = (5 \;\; 5 \;\; 10)^{\mathrm{T}}$, controls, $\mathbf{c}^1\mathbf{x}(k)$ and $\mathbf{c}^2\mathbf{x}(k)$ are given in figures 4.1, 4.2, 4.3 and 4.4 respectively. Evidently at time $k = 10$ working point reaches S, staying afterwards in it for ever.

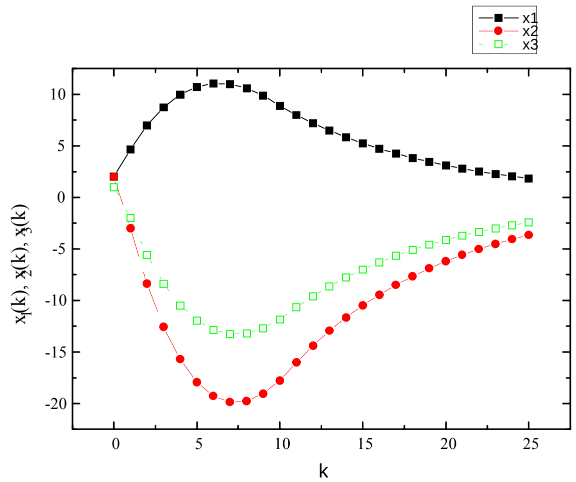

Fig.4.1

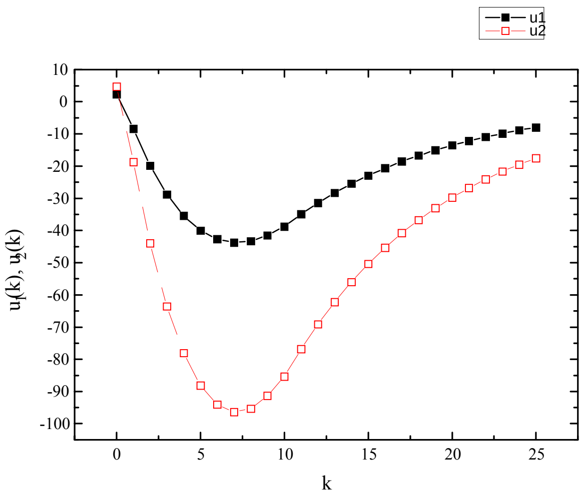

Fig.4.2

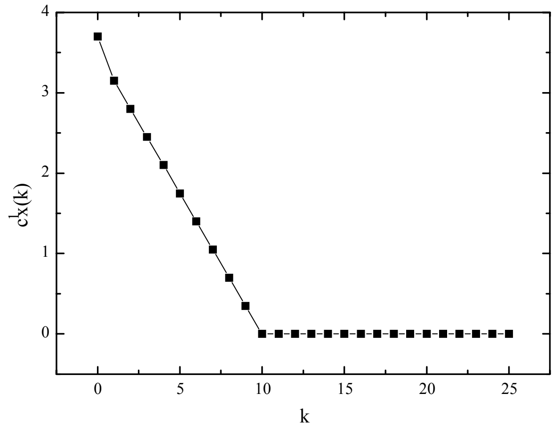

Fig.4.3

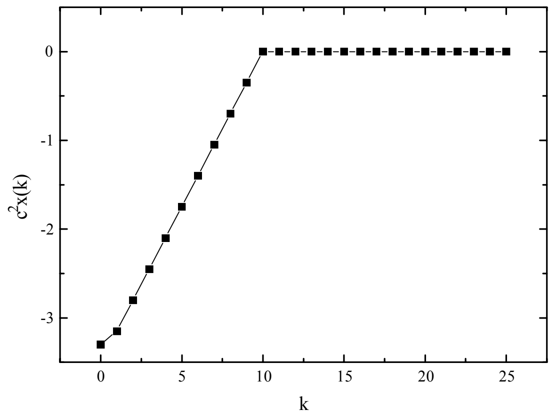

Fig.4.4

6. CONCLUSION

In the paper, discrete digital type of VSCS with sliding mode is considered but this time related to the most general case of linear time invariant plant with no restrictions on the form of state equation and number of controls.

The following phenomenon, characteristic only for this type of VSCS with sliding modes, is discovered: it is necessary to apply actual control so-called sliding control u^{sl} to the plant during sliding mode regime, what is not characteristic for continuous time VSCS with sliding modes.

Solution of this type of problem makes place for microprocessor compensator application as compensator in the systems.

Acknowledgment. *The author would like to express deep gratitude to Prof. Lj.T.Grujic for all sorts of support in research the paper resulted from*.

REFERENCES

1. Ambrosino, G., Gelentano, G., Garofalo, F.: *Variable Structure Model Reference Adaptive Control Systems*, International Journal of Control, Vol. 39, pp.1339-1349, 1984.
2. Bartolini, G., Zolezzi, T.: *Some Developments of Variable Structure Control for Nonlinear Systems*, Preprints of 25 IEEE Conference on Decision and Control, Athens, Greece, 1986.
3. Bartolini, G., Ferrara, A., Utkin, V. I.: *Adaptive Sliding Mode Control in Discrete Time Systems*, Automatica, Vol. 31, No. 5, pp. 769-773, 1995.
4. Bucevac, Z.: *Design of Digital Discrete Control Systems with Sliding Mode* - Ph. D. Dissertation (in Serbian), Mechanical Engineering Department, Belgrade University, Belgrade, Yugoslavia, 1985.
5. Bucevac, Z.: *Lyapunov's Method Approach in a Stabilizing Control Algorithm Design for Digital Discrete VS Systems with Sliding Modes - Linear Plant Case*, Proceedings of the 7. IASTED International Symposium Modelling, Identification and Control, Grindelwald, Switzerland, l988.
6. El-Khazali, R., DeCarlo, R.: *Output Feedback VSCS Design*, Automatica, Vol.31, No.6, 805-816, 1995.
7. Emel'yanov, S.V.: *Variable Structure Automatic Control Systems* (in Russian), Nauka, Moscow, 1967.
8. Emel'yanov, S.V., Utkin, V.I.,Taran,V.A., Kostyleva,N.E., Subladze, A.M., Ezerov, V.B., Dubrovskij, E.N.: *Theory of Variable Structure Systems* (in Russian), Nauka, Moscow, 1970.
9. Fluge-Lotz, I.: *Discontinuous Automatic Control Systems*, Academic Press, Princeton, New Jersey, 1953.
10. Furuta, K.: *Sliding Mode Control of a Discrete System*, System and Control Letters, Vol. 14, pp. 145-152. 1990.
11. Gao, W., Wang, Y., Homaifa, A.: *Discrete Time Variable Structure Control Systems*, IEEE Transactionon on Industrial Electronics, Vol.42, No.2, 117-122, 1995.
12. Geering, H.P., Guzzella, L.: *Model Following Variable Structure Control for a Classof Uncertain Mechanicalth Systems*, Preprints of 25th IEEE Conferenceon Decision and Control, Athens, Grecce, 1986.
13. Gough, E.N., Ismail,M.Z., King,E.R.: *Analysis of Variable Structure Systems with Sliding Modes*, International Journal of Systems Science, Vol. 15, pp. 401- 409, 1984.
14. Grujic, Lj., Ribar, Z.: *Nonstationary Lourie Variable Structure Control System Design*, II SAUM conference, Belgrade, 1986.
15. Itkis, Y.: *Control Systems of Variable Structure*, New York, Wiley, 1976.
16. Itkis,Y.: *Dynamic Switching of Type-I/Type-II / Structures in Tracking Servosystems*, IEEE Transactionon Automatic Control, AC-28, pp. 531-534, 1983.
17. Karakasoglu, A., Sundareshau, M.K.: *A Recurrent Neural Network-based Adaptive VS Model Following Control of Robotic Manipulators*, Automatica, Vol.31, No.10, 1995.
18. Khurana, H., Ashon, S.I., Lamba, S.S.: *On Stabilization of Large-Scale Control Systems Using Variable Structure Systems Theory*, IEEE Transactionon Automatic Control, AC-31, pp. 176-178, 1986.
19. Kotta, U.: *Comments on "On the Stability of Discrete Time Sliding Mode Control Systems"*, IEEE Transactionon Automatic Control, Vol.-34, No.9, pp. 1021-1022, 1989.
20. Milosavljevic, C.: *General Conditions for the Existence of a Quasisliding Mode on the Switching Hyperplanein Discrete Variable Structure Systems*, Automation and Remote Control, No. 3, pp. 36-44, 1985.
21. Li, K-C., Leung, T-P., Hu, Y-M.: *Sliding Mode Control of Distributed Parameter Systems*, Automatica, Vol.30, No.12, 1994.
22. Ryan, E.P.: *A Variable Structure Approach Regulation of Uncertain Dynamical Systems*, International Journal of Control, Vol.38, pp 1121-1134, 1983.
23. Sarpturk, S. Z., Istefanopulos, Y., Kaynak, O.: *On the Stability of Discrete Time Sliding Mode Control Systems*, IEEE Transactionon Automatic Control, AC-32, No.10, pp. 930-932, 1987.
24. Sira-Ramirez, S.: *Non linear discrete variable structure systems in quasi-sliding mode*, Int. J. Control, Vol.54, pp.1171-1187, 1991.
25. Slotine ,J.J., Sastry, S.S.: *Tracking Control of Nonlinear Systems Using Sliding Surfaces with Application to Robot Manipulators*, International Journal of Control, Vol.38, pp.465-492, 1983.
26. Slotine, J.J.E., Hedrick, J.K., Misawa, E.: *On Sliding Observers for Nonlinear Systems*, Preprints of 25th IEEE, Conference on Decision and Control, Athens, Greece, 1986.
27. Utkin, V.I.: *Sliding Modes and Their Applicationsin Variable Structure Systems* (in Russian), Nauka, Moscow, 1974.
28. Utkin, V.I.: *Sliding Modes in Optimization and Control Tasks* (in Russian), Nauka, Moscow, 1981.
29. Xing-Huo, Y., Chun-Bo, F.: *A Lyapunov-function Approach for the Design of Variable-Systems*, Preprints of 10th World Congress for Structure Automatic Control, Munich,W.Germany,1987.

30. Young, K-K.D.: *Asymptotic Stability of Model Reference Systems with Variable Structure Control*, IEEE Transaction on Automatic Control, AC-22, pp. 279-281, 1977.
31. Young,K-K.D.: *Design of Variable Structure Model Following Central Systems*, IEEE Transaction on Automatic Control, AC-23, pp. 1079-1085, 1978.
32. Zheung, F., Chen, M., Gao, W.: *VSC of Time Delay Systems with a Simulation Study on Stabilizing Combustion in Liquid Propellant Rocket Motors*, Automatica, Vol.31, No.7, 1995.

DIGITALNI SISTEMI AUTOMATSKOG UPRAVLJANJA PROMENLJIVI STRUKTURE - SLUČAJ MULTIVARIJABILNOG LINEARNOG OBJEKTA -

Zoran M. Bučevac

Razmatra se linearni stacionarni diskretni digitalni objekt bez ograničenja na oblik njegove jednačine stanja i broj upravljanja. Razvijen je stabilizirajući algoritam upravljanja sa povratnom spregom po stanju pomoću Ljapunovljeve Druge metode koji daje sistem promenljive strukture sa kliznim radnim režimom. Ovaj algoritam upravljanja obezbeđuje da se stanje sistema kreće iz proizvoljnog početnog položaja do propisanog takozvanog podprostora klizanja S, za konačno vreme. Unutar podprostora klizanja S sistem prelazi u klizni radni režim i u njemu ostaje zauvek.