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PARAMETRIC METHOD IN THE THEORY OF NON-STATIONARY AXISYMMETRICAL MHD BOUNDARY LAYER ON A ROTARY BODY

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Abstract. *The paper discusses axisymmetrical non-stationary MHD boundary layer on a rotary body. Firstly, the Mangler-Stepan transformations are enlarged; it has been shown that equations of the axisymmetrical problems are reduced to equations of the respective plane problem. Then, with the use of the parametric method, the universal equation which turns out to be identical to the universal equation of the respective plane problem or plane analogy, is obtained. It has practically been shown that the solutions for plane non-stationary MHD boundary layer can be used for solving axisymmetrical problems of non-stationary MHD boundary layer on a rotary body.*

The parametric method [1, 2, 3] has given satisfactory results in the theory of the plane boundary layer, namely, both of the stationary and the non-stationary one [4, 5, 6]. This paper presents an attempt to form a parametric method for non-stationary axisymmetrical problems of boundary layer MHD on rotary bodies. The discussion will be focused on the boundary layer formed at the longitudinal rotary body flow in the presence of a homogenous external magnetic field perpendicular to the body. This is in fact a boundary layer that is formed at non-plane two-dimensional motions of a viscous fluid. The fluid is incompressible while its electric conductivity is invariable. The problem is discussed in inductionless approximation, that is, for small values of the magnetic Reynolds number. Thus, the hydrodynamic problem can be separated from the electrodynamic one.

The mathematical model of the described problem is represented by the equations

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$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}-N(u-U)$$
$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial y}=0 \qquad \left(N=\frac{\sigma B^2}{\rho}\right) \tag{1}$$

as well as by the boundary and the initial conditions

$$u=0,\ v=0 \ \text{ for } \ y=0;\quad u\to U(x,t) \ \text{ for } \ y\to\infty;$$
$$u=u_1(x,y) \ \text{ for } \ t=t_0;\quad u\to u_0(t,y) \ \text{ for } \ x\to x_0. \tag{2}$$

The notations in equations (1) and in boundary and initial conditions (2) common for the theory of MHD boundary layer are used, namely: t - time, x, y - longitudinal and transversal coordinate in the boundary layer, respectively, u, v - longitudinal and transversal velocity in the boundary layer, respectively, U- velocity on the external limit of the boundary layer, ν, ρ, σ - coefficient of kinematic viscosity, density and electric conductivity of the fluid, respectively, B - magnetic induction, r- radius of the transversal curve of the body surface, u_1- distribution of the longitudinal velocity in the boundary layer at the moment $t=t_0$, u_0 - distribution of the longitudinal velocity in the section $x=x_0$ of the boundary layer.

Mangler and Stepanov [7] have shown that the equations of the stationary axisymmetrical boundary layer on a rotary body can be reduced, by an appropriate choice of transformations - new variables, to equations corresponding to the stationary plane boundary layer. In this way the process of solving of the stationary axisymmetrical problems of the boundary layer on rotary bodies is reduced to solving equations of the respective plane problem - analogy. In Ref. [8] these transformations are enlarged to comprise equations of the problem of stationary axisymmetrical MHD boundary layer on the rotary body. In this paper they will be further enlarged to non-stationary axisymmetrical problems of boundary layer MHD on the rotary body, that is, to equations (1) with boundary and initial conditions (2). For this purpose we are introducing the transformations

$$\tilde{t}=\frac{r^2}{L^2}t,\ \ \tilde{x}=\frac{1}{L^2}\int_0^x r^2(x)dx,\ \ \tilde{y}=\frac{r}{L}y,\ \ \tilde{u}(\tilde{x},\tilde{y},\tilde{t})=u(x,y,t),$$
$$\tilde{U}(\tilde{x},\tilde{t})=U(x,t),\ \ \tilde{v}=\frac{L}{r}\left(v+\frac{1}{r}\frac{dr}{dx}yu\right),\ \ \tilde{N}(\tilde{x},\tilde{t})=\frac{L^2}{r^2}N(x,t) \tag{3}$$

with the use of the operator

$$\frac{\partial}{\partial t}=\frac{r^2}{L^2}\frac{\partial}{\partial\tilde{t}},\ \ \frac{\partial}{\partial x}=\frac{r^2}{L^2}\frac{\partial}{\partial\tilde{x}}+\frac{y}{L}\frac{dr}{dx}\frac{\partial}{\partial\tilde{y}},\ \ \frac{\partial}{\partial y}=\frac{r}{L}\frac{\partial}{\partial\tilde{y}} \tag{4}$$

we transform equations (1) into equations

$$\frac{\partial\tilde{u}}{\partial\tilde{t}}+\tilde{u}\frac{\partial\tilde{u}}{\partial\tilde{x}}+\tilde{v}\frac{\partial\tilde{u}}{\partial\tilde{y}}=\frac{\partial\tilde{U}}{\partial\tilde{t}}+\tilde{U}\frac{\partial\tilde{U}}{\partial\tilde{x}}+\nu\frac{\partial^2\tilde{u}}{\partial\tilde{y}^2}-\tilde{N}(\tilde{u}-\tilde{U})$$
$$\frac{\partial\tilde{u}}{\partial\tilde{x}}+\frac{\partial\tilde{v}}{\partial\tilde{y}}=0, \tag{5}$$

while boundary and initial conditions (2) are transformed into conditions

$$\begin{aligned} &\tilde{u}=0\,,\ \tilde{v}=0\,,\ \text{for}\ \ \tilde{y}=0\,; &&\tilde{u}\to\tilde{U}(\tilde{x},\tilde{t})\ \ \text{for}\ \ \tilde{y}\to\infty\,;\\ &\tilde{u}=\tilde{u}_1(\tilde{x},\tilde{t})\,,\ \text{for}\ \ \tilde{t}=\tilde{t}_0\,; &&\tilde{u}\to\tilde{u}_0(\tilde{t},\tilde{y})\ \ \text{for}\ \ \tilde{x}\to\tilde{x}_0\,. \end{aligned} \tag{6}$$

where "~", as has been done so far, denotes respective values of plane MHD boundary layer.

It can be noticed that equations (5) with boundary and initial conditions (6) are identical with equations and boundary and initial conditions of the respective plane problem - plane analogy.

In this way the solving of the axisymmetrical problem, that is, solving of equations (1) with boundary and initial conditions (2) is reduced to solving the respective plane problem, that is, to solving equations (5) with boundary and initial conditions (6). It virtually means that the solution of the respective plane analogy can be used for solving the axisymmetrical problem. Naturally, all this stands for the concrete velocity on the external limit of the boundary layer, for the concrete magnetic field and for the radius of the transversal curve of the body surface.

As a simple case of applying transformations (3) we will consider the axisymmetrical fluid flow in the boundary layer for which

$$U(x,t)=C_1xt,\quad r(x)=C_2x,\quad N(x,t)=C_3xt\ . \tag{7}$$

In that case, transformations (3) have the form

$$\begin{gathered} \tilde{t}=\frac{C_2^2}{L^2}x^2t\,,\ \ \tilde{x}=\frac{C_2^2}{3L^2}x^3\,,\ \ \tilde{y}=\frac{C_2}{L}xy\,,\ \ \tilde{u}=u\,,\\ \tilde{U}=\frac{C_1}{3^{1/3}}\left(\frac{L}{C_2}\right)^{4/3}\tilde{x}^{-1/3}\tilde{t}\,,\ \ \tilde{v}=\frac{L}{C_2x}\left(v+\frac{1}{x}yu\right),\ \ \tilde{N}=\frac{C_3}{3}\frac{L^2}{C_2^2}\tilde{x}^{-1}\tilde{t} \end{gathered} \tag{8}$$

while the problem is reduced to solving the respective plane problem in which

$$\tilde{U}(\tilde{x},\tilde{t})=C\tilde{x}^{-1/3}\tilde{t}\ ,\ \ \tilde{N}(\tilde{x},\tilde{t})=C^*\tilde{x}^{-1}\tilde{t} \tag{9}$$

where

$$C=\frac{C_1}{3^{1/3}}\left(\frac{L}{C_2}\right)^{4/3},\quad C^*=\frac{C_3}{3}\frac{L^2}{C_2^2}\,. \tag{10}$$

The characteristic parameters of the boundary layer of the axisymmetrical problem and of the respective plane problem are connected by the following relations:

- for tangential stress upon the wall

$$\tau_w=\frac{r}{L}\tilde{\tau}_w \tag{11}$$

- for thickness of extrusion

$$\delta^*=\frac{L}{r}\tilde{\delta}^* \tag{12}$$

- for thickness of the impulse loss

$$\delta^{**}=\frac{L}{r}\tilde{\delta}^{**}\,. \tag{13}$$

Now some of the methods of the plane boundary layer theory can be applied to solving this problem. Likewise, the paper might as well end here with the already-presented conclusion.

Still, we will further see what the application of the generalized similarity method is like - that is, the parametric method in the Lojcjansky version applied to these problems. For this purpose we are taking into consideration flow function $\Psi(x,y,t)$ in the relations

$$\frac{1}{r}\frac{\partial(r\Psi)}{\partial y}=u\,,\quad \frac{1}{r}\frac{\partial(r\Psi)}{\partial x}=-v \tag{14}$$

and thus the system of equations (1) is transformed into the equation

$$\frac{\partial^2\Psi}{\partial t\partial y}+\frac{\partial\Psi}{\partial y}\frac{\partial^2\Psi}{\partial x\partial y}-\left(\frac{\partial\Psi}{\partial x}+\frac{\Psi}{r}\frac{dr}{dx}\right)\frac{\partial^2\Psi}{\partial y^2}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^3\Psi}{\partial y^3}-N\left(\frac{\partial\Psi}{\partial y}-U\right) \tag{15}$$

while boundary and initial conditions (2) are transformed into conditions

$$\begin{aligned}&\Psi=0\,,\ \frac{\partial\Psi}{\partial y}=0\ \text{ for }\ y=0\,;\quad \frac{\partial\Psi}{\partial y}\to U(x,t)\ \text{ for }\ y\to\infty\,;\\&\frac{\partial\Psi}{\partial y}=u_1(x,y)\ \text{ for }\ t=t_0\,;\quad \frac{\partial\Psi}{\partial y}\to u_0(t,y)\ \text{ for }\ x\to x_0\,.\end{aligned} \tag{16}$$

While using further Lojcjansky's variables [2] as well as transformations (3) we are introducing new variables

$$\widetilde{t}=\frac{r^2}{L^2}t\,,\ \widetilde{x}=\frac{1}{L^2}\int_0^x r^2(x)dx\,,\ \eta=D\frac{r}{L}\frac{y}{h(\widetilde{x},\widetilde{t})}\,,\ \varphi(\widetilde{x},\eta,\widetilde{t})=D\frac{r}{L}\frac{\Psi(x,y,t)}{U(x,y)h(\widetilde{x},\widetilde{t})} \tag{17}$$

in which D - standarizing constant, $h(\widetilde{x},\widetilde{t})$-linear size; thus, equation (15) is transformed into equation

$$\begin{aligned}&D^2\frac{\partial^3\varphi}{\partial\eta^3}+z\frac{\partial\widetilde{U}}{\partial\widetilde{x}}\left[1-\left(\frac{\partial\varphi}{\partial\eta}\right)^2+\varphi\frac{\partial^2\varphi}{\partial\eta^2}\right]+\left(\frac{z}{\widetilde{U}}\frac{\partial\widetilde{U}}{\partial\widetilde{t}}+\widetilde{N}z\right)\left(1-\frac{\partial\varphi}{\partial\eta}\right)+\\&+\frac{1}{2}\eta\frac{\partial z}{\partial\widetilde{t}}\frac{\partial^2\varphi}{\partial\eta^2}+\frac{1}{2}\widetilde{U}\frac{\partial z}{\partial\widetilde{x}}\varphi\frac{\partial^2\varphi}{\partial\eta^2}-z\frac{\partial^2\varphi}{\partial\widetilde{t}\partial\eta}+\widetilde{U}zX(\widetilde{x};\eta)=0\end{aligned} \tag{18}$$

where the notations are introduced

$$z=\frac{h^2}{\nu}\,,\quad X(x_1;x_2)=\frac{\partial\varphi}{\partial x_1}\frac{\partial^2\varphi}{\partial\eta\partial x_2}-\frac{\partial\varphi}{\partial x_2}\frac{\partial^2\varphi}{\partial x_1\partial\eta}\,. \tag{19}$$

The boundary conditions corresponding to equation (18) are obtained from conditions (16) and are of the following form

$$\varphi=0,\quad \frac{\partial\varphi}{\partial\eta}=0\ \text{ for }\ \eta=0\,;\frac{\partial\varphi}{\partial\eta}\to1\ \text{ for }\ \eta\to\infty\,. \tag{20}$$

Further, using the ideas of Bušmarin and Saraev, we are taking into consideration sets of parameters

$$f_{k,n}=\widetilde{U}^{k-1}\frac{\partial^{k+n}\widetilde{U}}{\partial\widetilde{x}^{k}\partial\widetilde{t}^{n}}z^{k+n}\qquad(k,n=0,1,2,...;kvn\neq 0)$$
$$g_{k,n}=\widetilde{U}^{k-1}\frac{\partial^{k-1+n}\widetilde{N}}{\partial\widetilde{x}^{k-1}\partial\widetilde{t}^{n}}z^{k+n}\qquad(k,n=0,1,2,...;k\neq 0)\tag{21}$$

as well as constant parameter

$$p=\frac{\partial z}{\partial\widetilde{t}}=const.\tag{22}$$

It can be noticed that the first parameters are of the forms

$$f_{1,0}=z\frac{\partial\widetilde{U}}{\partial\widetilde{x}};\quad f_{0,1}=\frac{z}{\widetilde{U}}\frac{\partial\widetilde{U}}{\partial\widetilde{t}};\quad g_{1,0}=\widetilde{N}z\,.\tag{23}$$

In further use of parameters as new independent variables instead of $\widetilde{x}$ and $\widetilde{t}$ as well as differential operators:

$$\frac{\partial}{\partial\widetilde{x}}=\sum_{\substack{k,n=0\\kvn\neq0}}^{\infty}\frac{\partial f_{k,n}}{\partial\widetilde{x}}\frac{\partial}{\partial f_{k,n}}+\sum_{\substack{k=1\\n=0}}^{\infty}\frac{\partial g_{k,n}}{\partial\widetilde{x}}\frac{\partial}{\partial g_{k,n}}$$
$$\frac{\partial}{\partial\widetilde{t}}=\sum_{\substack{k,n=0\\kvn\neq0}}^{\infty}\frac{\partial f_{k,n}}{\partial\widetilde{t}}\frac{\partial}{\partial f_{k,n}}+\sum_{\substack{k=1\\n=0}}^{\infty}\frac{\partial g_{k,n}}{\partial\widetilde{t}}\frac{\partial}{\partial g_{k,n}},\tag{24}$$

we are transforming equation (18) into the equation

$$D^{2}\frac{\partial^{3}\varphi}{\partial\eta^{3}}+\left(\frac{1}{2}F+f_{1,0}\right)\varphi\frac{\partial^{2}\varphi}{\partial\eta^{2}}+f_{1,0}\left[1-\left(\frac{\partial\varphi}{\partial\eta}\right)^{2}\right]+(f_{0,1}+g_{1,0})\left(1-\frac{\partial\varphi}{\partial\eta}\right)+\frac{1}{2}p\eta\frac{\partial^{2}\varphi}{\partial\eta^{2}}=$$
$$\sum_{\substack{k,n=0\\kvn\neq0}}^{\infty}\left[C^{*}_{k,n}X(\eta;f_{k,n})+A^{*}_{k,n}\frac{\partial^{2}\varphi}{\partial\eta\partial f_{k,n}}\right]+\sum_{\substack{k=1\\n=0}}^{\infty}\left[D^{*}_{k,n}X(\eta;g_{k,n})+B^{*}_{k,n}\frac{\partial^{2}\varphi}{\partial\eta\partial g_{k,n}}\right]\tag{25}$$

where, for the sake of brevity, the following notations are introduced

$$C^{*}_{k,n}=C_{k,n}+(k+n)Ff_{k,n};\quad C_{k,n}=(k-1)f_{1,0}f_{k,n}+f_{k+1,n};\quad F=\widetilde{U}\frac{\partial z}{\partial\widetilde{x}};$$
$$A^{*}_{k,n}=A_{k,n}+(k+n)pf_{k,n};\quad A_{k,n}=(k-1)f_{0,1}f_{k,n}+f_{k,n+1};$$
$$D^{*}_{k,n}=D_{k,n}+(k+n)Fg_{k,n};\quad D_{k,n}=(k-1)f_{1,0}g_{k,n}+g_{k+1,n};$$
$$B^{*}_{k,n}=B_{k,n}+(k+n)pg_{k,n};\quad B_{k,n}=(k-1)f_{0,1}g_{k,n}+g_{k,n+1}\,.\tag{26}$$

In order to make equation (25) universal, that is, explicitly independent of characteristics of the external flow, it is necessary to show that function F also depends only on the introduced parameters. For this purpose, let's start from the impulse equation of the plane analogy of the observed problem, that is, from the impulse equation of non-stationary plane MHD boundary layer

$$\frac{\partial}{\partial \tilde{t}}(\tilde{U}\tilde{\delta}^*)+\frac{\partial}{\partial \tilde{x}}(\tilde{U}^2\tilde{\delta}^{**})+\tilde{U}\left(\frac{\partial \tilde{U}}{\partial \tilde{x}}+\tilde{N}\right)\tilde{\delta}^*-\frac{\tilde{\tau}_w}{\rho}=0 \tag{27}$$

where

$$\tilde{\delta}^*=\int_0^\infty\left(1-\frac{\tilde{u}}{\tilde{U}}\right)d\tilde{y}\,,\quad \tilde{\delta}^{**}=\int_0^\infty\frac{\tilde{u}}{\tilde{U}}\left(1-\frac{\tilde{u}}{\tilde{U}}\right)d\tilde{y}\,,\quad \tilde{\tau}_w=\rho\nu\left(\frac{\partial \tilde{u}}{\partial \tilde{y}}\right)_{\tilde{y}=0}. \tag{28}$$

Further on, by taking into consideration the parameters

$$\begin{gathered} H^*=\frac{\tilde{\delta}^*}{h}=\frac{1}{D}\int_0^\infty\left(1-\frac{\partial\varphi}{\partial\eta}\right)d\eta\,, \\ H^{**}=\frac{\tilde{\delta}^{**}}{h}=\frac{1}{D}\int_0^\infty\frac{\partial\varphi}{\partial\eta}\left(1-\frac{\partial\varphi}{\partial\eta}\right)d\eta\,,\quad \zeta=\frac{\tilde{\tau}_w h}{\rho\nu\tilde{U}}=D\left.\frac{\partial^2\varphi}{\partial\eta^2}\right|_{\eta=0} \end{gathered} \tag{29}$$

and moving in equation (27) to the parameters as new independent variables, instead of $\tilde{x}$ and $\tilde{t}$, the equation is obtained from which it is

$$F=\frac{\zeta-f_{1,0}(2H^{**}+H^*)-\left(f_{0,1}+g_{1,0}+\dfrac{p}{2}\right)H^*-M}{\dfrac{1}{2}H^{**}+\sum\limits_{\substack{k,n=0\\kvn\neq 0}}^{\infty}(k+n)f_{k,n}\dfrac{\partial H^{**}}{\partial f_{k,n}}+\sum\limits_{\substack{k=1\\n=0}}^{\infty}(k+n)g_{k,n}\dfrac{\partial H^{**}}{\partial g_{k,n}}} \tag{30}$$

where, for the sake of brevity, the notation is introduced

$$M=\sum_{\substack{k,n=0\\kvn\neq 0}}^{\infty}\left(A^*_{k,n}\frac{\partial H^*}{\partial f_{k,n}}+C_{k,n}\frac{\partial H^{**}}{\partial f_{k,n}}\right)+\sum_{\substack{k=1\\n=0}}^{\infty}\left(B^*_{k,n}\frac{\partial H^*}{\partial g_{k,n}}+D_{k,n}\frac{\partial H^{**}}{\partial g_{k,n}}\right). \tag{31}$$

This has shown that function F explicitly depends only on parameters (21) and (22) and such is also equation (25). Therefore, equation (25) does not explicitly depend on characteristics of the external flow and, in that sense, it represents a universal equation of the observed problem. The boundary conditions that are also universal are of the form

$$\begin{gathered} \varphi=0\,,\ \frac{\partial\varphi}{\partial\eta}=0\ \text{ for }\ \eta=0\,;\qquad \frac{\partial\varphi}{\partial\eta}\to 1\ \text{ for }\ \eta\to\infty \\ \varphi=\varphi_0(n)\ \text{ for }\ \begin{cases} f_{k,n}=0\ \ (k,n=0,1,2,...;kvn\neq 0) \\ g_{k,n}=0\ \ (k,n=0,1,2,...;\neq 0) \\ p=0 \end{cases} \end{gathered} \tag{32}$$

where $\varphi_0(\eta)$is Blasius's solution for the boundary layer on the plate.

Linear size *h* is still arbitrary and it is the one to be chosen before integrating equation (25). On principle, it can be chosen in various ways; here, according to the ideas of

Bušmarin and Saraev, it is thus chosen so that it turns out to be identical with the thickness of the plane analogy impulse loss, that is, $h = \tilde{\delta}^{**}$. Then from (29) we obtain that

$$H^{**} = 1, \quad H^{*} = \frac{\tilde{\delta}^{*}}{\tilde{\delta}^{**}} = \tilde{H} \tag{33}$$

while expression (30) for function F is reduced to the expression

$$F = 2\left[\zeta - f_{1,0}(2+\tilde{H}) - \left(f_{0,1} + g_{1,0} + \frac{1}{2}p\right)\tilde{H} - \sum_{\substack{k,n=0 \\ kvn\neq 0}}^{\infty} A^{*}_{k,n}\frac{\partial \tilde{H}}{\partial f_{k,n}} - \sum_{\substack{k=1 \\ n=0}}^{\infty} B^{*}_{k,n}\frac{\partial \tilde{H}}{\partial g_{k,n}}\right] \tag{34}$$

while equation (25) and boundary conditions (32) do not change their form.

It can be noticed that universal equation (25) is completely identical with the universal equation of non-stationary plane MHD boundary layer obtained in Ref. [9], as was expected regarding all that has previously been said.

Equation (25) with boundary conditions (32) should, in particular approximation, be solved once and for all; the obtained universal results should be preserved in an appropriate way and they should be used both for the plane problem - analogy and for the described axisymmetrical problem. The obtained universal results can be used for making general conclusions about the boundary layer development as well as calculations of concrete cases.

If, for instance, the influence of parameters $f_{1,0}$ and $f_{0,1}$, $g_{1,0}$ and $p$is preserved in equation (25), as well as the influence of the derivatives with respect to $f_{1,0}$ and $g_{1,0}$, while the influence of the other parameters and their derivatives is neglected, the universal equation in four-parameter twice-localized approximation is obtained

$$\Im = f_{1,0}FX(\eta; f_{1,0}) + pf_{1,0}\frac{\partial^2 \varphi}{\partial \eta \partial f_{1,0}} + g_{1,0}FX(\eta; g_{1,0}) + pg_{1,0}\frac{\partial^2 \varphi}{\partial \eta \partial g_{1,0}} \tag{35}$$

where $\Im$ denotes the left side of equation (25). Function F, in the same approximation, is obtained from expression (34) and is of the form

$$F = 2\left[\zeta - f_{1,0}(2+\tilde{H}) - \left(f_{0,1} + g_{1,0} + \frac{1}{2}p\right)\tilde{H} - pf_{1,0}\frac{\partial \tilde{H}}{\partial f_{1,0}} - pg_{1,0}\frac{\partial \tilde{H}}{\partial g_{1,0}}\right]. \tag{36}$$

The universal boundary conditions, in the same approximation, are

$$\begin{gathered} \varphi = 0, \ \frac{\partial \varphi}{\partial \eta} = 0 \ \text{ for } \ \eta = 0; \ \frac{\partial \varphi}{\partial \eta} \to 1 \ \text{ for } \ \eta \to \infty \\ \varphi = \varphi_0(\eta) \ \text{ for } \ f_{1,0} = 0; \ f_{0,1} = 0; \ g_{1,0} = 0; \ p = 0. \end{gathered} \tag{37}$$

Equation (35) with boundary conditions (37) has already been solved in Ref. [9]; these results can be used both for plane problems and for axisymmetrical non-stationary problems of boundary layer MHD on the rotary body. As for calculation of concrete problems of the axisymmetrical non-stationary boundary layer on the rotary body it is simply realized by using expressions (11), (12) and (13).

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PARAMETARSKA METODA U TEORIJI NESTACIONARNOG OSNOSIMETRIČNOG MHD GRANIČNOG SLOJA NA OBRTNOM TELU

Zoran B. Boričić, Dragiša D. Nikodijević, Dragica R. Milenković

U radu se razmatra osnosimetrični nestacionarni MHD granični sloj na obrtnim telima. Prvo je izvršeno proširenje Mangler-Stepanovih transformacija i pokazano da se jednačine osnosimetričnog problema svode na jednačine odgovarajućeg ravanskog problema. Zatim je korišćenjem parametarske metode dobijena univerzalna jednačina koja se poklapa sa univerzalnom jednačinom odgovarajućeg ravanskog problema - ravanske analogije. Praktično je pokazano da se za rešavanje osnosimetričnog problema nestacionarnog MHD graničnog sloja na obrtnom telu mogu koristiti rešenja ravanskog nestacionarnog MHD graničnog sloja.