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BOUNDARY LAYER OF IONIZED GAS IN THE CASE OF CHANGEABLE ELECTROCONDUCTIVITY

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Abstract. *This paper studies ionized gas flow in the case of a concrete form of the law of its electroconductivity change. The corresponding boundary layer equations are first by means of suitable transformations brought to a universal form. Then the equations are numerically solved in three-parametric approximation. Physical values and some boundary layer characteristics are graphically presented.*

1. INTRODUCTORY STUDIES, STARTING EQUATIONS

This paper studies ionized gas flow in the boundary layer on the body of arbitrary shape. The study represents the sequence of our previous analysis of this extremely complex case of the fluid flow. Ionized gas, that is plasma, is essentially different from the three known aggregate states; therefore it is usually called the fourth aggregate state.

One of the main features of ionized gas is its electroconductivity. As a matter of fact, due to ionization, and under the influence of the outer magnetic field, an electric stream appears in the gas. The electric stream causes the appearance of Lorentz's force and Joule's heat. Because of these two effects new members appear in the corresponding boundary layer equations, which is not the case in the equations of homogeneous gas which is not ionized. Therefore, in the case of ionized gas flow, equations of laminar, steady and plane boundary layer [1], [2], in the conditions of so-called equilibrium ionization, have the following form:

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$$
\begin{gathered}
\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0\,,\\
\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\sigma B_m^2 u\,,\\
\rho u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial y}=u\frac{dp}{dx}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\frac{\partial h}{\partial y}\right)+\sigma B_m^2 u^2;\\
u=v=0\quad h=h_w\qquad \text{for}\quad y=0\,,\\
u\to u_e(x)\,,\quad h\to h_e(x)\quad \text{for}\quad y\to\infty\,,\\
(u=u_0(y)\,,\quad h=h_0(y)\quad \text{for}\quad x=x_0)
\end{gathered}
\tag{1}
$$

In these equations as well as in the corresponding boundary conditions, the symbols common in the boundary layer theory are used for certain physical values. So, $\sigma B_m^2 u^2$ represents Joule's heat. It is considered, as usually [1], that the outer magnetic field is perpendicular to the contour of the body, and that because of a relatively small thickness of the boundary layer the equation of this field is $B_m = B_m(x)$.

In our earlier studies it was presumed that the electroconductivity of ionized gas could also be presented in a form of a function only of longitudinal coordinate x, i.e. $\sigma = \sigma(x)$. However, some of the characteristics and parametric solutions of the ionized gas boundary layer equations, obtained according to this form of electroconductivity law [3], show that, among other things, it is necessary to have a concrete form of this law. Therefore, in this paper and in this phase of our studies, it is considered that the law of electroconductivity change is determined by the expression [4, 6]:

$$
\sigma=\sigma_0\left(1-\frac{u}{u_e}\right),\qquad \sigma_0=const. \tag{2}
$$

According to the form of the law (2), it is concluded that electroconductivity disappears at the outer boundary of the boundary layer, i.e. $\sigma = \sigma_e = 0$ at this boundary.

If, by the known procedure, we exclude the pressure from the equation system (1) then the system can take the following form:

$$
\begin{gathered}
\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)=0\,,\\
\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=\rho_e u_e\frac{du_e}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\sigma B_m^2 u\,,\\
\rho u\frac{\partial h}{\partial x}+\rho v\frac{\partial h}{\partial y}=-u\,\rho_e u_e\frac{du_e}{dx}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\frac{\partial}{\partial y}\left(\frac{\mu}{\Pr}\frac{\partial h}{\partial y}\right)+\sigma B_m^2 u^2;
\end{gathered}
\tag{1'}
$$

with the unchanged boundary conditions. Here, the index $''e''$ presents physical values at the edge of the boundary layer.

2. Introduction of New Variables

Modern parametric methods of solution of boundary layer equations are based, as it is known, on the application of an impulse equation. In order for an impulse equation to have the simplest form we introduce, as with similar flow problems [2], new variables s, z and the stream function ψ in the form of:

$$s(x)=\frac{1}{\rho_0\,\mu_0}\int_0^x \rho_w\mu_w\;dx, \qquad z(x,y)=\frac{1}{\rho_0}\int_0^y \rho\;dy$$
$$u=\frac{\partial\psi}{\partial z}, \qquad \tilde{v}=\frac{\rho_0\,\mu_0}{\rho_w\mu_w}\left(u\frac{\partial z}{\partial x}+v\frac{\rho}{\rho_0}\right)=-\frac{\partial\psi}{\partial s} \tag{3}$$

where the index $''w''$ represents conditions at the wall of the body rounded with fluid, while ρ_0, $\mu_0=\rho_0\nu_0$ represent the known constant values of density and dynamic viscosity of ionized gas.

By means of newly introduced transformations (3) the starting equation system (1') comes down to:

$$\frac{\partial\psi}{\partial z}\frac{\partial^2\psi}{\partial s\partial z}-\frac{\partial\psi}{\partial s}\frac{\partial^2\psi}{\partial z^2}=\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}+\nu_0\frac{\partial}{\partial z}\left(Q\frac{\partial^2\psi}{\partial z^2}\right)-\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\frac{\sigma B_m^2}{\rho}\frac{\partial\psi}{\partial z},$$
$$\frac{\partial\psi}{\partial z}\frac{\partial h}{\partial s}-\frac{\partial\psi}{\partial s}\frac{\partial h}{\partial z}=-\frac{\rho_e}{\rho}u_e\frac{du_e}{ds}\frac{\partial\psi}{\partial z}+\nu_0\,Q\left(\frac{\partial^2\psi}{\partial z^2}\right)^2+\nu_0\frac{\partial}{\partial z}\left(\frac{Q}{\Pr}\frac{\partial h}{\partial z}\right)+\frac{\rho_0\,\mu_0}{\rho_w\,\mu_w}\frac{\sigma B_m^2}{\rho}\left(\frac{\partial\psi}{\partial z}\right)^2;$$
$$\psi=\frac{\partial\psi}{\partial z}=0, \quad h=h_w \quad for \quad z=0,$$
$$\frac{\partial\psi}{\partial z}\to u_e(s) \quad h\to h_e(s) \quad for \quad z\to\infty,$$
$$\left(\frac{\partial\psi}{\partial z}=u_0(z) \quad h=h_0(z) \quad for \quad s=s_0\right). \tag{4}$$

In the equation system (4) the non-dimensional function Q and Prandtl's number Pr are determined by the following expressions:

$$Q=\frac{\rho\,\mu}{\rho_w\mu_w}, \qquad \Pr=\frac{\mu\,c_p}{\lambda}. \tag{5}$$

By means of the variables (3) and by the known procedure it is relatively easy to obtain the impulse equation based on the two equations of the system (1'). This impulse equation can be written in its three form as:

$$\frac{dZ^{**}}{ds}=\frac{F_m}{u_e}, \qquad \frac{df}{ds}=\frac{u_e'}{u_e}F_m+\frac{u_e''}{u_e'}f, \qquad \frac{\Delta^{**'}}{\Delta^{**}}=\frac{u_e'}{u_e}\frac{F_m}{2f}; \tag{6}$$

where the apostrophe represents the derivative per the longitudinal variable s.

While obtaining the impulse equation we take into consideration the parameter f, the magnetic parameter g, the conditional thickness Δ_1^{**} as well as the other common values and characteristics of the boundary layer, which are:

$$
\begin{aligned}
&Z^{**}=\frac{\Delta^{**2}}{\nu_0}, \qquad f(s)=f_1=u_e' Z^{**}, \qquad g(s)=g_1=N_\sigma Z^{**},\\
&F_m=2[\zeta-(2+H)f]+2gH_1, \qquad \zeta=\left[\frac{\partial(u/u_e)}{\partial(z/\Delta^{**})}\right]_{z=0}, \qquad H=\frac{\Delta^*}{\Delta^{**}},\\
&N_\sigma(s)=\frac{\rho_0\mu_0}{\rho_w\mu_w}N, \quad N=\frac{\sigma_0 B_m^2}{\rho_e}; \quad \Delta^*(s)=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{u}{u_e}\right)dz, \quad H_1=\frac{\Delta_1^{**}}{\Delta^{**}},\\
&\Delta^{**}(s)=\int_0^\infty\frac{u}{u_e}\left(1-\frac{u}{u_e}\right)dz; \qquad \Delta_1^{**}(s)=\int_0^\infty\frac{u}{u_e}\left(1-\frac{u}{u_e}\right)\frac{\rho_e}{\rho}\,dz.
\end{aligned}
\tag{7}
$$

The tangential stress at the wall of the body rounded with fluid, in this case of flow, is determined by the non-dimensional function of friction ζ as:

$$
\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}=\frac{\rho_w\mu_w}{\rho_0}\,\frac{u_e}{\Delta^{**}}\,\zeta\,. \tag{7'}
$$

Using the ideas from [2] we apply another transformation of variables to the equation system (4):

$$
\begin{aligned}
&s=s, \qquad \eta(s,z)=\frac{B(s)}{\Delta^{**}(s)}\,z, \qquad \psi(s,z)=\frac{u_e(s)\,\Delta^{**}(s)}{B(s)}\,\Phi(s,\eta),\\
&h(s,z)=h_1\cdot\bar{h}(s,\eta); \qquad h_e+\frac{u_e^2}{2}=h_1=const.;
\end{aligned}
\tag{8}
$$

where $\Phi(s,z)$ and $\bar{h}(s,\eta)$ stand for the conditional stream function and the non-dimensional enthalpy.

The previously introduced relations and characteristics (7) can be, by means of the newly introduced transformations (8), written as:

$$
\begin{aligned}
&\zeta=B\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)_{\eta=0}; \qquad H=\frac{A}{B}, \qquad H_1=\frac{A_1}{B}, \qquad B=\int_0^\infty\frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)d\eta,\\
&A=\int_0^\infty\left(\frac{\rho_e}{\rho}-\frac{\partial\Phi}{\partial\eta}\right)d\eta, \qquad A_1=\int_0^\infty\frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\rho_e}{\rho}\,d\eta;
\end{aligned}
\tag{9}
$$

where it is presumed that the values A, A_1 and B are continuous functions of the coordinate s.

The equation system (4), after a complex derivation and by means of the transformations (8), comes down to:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]-\underline{\frac{g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}}=$$
$$=\frac{u_eZ^{**}}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial s\,\partial\eta}-\frac{\partial\Phi}{\partial s}\frac{\partial^2\Phi}{\partial\eta^2}\right),$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+\underline{\frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^2}=$$
$$=\frac{u_eZ^{**}}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial s}-\frac{\partial\Phi}{\partial s}\frac{\partial\bar{h}}{\partial\eta}\right); \tag{10}$$

$$\Phi=\frac{\partial\Phi}{\partial\eta}=0\,,\qquad \bar{h}=\bar{h}_w\qquad for\qquad \eta=0\,,$$
$$\frac{\partial\Phi}{\partial\eta}\to 1\,,\qquad \bar{h}=\bar{h}_e=1-\kappa\qquad for\qquad \eta\to\infty\,,$$
$$(\Phi=\Phi_0(\eta)\,,\qquad \bar{h}=\bar{h}_0(\eta)\qquad for\qquad s=s_0)$$

It is pointed out that the obtained equation system (10) is different from the corresponding equations [3], in the case when $\sigma=\sigma(x)$, only in sign and in the form of the last member on the left handside of the sign of equality of the both equations (underlined members).

Further on, the local parameter of compressibility $\kappa=f_0$ and constants a, b exist in the equation systems (10) and in the corresponding boundary conditions. The parameter of compressibility satisfies the simple differential equation:

$$\kappa=\frac{u_e^2}{2h_1}\,,\qquad u_eZ^{**}\frac{d\kappa}{ds}=2\kappa f\equiv\theta_0\,. \tag{11}$$

3. Generalization of Boundary Layer Equations of the Considered Problem and Their Solutions

It is also noticed that, besides the parameters and their distributions: κ, f, g, Pr, Q, ρ_e/ρ, the outer velocity $u_e(s)$ clearly exists in the obtained equation system (10). Therefore the solution of the system depends on each concrete form of distribution of this velocity. In order to avoid the noticed disadvantage instead of the transformations (8) we apply the similarity transformations in the following form:

$$\psi(s,z)=\frac{u_e\,\Delta^{**}}{B}\,\Phi(\eta,\kappa,f,g)\,;\qquad h=h_1\cdot\bar{h}\,(\eta,\kappa,f,g)\,. \tag{12}$$

With the new transformations (12), the function Φ and the non-dimensional enthalpy $\bar{h}$ are not directly dependent on the longitudinal variable s but indirectly by means of the parameters κ, f and g.

By means of the relations (12) the basic equation system (4) is transformed into this form:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]-\frac{g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}=$$

$$=\frac{u_eZ^{**}}{B^2}\left[\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial\kappa}-\frac{\partial\Phi}{\partial\kappa}\frac{\partial^2\Phi}{\partial\eta^2}\right)\kappa'+\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial^2\Phi}{\partial\eta^2}\right)f'+\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial g}-\frac{\partial\Phi}{\partial g}\frac{\partial^2\Phi}{\partial\eta^2}\right)g'\right], \quad (13)$$

$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+\frac{2\kappa g}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^2=$$

$$=\frac{u_eZ^{**}}{B^2}\left[\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial\kappa}-\frac{\partial\Phi}{\partial\kappa}\frac{\partial\bar{h}}{\partial\eta}\right)\kappa'+\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f}-\frac{\partial\Phi}{\partial f}\frac{\partial\bar{h}}{\partial\eta}\right)f'+\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial g}-\frac{\partial\Phi}{\partial g}\frac{\partial\bar{h}}{\partial\eta}\right)g'\right].$$

As it is seen not even these equations can satisfy the conditions of generalized similarity because the members $u_eZ^{**}f'$ and $u_eZ^{**}g'$ contain $u_e(s)$ on their right hand sides and cannot be expressed by means of κ, f, g and F_m. Therefore in further researches, new parameters are introduced. As a matter of fact, the parameters f and g, used so far, were considered to be the first in order $f=f_1$ and $g=g_1$, while the introduced parameters f_2 and g_2 are the second order and they contain higher derivatives of the outer velocity u_e and of the function N_σ. So, we have applied similarity transformations in this form:

$$\psi(s,z)=\frac{u_e\Delta^{**}}{B}\Phi(\eta,\ \kappa,\ f_1,\ f_2,\ g_1,\ g_2), \qquad h(s,z)=h_1\cdot\bar{h}(\eta,\ \kappa,\ f_1,\ f_2,\ g_1,\ g_2). \quad (12')$$

It has been shown that, in this case as well, the obtained equations contain new factors among which there are the outer velocity and its derivatives. Further detailed analysis has determined that also in this case of ionized gas flow it is necessary to introduce two sets of parameters in the following form:

$$f_k=u_e^{k-1}u_e^{(k)}Z^{**k}, \quad f_0=\kappa=\frac{u_e^2}{2h_1}, \quad g_k=u_e^{k-1}N_\sigma^{(k-1)}Z^{**k}, \quad (k=1,\ 2,\ 3,\ ...) \quad (14)$$

These parameters satisfy the next known recurring simple differential equations:

$$\begin{aligned} u_e Z^{**} f_k' &= [(k-1)f_1+kF_m]\,f_k+f_{k+1}\equiv\theta_k\,, \\ u_e Z^{**} g_k' &= [(k-1)f_1+kF_m]\,g_k+g_{k+1}\equiv G_k\,. \\ &(\ k=1,\ 2,\ 3,\ ...\ ) \end{aligned} \quad (15)$$

If at the very beginning of the similarity transformations (12) we introduce the two sets of parameters (f_k), (g_k); i.e. if we apply the transformations to the system (4):

$$\psi(s,z)=\frac{u_e\Delta^{**}}{B}\Phi[\eta,\ \kappa,\ (f_k),\ (g_k)], \qquad h=h_1\cdot\bar{h}\ [\eta,\ \kappa,\ (f_k),\ (g_k)] \quad (16)$$

then the system takes this form:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f_1}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]-\frac{g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}=$$
$$=\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)+\sum_{k=1}^{\infty}G_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial g_k}-\frac{\partial\Phi}{\partial g_k}\frac{\partial^2\Phi}{\partial\eta^2}\right)\right],$$
$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+\frac{2\kappa g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^2=$$
$$=\frac{1}{B^2}\left[\sum_{k=0}^{\infty}\theta_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f_k}-\frac{\partial\Phi}{\partial f_k}\frac{\partial\bar{h}}{\partial\eta}\right)+\sum_{k=1}^{\infty}G_k\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial g_k}-\frac{\partial\Phi}{\partial g_k}\frac{\partial\bar{h}}{\partial\eta}\right)\right];\tag{17}$$
$$\Phi=\frac{\partial\Phi}{\partial\eta}=0,\quad \bar{h}=\bar{h}_w=const.\quad for\quad \eta=0,$$
$$\frac{\partial\Phi}{\partial\eta}\to 1,\quad \bar{h}=\bar{h}_e=1-\kappa\quad for\quad \eta\to\infty,$$
$$(\Phi=\Phi_0(\eta),\quad \bar{h}=\bar{h}_0(\eta)\quad for\quad \kappa=const.,\ \ f_i=g_i=0).$$

The characteristic function F_m in this case is:

$$F_m=aB^2-bf_1+\frac{2}{B}\left(\sum_{k=0}^{\infty}\theta_k\frac{\partial B}{\partial f_k}+\sum_{k=1}^{\infty}G_k\frac{\partial B}{\partial g_k}\right).\tag{18}$$

Since the distribution of outer velocity $u_e(s)$ exists neither in the equation system (17) nor in the corresponding boundary conditions, the system (17) is in that sense universal, i.e. generalized.

The solution of this equation system is practically possible only when there is relatively small number of parameters. Therefore, as with similar flow problems [2], the solution is obtained by so called n - parametric approximation. If it is presumed that all the parameters equal zero, starting from the second one, and if the derivatives per the compressibility parameter and per magnetic parameter are neglected, which means that:

$$\kappa\neq 0,\quad f_1\neq 0,\quad g_1\neq 0,\quad f_2=f_3=\ ...\ =0,\quad g_2=g_3=\ ...\ =0,$$
$$\partial/\partial\kappa=0,\quad \partial/\partial g_1=0\tag{19}$$

the system (17) becomes much simpler. In the so-called three-parametric twice localized approximation the equation system (17) comes down to:

$$\frac{\partial}{\partial\eta}\left(Q\frac{\partial^2\Phi}{\partial\eta^2}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{f_1}{B^2}\left[\frac{\rho_e}{\rho}-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]-\frac{g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\frac{\partial\Phi}{\partial\eta}=$$
$$=\frac{F_m f_1}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial^2\Phi}{\partial\eta\partial f_1}-\frac{\partial\Phi}{\partial f_1}\frac{\partial^2\Phi}{\partial\eta^2}\right),\tag{20}$$
$$\frac{\partial}{\partial\eta}\left(\frac{Q}{\Pr}\frac{\partial\bar{h}}{\partial\eta}\right)+\frac{aB^2+(2-b)f_1}{2B^2}\Phi\frac{\partial\bar{h}}{\partial\eta}-\frac{2\kappa f_1}{B^2}\frac{\rho_e}{\rho}\frac{\partial\Phi}{\partial\eta}+2\kappa Q\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2+$$
$$+\frac{2\kappa g_1}{B^2}\frac{\rho_e}{\rho}\left(1-\frac{\partial\Phi}{\partial\eta}\right)\left(\frac{\partial\Phi}{\partial\eta}\right)^2=\frac{F_m f_1}{B^2}\left(\frac{\partial\Phi}{\partial\eta}\frac{\partial\bar{h}}{\partial f_1}-\frac{\partial\Phi}{\partial f_1}\frac{\partial\bar{h}}{\partial\eta}\right);$$

$$\Phi = \frac{\partial \Phi}{\partial \eta} = 0, \qquad \bar{h} = \bar{h}_w \qquad \mathit{for} \quad \eta = 0,$$
$$\frac{\partial \Phi}{\partial \eta} \to 1, \qquad \bar{h} = \bar{h}_e = 1 - \kappa \qquad \mathit{for} \quad \eta \to \infty, \tag{cont. 20}$$
$$(\Phi = \Phi_0(\eta), \qquad \bar{h} = \bar{h}_0(\eta) \qquad \mathit{for} \quad \kappa = const.; \quad f_1 = g_1 = 0).$$

Under the conditions (19) the expression for the characteristic function F_m is much simplified. In three-parametric unlocalized and twice localized approximation this function is determined by the expressions:

$$F_m \approx \frac{aB^2 - bf_1 + \dfrac{4\kappa f_1}{B}\dfrac{\partial B}{\partial \kappa}}{1 - \dfrac{2}{B} f_1 \dfrac{\partial B}{\partial f_1} - \dfrac{2}{B} g_1 \dfrac{\partial B}{\partial g_1}}, \qquad F_m \approx \frac{aB^2 - bf_1}{1 - \dfrac{2}{B} f_1 \dfrac{\partial B}{\partial f_1}}; \tag{21}$$

where the correct expression for the function F_m is determined by the relation (18), that is (7).

Numerical solution of the equation system (20) is performed by the method of finite differences, by the ″progonka″ procedure, where the order of the equation system was first reduced by the usual shift $u/u_e = \partial\Phi/\partial\eta = \varphi$. For the correct solution of the system, a program has been written in the Fortran program. A similar program developed and applied in [5] was used while writing it.

While solving the obtained equation system (20), for the non-dimensional function Q and for the relation of the densities ρ/ρ_e, the following approximate formulas have been used:

$$Q = Q(\bar{h}) \approx \left(\frac{\bar{h}_w}{\bar{h}}\right)^{1/3}, \quad \frac{\rho_e}{\rho} \approx \frac{\bar{h}}{1-\kappa}, \tag{22}$$

where the value of Prandtl's number, Pr = 0,712.

These formulas represent relatively rough approximation in the case of ionized gas flow. For the constants a, b, the usual values in the boundary layer theory have been accepted (a = 0,4408 , b = 5,7140).

Out of many obtained numerical results only some of them are shown in this paper, in a form of the corresponding diagrams. Diagrams of non-dimensional velocity (Fig. 1, Fig. 2, Fig. 3), non-dimensional enthalpy (Fig. 4, Fig. 5) and of some characteristics of boundary layer (Fig. 6, Fig. 7, Fig. 8) are given here.

First of all, it should be pointed out that, as with other problems of ionized (or dissociated) gas flow, the obtained solutions are expected, logical and acceptable.

As it is seen in the presented diagrams (Fig. 1, Fig. 2, Fig. 3), non-dimensional velocity, for different values of compressibility parameter, convergates fastly towards zero. The graphics non-dimensional enthalpy (Fig. 4, Fig. 5) are logical, but it is also pointed out that the compressibility parameter has a considerable influence on the distribution of enthalpy in the boundary conditions thus defining its value at the outer boundary.

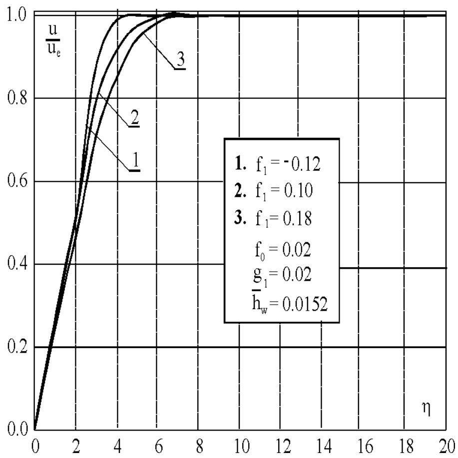

Fig. 1. Graphic of the non-dimensional velocity

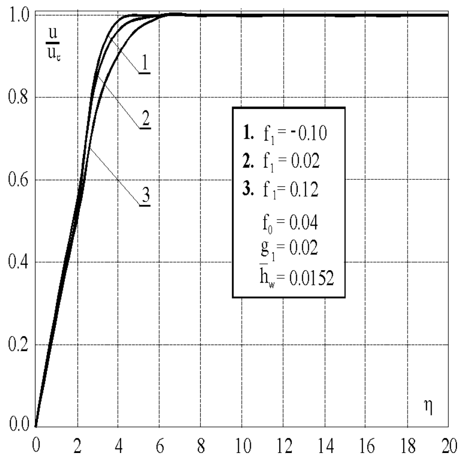

Fig. 2. Graphic of the non-dimensional velocity

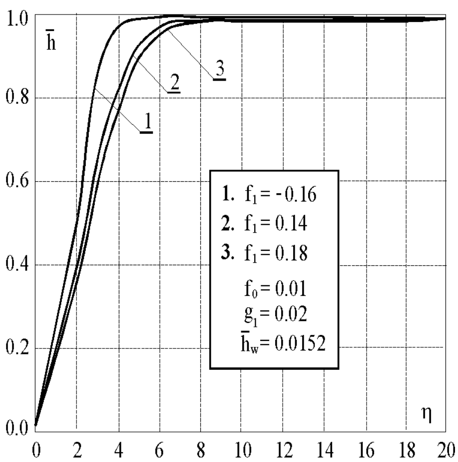

Fig. 3. Graphic of the non-dimensional velocity for different values of the parameter f_0

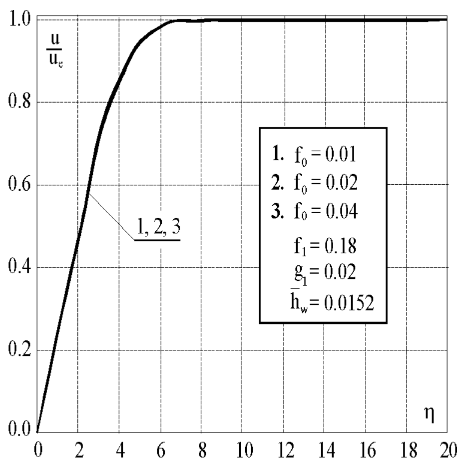

Fig. 4. Distribution of the non-dimensional enthalpy

The behaviour of the boundary layer characteristics (Fig. 6, Fig. 7, Fig. 8) is also expected. However, for some values of the input parameter, an unexpected behaviour of the characteristic F_m (7, 21) in the boundary layer has been noticed even besides making a concrete form of the law of electroconductivity (2).

Further studies should, among other things, give a precise answer to this behaviour of the function mentioned. That's why while solving the obtained equation system (20), as with [5], the correct expression (7) has been used for the characteristic function of the boundary layer F_m.

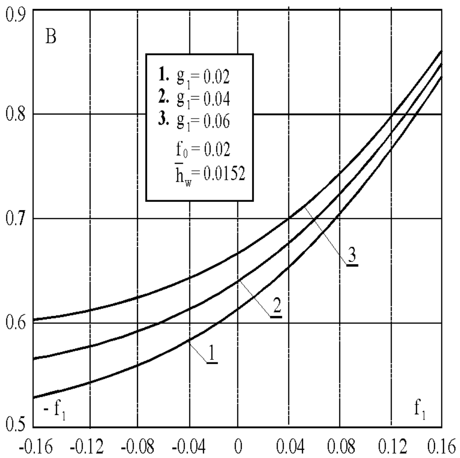

Fig. 5. Distribution of the non-dimensional enthalpy for different values of the parameter f_0

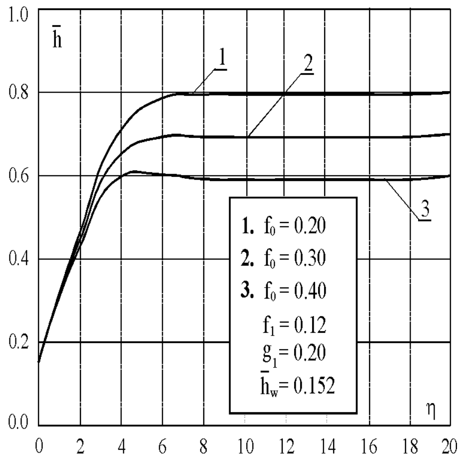

Fig. 6. The characteristic of the boundary layer B

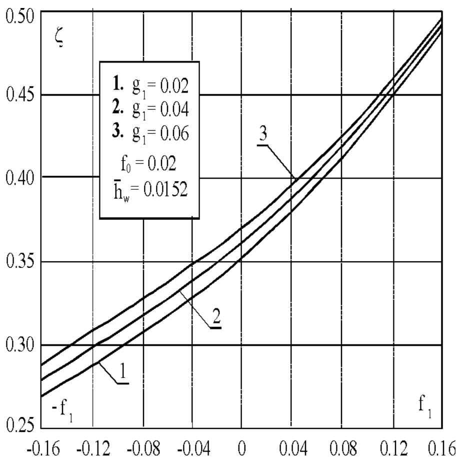

Fig. 7. The characteristic of the boundary layer F_m

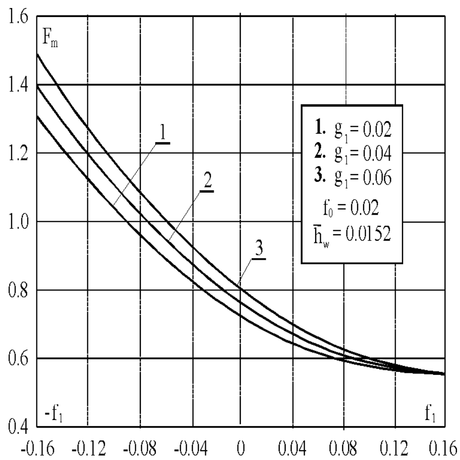

Fig. 8. The non-dimensional function of the friction ζ

REFERENCES

1. Loyc®nskiy, L.G., *Mehanika židkosti i gaza*, ″Nauka″, Moskva, 1973.
2. Obrović, B., *Boundary layer of disassociated gas*, Monograph, Faculty of Mechanical Engineering, Kragujevac, 1994.
3. Obrović, B., *Equations of ionized gas boundary layer and their parametric solutions*, Facta Universitatis, Vol. 1, No 5, Niš, 1995.
4. Boričić, Z., Nikodijević, D., Milenković, D., *Unsteady MHD boundary layer on a porous surface*, Facta Universitatis, Vol. 1, No 5, Niš, 1995.
5. Saljnikov, V., Dallmann, U., *Verallgemeinerte Ähnlichkeitslösungen für dreidimensionale, laminare, stationäre, kompressible Grenzschichtströmungen an schiebenden profilierten Zylindern*, Institut für

Theoretische Strömungsmechanik, DLR-FB89-34, Göttingen, 1989.
6. Saljnikov, V., Boričić, Z., Nikodijević, D., *Parametric method in unsteady mhd boundary layer theory of fluid with variable electroconductivity*, Facta Universitatis, Vol. 2, No 7/ 2, Niš, 1997.

GRANIČNI SLOJ JONIZOVANOG GASA ZA SLUČAJ PROMENLJIVE ELEKTROPROVODNOSTI

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U radu se istražuje strujanje jonizovanog gasa za slučaj konkretnog oblika zakona promene njegove elektroprovodnosti. Odgovarajuće jednačine graničnog sloja su prvo pogodnim transformacijama dovedene na univerzalni oblik. Zatim su jednačine numerički rešene u troparametarskom približenju. Grafički je prikazano ponašanje fizičkih veličina i nekih karakteristika graničnog sloja.