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APPLICATION ON A MORE ACCURATE BENDING THEORY OF A SANDWICH PLATE WITH A LIGHT CORE

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Abstract. *The application of the more accurate theory of bending thin homogenous plates in calculation of the sandwich nonhomogenous constructions is presented in the work. There were derived the basic equations for the sandwich plate of symmetrical structure with thin outer layers, that bend according to REISSNER-MINDLIN 's theory, and middle layer exposed only to transversal shear. The bending variational equation was derived by varying the possible displacements. In that the appropriate static boundary conditions were determined on each end of the plate and the basic equations of bending for the sandwich plate were confirmed.*

1. INTRODUCTION

There are two approaches for the calculation of multilayer constructions. In the first, the basic equations are obtained on the basis of kinematic conditions for each layer separately [4]. With this approach it is possible to describe, with high accuracy, the stress-strain state as well as the local influences in each layer of the construction. The number and order of the equations depend on the number of the construction layers. In the second approach, the calculation is performed on the basis of hypothesis about the straight line that is unique for all the package layers [4], [2]. The number and order of equations do not depend on the number of package layers.

The theory of sandwich constructions is based on different assumptions depending on stiffness and treatment of the middle layer (core). In constructions with the stiff core the calculation does not differ from the calculation of multilayer constructions. In constructions with the light core, the hypothesis of piecewise line is used. For the outer layers, the Kirchhoff's hypothesis of straight vertical line is used, and for the middle layer the hypothesis of straight line for two-dimensional body is used, considering only the transversal shear [7], [1], [5].

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The basic equations of bending for sandwich plates, on the basis of the more accurate theory for bending the outer layers, were derived in the work. These plate layers bend according to Reissner-Mindlin's theory, while the middle layer is exposed only to transversal shear. The bending of the plate is reduced on solving the system of seven partial differential equations for the displacement components of the outer layers middle plane and unknown functions $\theta_x(x,y)$ and $\theta_y(x,y)$ that determines the rotation of the vertical line during deformation. The number of unknown values, as well as the number of differential equations is for two higher than the classical theory.

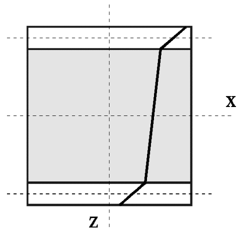

Fig. 1.

In the work it is also derived the variational equation, where, once again, the differential equations of bending and corresponding static boundary conditions on each plate end were presented. The number of the boundary conditions is for one higher in relation to the classical theory, it enables for all kinematic conditions on the plate contour to be satisfied, that presents the advantage of the more accurate theory comparing with the classical one.

2. COMPONENT DISPLACEMENTS

Applying the Reissner-Mindlin's theory [4], [3] on bending the outer layers, line perpendicular on middle plane of the plate changes into the piecewise line (Fig. 1). Different from the classical theory, the rotation of perpendicular line during the deformation will be determined using the unknown functions $\theta_x(x,y)$ and $\theta_y(x,y)$. Component displacements of the random point of the outer layer were determined with relations (for $-h - t \le z \le -h$):

$$u_g = u_1 - \left(z + h + \frac{t}{2}\right)\theta_x\,,\quad v_g = v_1 - \left(z + h + \frac{t}{2}\right)\theta_y\,,\quad w_g = w\,, \tag{1}$$

where u_1, v_1, w are component displacements of the middle plane point of the plate upper layer.

For the bottom layer (for $h \le z \le h + t$) it is:

$$u_d = u_2 - \left(z - h - \frac{t}{2}\right)\theta_x\,,\quad v_d = v_2 - \left(z - h - \frac{t}{2}\right)\theta_y\,,\quad w_d = w\,, \tag{2}$$

where u_2, v_2, w are component displacements of the middle plane point of the plate bottom layer.

The displacements of the middle layer (for $-h \le z \le h$) are:

$$u_s = \frac{1}{2}(u_1 + u_2) - \frac{z}{2h}(u_1 - u_2 - t\theta_x)\,, v_s = \frac{1}{2}(v_1 + v_2) - \frac{z}{2h}(v_1 - v_2 - t\theta_y)\,,\ w_s = w\,. \tag{3}$$

3. FORCES AND MOMENTS

The component deformations and stresses in the outer layers can be calculated by the known formulas of elastic theory [6]:

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \; \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \; \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, \; \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\sigma_x = \frac{E}{1-\mu^2}(\varepsilon_x + \mu\varepsilon_y), \; \sigma_y = \frac{E}{1-\mu^2}(\varepsilon_y + \mu\varepsilon_x), \tag{4}$$

$$\tau_{xy} = \frac{E}{2(1+\mu)}\gamma_{xy}, \quad \tau_{xz} = \frac{E}{2(1+\mu)}\gamma_{xz}, \quad \tau_{yz} = \frac{E}{2(1+\mu)}\gamma_{yz},$$

where E is the elasticity modulus and μ is Poisson's coefficient for the outer layers material.

In the plate middle layer only the shear stresses will act:

$$\tau_{xz} = G_3\gamma_{xz}, \quad \tau_{yz} = G_3\gamma_{yz}, \tag{5}$$

where G_3 is the shearing modulus of elasticity of the middle layer material.

Forces and moments in the plate layers cross-sections are calculated integrating the stresses in relation to thickness of the corresponding layer, i.e. by integrals of the following form:

$$N_x, N_y, \; T_{xy} = \int(\sigma_x, \sigma_y, \tau_{xy})dz,$$

$$Q_x, \; Q_y = \int(\tau_{xz}, \tau_{yz})dz, \tag{6}$$

$$M_x, \; M_y, \; H = \int(\sigma_x, \sigma_y, \tau_{xy})zdz,$$

By using the formulas (1) to (6) we obtain the moments on the unit length of outer layers in relation to their middle plane

$$\begin{gathered} M_{x1} = M_{x2} = -D\left(\frac{\partial\theta_x}{\partial x} + \mu\frac{\partial\theta_y}{\partial y}\right), \quad M_{y1} = M_{y2} = -D\left(\frac{\partial\theta_y}{\partial y} + \mu\frac{\partial\theta_x}{\partial x}\right), \\ H_1 = H_2 = -D\frac{1-\mu}{2}\left(\frac{\partial\theta_x}{\partial y} + \frac{\partial\theta_y}{\partial x}\right), \end{gathered} \tag{7}$$

normal and shear forces

$$\begin{gathered} N_{x1} = B\left(\frac{\partial u_1}{\partial x} + \mu\frac{\partial v_1}{\partial y}\right), \quad N_{y1} = B\left(\frac{\partial v_1}{\partial y} + \mu\frac{\partial u_1}{\partial x}\right), \\ T_{xy1} = B\frac{1-\mu}{2}\left(\frac{\partial u_1}{\partial y} + \frac{\partial v_1}{\partial x}\right), \\ Q_{x1} = Q_{x2} = B\frac{1-\mu}{2}\left(-\theta_x + \frac{\partial w}{\partial x}\right), \quad Q_{y1} = Q_{y2} = B\frac{1-\mu}{2}\left(-\theta_y + \frac{\partial w}{\partial y}\right), \end{gathered} \tag{8}$$

where $D = \frac{Et^3}{12(1-\mu^2)}$ is flexural and $B = \frac{Et}{(1-\mu^2)}$ is axial rigidity of this layers, and shear forces of middle layer

$$Q_{x3} = -G_3\left(u_1 - u_2 - t\theta_x - 2h\frac{\partial w}{\partial x}\right), Q_{y3} = -G_3\left(v_1 - v_2 - t\theta_y - 2h\frac{\partial w}{\partial y}\right). \tag{9}$$

Positive moments and forces that act on the element of plate are shown in Fig. 2 and 3. In all formulas values with index "1" correspond to upper, with index "2" to bottom, and with index "3" to the middle plate layers.

4. EQUILIBRIUM EQUATIONS

The basic equations of bending are obtained from the equilibrium equations of plate layers' elements. Equations of forces equilibrium for the plate upper layer (Fig. 2) are reduced to the following form:

$$\begin{gathered}\frac{\partial N_{x1}}{\partial x} + \frac{\partial T_1}{\partial y} + \tau_{zx1} = 0\,, \quad \frac{\partial T_1}{\partial x} + \frac{\partial N_{y1}}{\partial y} + \tau_{zy1} = 0\,, \quad \frac{\partial Q_{x1}}{\partial x} + \frac{\partial Q_{y1}}{\partial y} + q = 0\,, \\ \frac{\partial M_{x1}}{\partial x} + \frac{\partial H_1}{\partial y} + \frac{t}{2}\tau_{zx1} - Q_{x1} = 0\,, \quad \frac{\partial H_1}{\partial x} + \frac{\partial M_{y1}}{\partial y} + \frac{t}{2}\tau_{zy1} - Q_{y1} = 0\,.\end{gathered} \tag{10a}$$

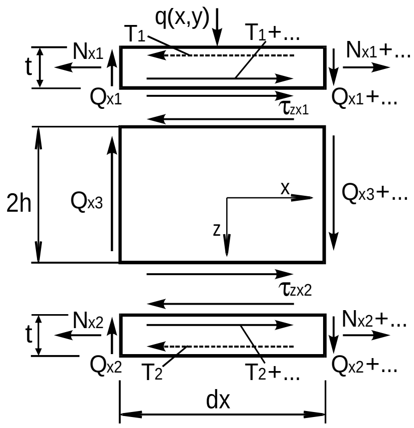

Fig. 2.

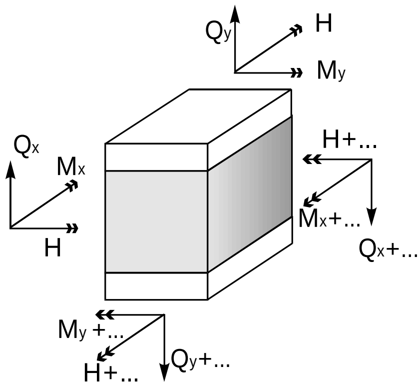

Fig. 3.

For the plate bottom layer these equations are:

$$\begin{gathered}\frac{\partial N_{x2}}{\partial x} + \frac{\partial T_2}{\partial y} - \tau_{zx2} = 0\,, \quad \frac{\partial T_2}{\partial x} + \frac{\partial N_{y2}}{\partial y} - \tau_{zy2} = 0\,, \quad \frac{\partial Q_{x2}}{\partial x} + \frac{\partial Q_{y2}}{\partial y} = 0\,, \\ \frac{\partial M_{x2}}{\partial x} + \frac{\partial H_2}{\partial y} + \frac{t}{2}\tau_{zx2} - Q_{x2} = 0\,, \quad \frac{\partial H_2}{\partial x} + \frac{\partial M_{y2}}{\partial y} + \frac{t}{2}\tau_{zy2} - Q_{y2} = 0\,.\end{gathered} \tag{10b}$$

The same equations for the plate middle layer will be:

$$\tau_{zx2}-\tau_{zx1}=0, \tau_{zy2}-\tau_{zy1}=0\,,\quad \frac{\partial Q_{x3}}{\partial x}+\frac{\partial Q_{y3}}{\partial y}=0\,,$$
$$Q_{x3}-h(\tau_{zx1}+\tau_{zx2})=0\,,\quad Q_{y3}-h(\tau_{zy1}+\tau_{zy2})\,. \tag{10c}$$

In the equilibrium equations (10) τ_{zx1}, τ_{zx2}, τ_{zy1}, τ_{zy2} are shear stresses that act in the planes of coupling the outer layers with the plate middle layer.

5. BASIC EQUATIONS

The equilibrium equations (10) can be reduced on the system of seven partial differential equations in relation to unknown functions $u_\alpha(x,y)$, $v_\alpha(x,y)$, $u_\beta(x,y)$, $v_\beta(x,y)$, plate flexure $w(x,y)$ and angles of outer layers perpendicular line rotation $\theta_x(x,y)$ and $\theta_y(x,y)$, where

$$u_\alpha=\frac{1}{2}(u_1+u_2)\,,\quad v_\alpha=\frac{1}{2}(v_1+v_2)\,,\quad u_\beta=\frac{1}{2}(u_1+u_2)\,,\quad v_\beta=\frac{1}{2}(v_1+v_2)\,. \tag{11}$$

Using the formulas (7), (8), (9) and (11), we reduce the system of equilibrium equations (10) to the following form:

$$\frac{\partial^2 u_\alpha}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2 u_\alpha}{\partial y^2}+\frac{1+\mu}{2}\frac{\partial^2 v_\alpha}{\partial x\partial y}=0\,,$$
$$\frac{\partial^2 v_\alpha}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2 v_\alpha}{\partial x^2}+\frac{1+\mu}{2}\frac{\partial^2 u_\alpha}{\partial x\partial y}=0\,. \tag{12}$$

$$\frac{Bh}{G_3}\left(\frac{\partial^2 u_\beta}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2 u_\beta}{\partial y^2}+\frac{1+\mu}{2}\frac{\partial^2 v_\beta}{\partial x\partial y}\right)-u_\beta+\frac{t}{2}\theta_x+h\frac{\partial w}{\partial x}=0\,,$$
$$\frac{Bh}{G_3}\left(\frac{\partial^2 v_\beta}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2 v_\beta}{\partial x^2}+\frac{1+\mu}{2}\frac{\partial^2 u_\beta}{\partial x\partial y}\right)-v_\beta+\frac{t}{2}\theta_y+h\frac{\partial w}{\partial y}=0\,, \tag{13}$$
$$2D\nabla^2\left(\frac{\partial\theta_x}{\partial x}+\frac{\partial\theta_y}{\partial y}\right)+2B\left(h+\frac{t}{2}\right)\nabla^2\left(\frac{\partial u_\beta}{\partial x}+\frac{\partial v_\beta}{\partial y}\right)=q(x,y)\,.$$

$$\frac{2Dh}{G_3 t}\left(\frac{\partial^2\theta_x}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2\theta_x}{\partial y^2}+\frac{1+\mu}{2}\frac{\partial^2\theta_y}{\partial x\partial y}\right)+u_\beta-\frac{t}{2}\theta_x-h\frac{\partial w}{\partial x}+\frac{Bh(1-\mu)}{G_3 t}\left(-\theta_x+\frac{\partial w}{\partial x}\right)=0\,,$$
$$\frac{2Dh}{G_3 t}\left(\frac{\partial^2\theta_y}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2\theta_y}{\partial x^2}+\frac{1+\mu}{2}\frac{\partial^2\theta_x}{\partial x\partial y}\right)+v_\beta-\frac{t}{2}\theta_y-h\frac{\partial w}{\partial y}+\frac{Bh(1-\mu)}{G_3 t}\left(-\theta_y+\frac{\partial w}{\partial y}\right)=0\,. \tag{14}$$

Equations (12), (13) and (14) are separated on two independent systems, system (12) that has trivial solutions on u_α, v_α and equations (13) and (14) coupled on the basis of the unknown values u_β , v_β , w, θ_x , θ_y. The system of partial differential equations (13) and (14) represents the basic system of equations for bending of sandwich plate with light core of symmetrical structure. This system of equations differs from the same system of the classical theory by the equations (14) and members that are defined by the functions $\theta_x(x,y)$, $\theta_y(x,y)$. The basic system of equations, as well as all the expressions, may be reduced on corresponding equations of the classical theory if we introduce the replacements $\theta_x = \frac{\partial w}{\partial x}, \theta_y = \frac{\partial w}{\partial y}$, see [1].

6. VARIATIONAL EQUATION OF BENDING. BOUNDARY CONDITIONS

Varying the possible displacements, u_α, v_α, u_β, v_β, w, θ_x, θ_y , by the energy method, as it is well known, we may obtain the basic equations (12), (13) and (14), and necessary boundary conditions. Energy bending equation of the sandwich plate is:

$$A_d + A = 0 \,, \tag{15}$$

where A_d is strain energy, and A is work of external forces. Strain energy of outer layers can be calculated by formula [6]

$$A_{d1,2} = \frac{1}{2}\int_V (\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz})dV \,, \tag{16}$$

and strain energy of the middle layer is

$$A_{d3} = \frac{1}{2}\int_V (\tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz})dV \tag{17}$$

Integrating the expressions (16) and (17) on the thickness of corresponding layer, taking care of expressions (1) to (5) and (11), we obtain:

$$\begin{aligned} A_d = \frac{1}{2}\iint \Bigg\{ & 2B\Bigg[\left(\frac{\partial u_\alpha}{\partial x}\right)^2 + \left(\frac{\partial v_\alpha}{\partial y}\right)^2 + 2\mu\frac{\partial u_\alpha}{\partial x}\frac{\partial v_\alpha}{\partial y} + \frac{1-\mu}{2}\left(\frac{\partial u_\alpha}{\partial y} + \frac{\partial v_\alpha}{\partial x}\right)^2 + \\ & + \left(\frac{\partial u_\beta}{\partial x}\right)^2 + \left(\frac{\partial v_\beta}{\partial y}\right)^2 + 2\mu\frac{\partial u_\beta}{\partial x}\frac{\partial v_\beta}{\partial y} + \frac{1-\mu}{2}\left(\frac{\partial u_\beta}{\partial y} + \frac{\partial v_\beta}{\partial x}\right)^2\Bigg] + \\ & + 2D\left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2 + 2\mu\frac{\partial \theta_x}{\partial x}\frac{\partial \theta_y}{\partial y} + \frac{1-\mu}{2}\left(\frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}\right)^2\right] + \\ & + \frac{2Q_{x3}}{h}\left(-u_\beta + \frac{t}{2}\theta_x + h\frac{\partial w}{\partial x}\right) + \frac{2Q_{y3}}{h}\left(-v_\beta + \frac{t}{2}\theta_y + h\frac{\partial w}{\partial y}\right)\Bigg\} dxdy \,. \end{aligned} \tag{18}$$

The work of uniformly distributed load is determined by the expression:

$$A=\frac{1}{2}q\iint wdxdy\,. \tag{19}$$

Varying the possible displacements in the energy equation (15), we obtain:

$$\delta A_d+\delta A=0\,. \tag{20}$$

Hence, if the plate is bounded by edges x = 0, x = a and y = 0, y = b, varying the possible displacements u_α, v_α, u_β, v_β, w, θ_x, θ_y, variational equation (20) is:

$$\begin{aligned}
&-\int_0^a\int_0^b\left[2B\left(\frac{\partial^2 u_\alpha}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2 u_\alpha}{\partial y^2}+\frac{1+\mu}{2}\frac{\partial^2 v_\alpha}{\partial x\partial y}\right)\right]\delta u_\alpha dxdy-\\
&-\int_0^a\int_0^b\left[2B\left(\frac{\partial^2 v_\alpha}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2 v_\alpha}{\partial x^2}+\frac{1+\mu}{2}\frac{\partial^2 u_\alpha}{\partial x\partial y}\right)\right]\delta v_\alpha dxdy-\\
&-\int_0^a\int_0^b\left[2B\left(\frac{\partial^2 u_\beta}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2 u_\beta}{\partial y^2}+\frac{1+\mu}{2}\frac{\partial^2 v_\beta}{\partial x\partial y}\right)+\frac{2G_3}{h}\left(-u_\beta+\frac{t}{2}\theta_x+h\frac{\partial w}{\partial x}\right)\right]\delta u_\beta dxdy\\
&-\int_0^a\int_0^b\left[2B\left(\frac{\partial^2 v_\beta}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2 v_\beta}{\partial x^2}+\frac{1+\mu}{2}\frac{\partial^2 u_\beta}{\partial x\partial y}\right)+\frac{2G_3}{h}\left(-v_\beta+\frac{t}{2}\theta_y+h\frac{\partial w}{\partial y}\right)\right]\delta v_\beta dxdy-\\
&-\int_0^a\int_0^b\left[2D\left(\frac{\partial^2 \theta_x}{\partial x^2}+\frac{1-\mu}{2}\frac{\partial^2 \theta_x}{\partial y^2}+\frac{1+\mu}{2}\frac{\partial^2 \theta_y}{\partial x\partial y}\right)-\frac{G_3 t}{h}\left(-u_\beta+\frac{t}{2}\theta_x+h\frac{\partial w}{\partial x}\right)+\right.\\
&\left.+B(1-\mu)\left(-\theta_x+\frac{\partial w}{\partial x}\right)\right]\delta\theta_x dxdy-\\
&-\int_0^a\int_0^b\left[2D\left(\frac{\partial^2 \theta_y}{\partial y^2}+\frac{1-\mu}{2}\frac{\partial^2 \theta_y}{\partial x^2}+\frac{1+\mu}{2}\frac{\partial^2 \theta_x}{\partial x\partial y}\right)-\frac{G_3 t}{h}\left(-v_\beta+\frac{t}{2}\theta_y+h\frac{\partial w}{\partial y}\right)+\right.\\
&\left.+B(1-\mu)\left(-\theta_y+\frac{\partial w}{\partial y}\right)\right]\delta\theta_y dxdy-\\
&-\int_0^a\int_0^b\left\{2G_3\left[-\left(\frac{\partial u_\beta}{\partial x}+\frac{\partial v_\beta}{\partial y}\right)+\frac{t}{2}\left(\frac{\partial \theta_x}{\partial x}+\frac{\partial \theta_y}{\partial y}\right)+h\nabla^2 w\right]+\right.\\
&\left.+B(1-\mu)\left[-\left(\frac{\partial \theta_x}{\partial x}+\frac{\partial \theta_y}{\partial y}\right)+\nabla^2 w\right]+q\right\}\delta w dxdy+\\
&+2\int_0^b B\left[\left(\frac{\partial u_\alpha}{\partial x}+\mu\frac{\partial v_\alpha}{\partial y}\right)\delta u_\alpha+\frac{1-\mu}{2}\left(\frac{\partial u_\alpha}{\partial y}+\frac{\partial v_\alpha}{\partial x}\right)\delta v_\alpha\right]_0^a dy+\\
&+2\int_0^b B\left[\left(\frac{\partial u_\beta}{\partial x}+\mu\frac{\partial v_\beta}{\partial y}\right)\delta u_\beta+\frac{1-\mu}{2}\left(\frac{\partial u_\beta}{\partial y}+\frac{\partial v_\beta}{\partial x}\right)\delta v_\beta\right]_0^a dy+
\end{aligned} \tag{21}$$

$$+2\int_0^b D\left[\left(\frac{\partial\theta_x}{\partial x}+\mu\frac{\partial\theta_y}{\partial y}\right)\delta\theta_x+\frac{1-\mu}{2}\left(\frac{\partial\theta_x}{\partial y}+\frac{\partial\theta_y}{\partial x}\right)\delta\theta_y\right]_0^a dy+$$

$$+2\int_0^b\left\{\left[G_3\left(-u_\beta+\frac{t}{2}\theta_x+h\frac{\partial w}{\partial x}\right)+B\frac{1-\mu}{2}\left(-\theta_x+\frac{\partial w}{\partial x}\right)\right]\delta w\right\}_0^a dy+$$

$$+2\int_0^a B\left[\left(\frac{\partial v_\alpha}{\partial y}+\mu\frac{\partial u_\alpha}{\partial x}\right)\delta v_\alpha+\frac{1-\mu}{2}\left(\frac{\partial u_\alpha}{\partial y}+\frac{\partial v_\alpha}{\partial x}\right)\delta u_\alpha\right]_0^b dx+ \qquad \text{(21 cont.)}$$

$$+2\int_0^a B\left[\left(\frac{\partial v_\beta}{\partial y}+\mu\frac{\partial u_\beta}{\partial x}\right)\delta v_\beta+\frac{1-\mu}{2}\left(\frac{\partial u_\beta}{\partial y}+\frac{\partial v_\beta}{\partial x}\right)\delta u_\beta\right]_0^b dx+$$

$$+2\int_0^a D\left[\left(\frac{\partial\theta_y}{\partial y}+\mu\frac{\partial\theta_x}{\partial x}\right)\delta\theta_y+\frac{1-\mu}{2}\left(\frac{\partial\theta_x}{\partial y}+\frac{\partial\theta_y}{\partial x}\right)\delta\theta_x\right]_0^b dx+$$

$$+2\int_0^a\left\{\left[G_3\left(-v_\beta+\frac{t}{2}\theta_y+h\frac{\partial w}{\partial y}\right)+B\frac{1-\mu}{2}\left(-\theta_y+\frac{\partial w}{\partial y}\right)\right]\delta w\right\}_0^b dx=0\,.$$

All double integrals in equation (21) are equal to zero, the terms by the variation of possible displacements δu_α, ..., $\delta\theta_y$ obviously correspond to the equilibrium equations (12), (13) and (14), while definite integrals define the static boundary conditions on each side of the plate. Subintegral function of the last double integral can be reduced to the form of the third equation of the system (13) using the first two equations of this system and equation (14).

7. EXAMPLE

Rectangular sandwich plate is subjected to uniformly continuos load on the whole surface. The plate is fixed by joints and reinforced by diaphragm of infinite rigidity in the support plane (Fig. 4). The boundary conditions of such supported plate, according to (21), are:

$$\frac{\partial u_\beta}{\partial x}=\frac{\partial\theta_x}{\partial x}=w=v_\beta=\theta_y=0 \qquad (22)$$

for $x = 0$ and $x = a$, and

$$\frac{\partial v_\beta}{\partial y}=\frac{\partial\theta_y}{\partial y}=w=u_\beta=\theta_x=0 \qquad (23)$$

for $y = 0$ and $y = b$.

If the solution of the basic system is searched in the form:

$$w(x,y)=C_1\sin(\alpha x)\sin(\beta y)\,,$$
$$u_\beta(x,y)=C_2\cos(\alpha x)\sin(\beta y)\,,\ v_\beta(x,y)=C_3\sin(\alpha x)\cos(\beta y)\,, \qquad (24)$$
$$\theta_x(x,y)=C_4\cos(\alpha x)\sin(\beta y)\,,\ \theta_y(x,y)=C_5\sin(\alpha x)\cos(\beta y)\,,$$

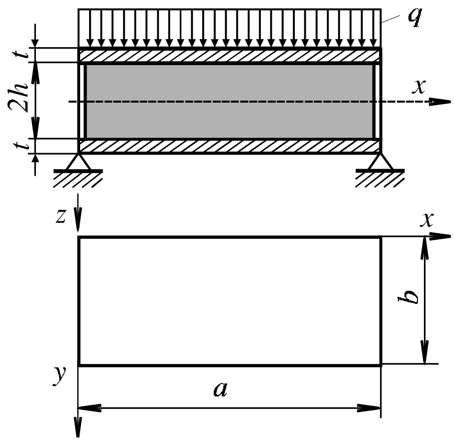

Fig. 4.

where $\alpha = \frac{m\pi}{a}$, $\beta = \frac{n\pi}{b}$, $m, n = 1,2,...$, the boundary conditions (22) and (23) will be fulfilled, and the system of partial differential equations (13) and (14) will be reduced to the system of algebraic equations on unknown coefficients C_1, ..., C_5. For $\alpha = \beta$ and $h = 4t$, the desired coefficients are:

$$C_1 = \frac{q[2t^2\alpha^2(2 + k_1\alpha^2) + 3(1-\mu)(1 + 2k_1\alpha^2)]}{G_3 k k_1 t \alpha^4}$$
$$C_2 = C_3 = \frac{q[8t^2\alpha^2 + 27(1-\mu)]}{G_3 k k_1 t \alpha^4} \tag{25}$$
$$C_4 = C_5 = \frac{-3q[8t^2\alpha^2 - (1-\mu)(1 + 2k_1\alpha^2)]}{G_3 k k_1 t \alpha^4}$$

where $k = 32t^2\alpha^2 + (1-\mu)(122 + k_1\alpha^2)$, $k_1 = \frac{bh}{G_3}$. (26)

In that way the problem is solved. With the relations (24) we may estimate all forces and displacements in any point of the plate with the fulfillment of the boundary conditions.

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PRIMENA STROŽIJE TEORIJE PRI SAVIJANJU TROSLOJNE PLOČE SA LAKIM JEZGROM

Zlatibor Vasić

U radu je prikazana primena strožije teorije savijanja tankih homogenih ploča kod proračuna troslojnih nehomogenih konstrukcija. Izvedene su osnovne jednačine troslojne ploče simetrične strukture sa tankim spoljašnim slojevima, koji se savijaju prema RESSNER - MINDLIN - ovoj teoriji i srednjim slojem koji je izložen samo poprečnom smicanju. Metodom energije, varirajući moguća pomeranja, izvedena je varijaciona jednačina savijanja. Pri tome su odredjeni odgovarajući statički konturni uslovi i potvrdjene osnovne jednačine savijanja troslojne ploče