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ONE EXPERIENCE IN THE VERIFICATION OF MODEL VALIDITY IN SPHERICAL SHELL FINITE ELEMENT ANALYSIS *

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Abstract. *The purpose of this article is to reconsider an example of the spherical shell, which has been used for more than ten years in monlinear finite element analysis tests. However, the numerical results were compared, as a rule, with the analytical results concerning the shell (a rotational paraboloid) having only one spherical point.*

Key words: *Finite element model validity, spherical shell, shell with spherical point*

INTRODUCTION

The purpose of this paper is to point out a dilemma we were faced with during validity verification of mathematical model itself, before its finite element idealization. Namely, bearing in mind that "there are many examples of ... making serious simulation errors... which occur ... because the real ... structure has not been adequately represented" by the FE model[1] (s. [18], pp. 17-18), we believe it to be of benefit to reconsider an example of a *spherical shell*, which has been used for more than ten years in nonlinear finite element analysis tests; however, the numerical results were compared, as a rule, with analytical results pertaining to a shell having *only one spherical point*.

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[1] We remember, for example, the discovery of an inconsistency in the finite element model from User project "Cantilevered curved beam problem for evaluating shell elements" in FINITE ELEMENT NEWS - 1987, Issue No. 4 (August).

SPHERICAL SHELL VERSUS SHELL WITH SPHERICAL POINT

During the quadrilateral shell element testing using the code STATA (Static Analysis program) of an in-house structural analysis package [14] (the shell element used is four noded linear isoparametric, based on the Cosserat shell theory and selective integration; the details are presented in [5] and [6]) we have utilized some of the popular benchmarks: the pinched cylinder problem, the barrel vault problem, the hemisphere with pinch loading, etc. One of fhese was the example of the shallow spherical shell (Fig. 1), which we first encountered in [13]; it is typical for nonlinear analysis, but we considered it might be useful to apply this shallow doubly-curved shell as a test in linear static analysis, too, However, first hesitation arised during the preparation of the code for automatic mesh generation – is $R_1 = R_2$ the sphere radius, so that the spherical sector surface is in question (s. Fig. 1, taken from [13], i.e. [12])? This follows from the fact that the intersection of these radia continuations (Fig. 1) seemingly lies on the concentrated load direction. (Table 1 contains part of a code for node coordinate generation in the case of quadrilateral finite shell elements.)

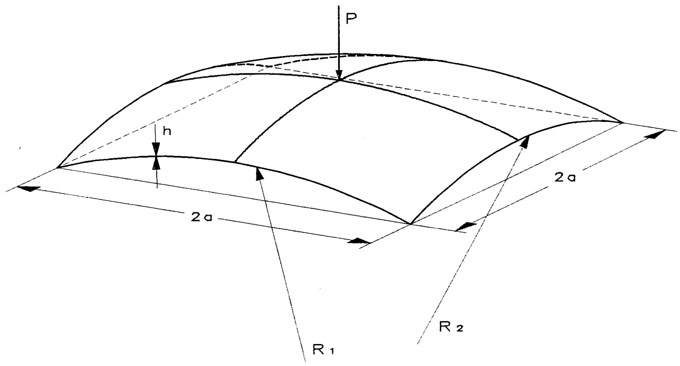

Fig. 1. Model 1: $R_1 = R_2 = 2540.$ mm, $a = 784.90$ mm, $h = 99.45$ mm

Table 1.

```
          .                  ngrid - number of elements
          .                          per boundary of one
          .                          quarter of the shell
     Phy_0=asin(a/sqrt(R*R-a*a))
   Theta_0=asin(a/sqrt(R*R-a*a))
do i=1,ngrid+1
     do j=1,ngrid+1
          k=j+(ngrid+1)*(i-1)
        Phy=tan((j-1)*Phy_0/ngrid)*cos((i-1)*Theta_0/ngrid)
      Theta=tan((i-1)*Theta_0/ngrid)*cos((j-1)*Phy_0/ngrid)
       X(k)=R*Phy/sqrt(1.+Phy*Phy)
       Y(k)=R*Theta/sqrt(1.+Theta*Theta)
       Z(k)=sqrt(R*R-X(k)*X(k)-Y(k)*Y(k))
     enddo
enddo     .
          .
          .
```

In view of the fact that, for this example, a reference (in [12] and [13]) is made to [7], we browsed through this paper too, and we saw the situation presented in Fig. 2 (s. [7],

p. 46; the same is in [4], p. 163 and [9], p. 201) – from there we concluded that R_1 and R_2 are the radia of the circles obtained by cutting of sphere $R = \sqrt{R_1^2 + a^2}$ with a plane orthogonal to the model square planform. The fact that paper [9] deals with "a spherical dome segment on a square base" leads to this conclusion, too. (The corresponding code for node coordinate determination is presented in Table 2.)

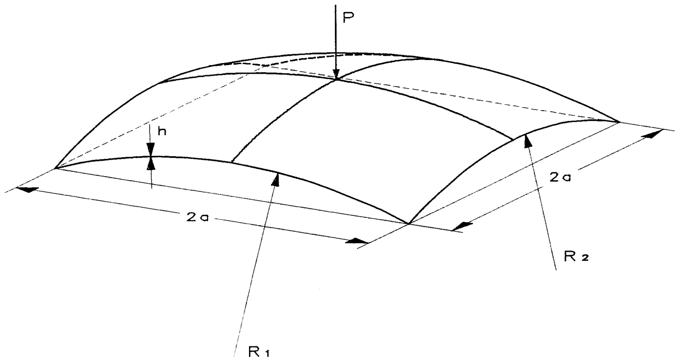

Fig. 2. Model 2: $R_1 = R_2 = 2540.$ mm, $R = \sqrt{R_1^2 + a^2} = 2658.5$mm , a = 784.90 mm, h = 99.45 mm

Table 2.

```
        .
        .
        .
     R=sqrt(R1*R1+a*a)
    Phy=asin(a/R)
   Theta=asin(a/R)
do i=1,ngrid+1
   do j=1,ngrid+1
         k=j+(ngrid+1)*(i-1)
      X(k)=R*sin((j-1)*Phy/ngrid)
      Y(k)=R*sin((i-1)*Theta/ngrid)
      Z(k)=sqrt(R*R-X(k)*X(k)-Y(k)*Y(k))
   enddo
enddo   .
        .
        .
```

However, because of the non-negligible difference between the central deflections (s. Table 5) during the linear analysis of the models from Fig. 1 and Fig. 2, we could not be certain how to use and with what to compare the obtained results. Therefore, we searched for another reference in connection with the example under consideration. In this way, we arrived at Fig. 3 (s. [10], p. 409; [8], p. 168; [3], pp. 47–49); it follows from there that the prescribed R = 2540 mm is the radius of curvature for a spherical shell on a quadratic base (this is unambiguously clear from the analytical point of view in [8], where the angle $\alpha = 36°$ is quoted – α is the "measure" of the part of the sphere in question: sin $1/2\alpha$ = sin 18° = 0.30902 = a / R). (Table 3 presents the code for node coordinate determination in this case.)

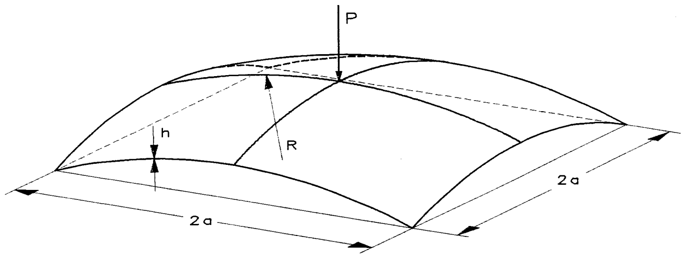

Fig. 3. Model 3: R = 2540. mm, $R_1 = R_2 = \sqrt{R^2 - a^2}$ = 2415.7mm, a = 784.90 mm, h = 99.45 mm

Table 3.

```
          .
          .
          .
     R=2540.
    Phy=asin(a/R)
   Theta=asin(a/R)
do i=1,ngrid+1
    do j=1,ngrid+1
         k=j+(ngrid+1)*(i-1)
       X(k)=R*sin((j-1)*Phy/ngrid)
       Y(k)=R*sin((i-1)*Theta/ngrid)
       Z(k)=sqrt(R*R-X(k)*X(k)-Y(k)*Y(k))
    enddo
enddo     .
          .
          .
```

At this point of our investigation, we were in doubt which of the models: in Fig. 1, Fig. 2 or Fig. 3, is to be considered as competent. There was nothing left for us to do but to find references [1] and [2], quoted by all others, where Leicester studied the deformation of shallow shells. When we received them, the true surprise was in store - what all authors reference as a spherical shell, and Leicester calls the spherical shell too, in [1] and [2] is, in fact, a rotational paraboloid (Fig. 4):

$$\bar{z} = \frac{1}{2}k_1\bar{x}(\bar{x}-\bar{a}) + \frac{1}{2}k_2\bar{y}(\bar{y}-\bar{b}) = \frac{1}{2R_0}\bar{x}(\bar{x}-\bar{a}) + \frac{1}{2R_0}\bar{y}(\bar{y}-\bar{b}),$$

where $\bar{x}, \bar{y}, \bar{z}$ are Cartesian coordinates, $\bar{a} = \bar{b} = \mathrm{const}$ const are the length and width of the shell (the shell is square in planform; in the case a rectangular shell planform, elliptic paraboloid is in question), $k_1 = k_2 = k$ are the principal curvatures at the center of the shell, and $R_0 = 1/k$ is the radius of curvature in this point. This shell has only one spherical (or umbilical) point – at the paraboloid head.

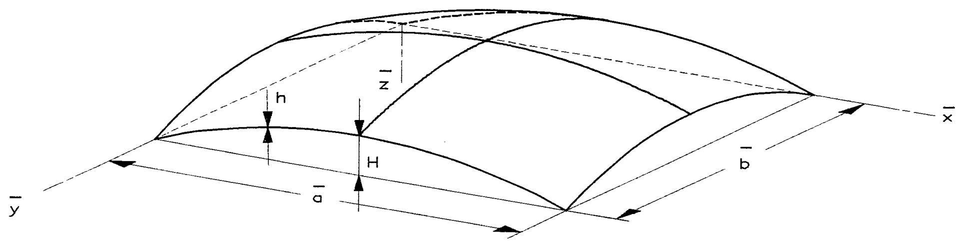

Fig. 4. Model 4: $a = \bar{a}/2 = \bar{b}/2 = 784.90\,\text{mm}$, h = 99.45 mm, H/h =1.2500,

$$R_0 = \frac{a^2}{2H} = 2477.9\,\text{mm}$$

Table 4.

```
          .
          .
          .
      Phy=asin(a/R_o)
     Theta=asin(a/R_o)
  do i=1,ngrid+1
      do j=1,ngrid+1
           k=j+(ngrid+1)*(i-1)
        X(k)=R_o*sin((j-1)*Phy/ngrid)
        Y(k)=R_o*sin((i-1)*Theta/ngrid)
        Z(k)=+0.5*(-(X(k)+a)*(X(k)-a)
1                     -(Y(k)+a)*(Y(k)-a))/R_o
      enddo
  enddo    .
           .
           .
```

In our Cartesian coordinates x, y, z, where:

$$\bar{y} = \frac{\bar{a}}{2} + x = a + x$$

$$\bar{x} = \frac{\bar{a}}{2} + y = a + y$$

$$\bar{z} = -z\,,$$

the equation of this rotational paraboloid reads:

$$z = -\frac{1}{2R_0}(x^2 + y^2) + \frac{a^2}{R_0}.$$

In [1] (and [2]) this shell is defined by its thickness parameter H/h = 1.25, in which H is the rise of the shell along the boundaries (s. Fig. 4), and h is its thickness. If the shell thickness is h = 99.45 mm, it follows that: H = 1.25 h = 124.3125 mm, and, having in mind that:

$$H = z(a,0) = \frac{a^2}{2R_0},$$

finally:

$$R_0 = \frac{a^2}{2H} = 2477.9\,\text{mm}\,.$$

(This figure is used in the corresponding code for node coordinate determination presented in Table 4.)

NUMERICAL RESULTS

Linear static analysis was performed for all four thin shell models. Only a uniform pressure $q = 0.1$ N/mm^2 normal to the shell surface has been considered. The shell material has a Poisson's ratio of 0.3 and Young's modulus of 68.95 N/mm^2; all shell boundaries are hinged. Because of double symmetry, only a quarter of the shell has been analyzed (with 3×3 mesh size; Fig. 5) and the results obtained using STATA code are quoted in Table 5.

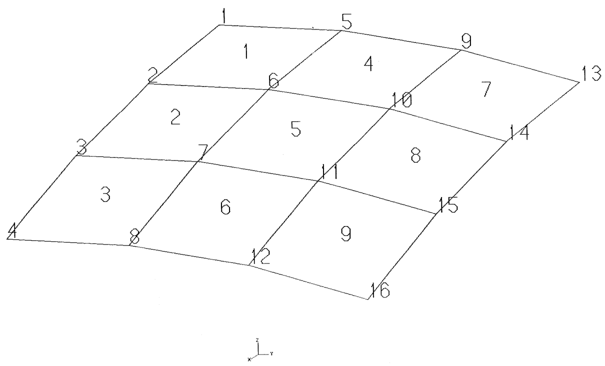

Fig. 5. Finite element discretization of one quarter of the shell

Table 5

Central deflection (mm)	
Model 1	-56.03
Model 2	-60.19
Model 3	-55.38
Model 4	-56.10

CONCLUDING REMARKS

Note that the above example is also fairly thick shell. Hence, if a concentrated load and the degenerate solid or Reissner-Mindlin shell formulations are used (as it is a very common practice indeed), one can not speak about the finite value of the displacement under the load (s. for example [19], either in linear or nonlinear analyses.

However, in this paper we were interested primarily in pointing out one example of uncritical substitution[2] of the notion of a spherical surface with a surface having only one spherical point (a rotational paraboloid); however small the difference between the various models (s. Fig. 6), i.e. between the corresponding central deflections (s. Table 5), their comparison is, in principle, unacceptable.

Finally, we emphasize that only recently we have encountered the paper [16], where *two models* of the spherical shell have been distinguished and analysed[3], as well the

[2] Still present in the literature (s., for example, [15].

[3] The first model ("vertical cutting") corresponds to model 3, and the second one ("pyramid cutting") to Model

thesis of Ma [17], where we found confirmation for our precaution during model validity verification for shallow spherical shells with square planform ([17], p. 109: "For this problem there is a conflict in the current literature concerning the dimensions in the structure model.").

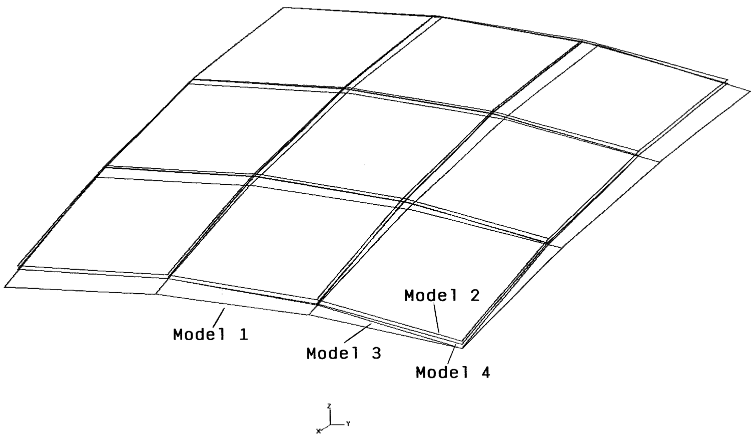

Fig. 6. Mesh of quadrilateral elements for one quarter of the shell (Model 1 - Model 4)

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1. It should be noted that the reference in [16], concerning the spherical shell example, is made to paper [11].

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JEDNO ISKUSTVO U PROVERI VALJANOSTI MODELA PRI ANALIZI SFERNE LJUSKE METODOM KONAČNIH ELEMENATA

Zoran Drašković, Mladen Berković

U radu je analiziran primer sferne ljudske, koji se više od deset godina koristi u testovima iz nelinearne analize konačnim elementima. Međutim, rezultati proračuna se, po pravilu, porede sa analitičkim rezultatima koji se zapravo odnose na ljusku (rotacioni paraboloid) sa samo jednom sfernom tačkom.

Ključne reči: *Valjanost KE modela, sferna ljuska, ljuska sa sfernom tačkom.*