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ALGEBRAIC CRITERIA FOR ASYMPTOTIC STABILITY AT 1:1 RESONANCE IN THE CASE OF SIGN-CONSTANT LYAPUNOV FUNCTION

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Abstract. *In the critical case of two pairs of purely imaginary eigenvalues at 1:1 resonance, the asymptotic stability of a system with two degrees of freedom is investigated. It is assumed that eigenvalues have simple elementary divisors. By means of sign-constant Lyapunov function which has sign-constant derivative, the new algebraic criteria of asymptotic stability is obtained.*

It is well known, the problem of stability of equilibrium at 1:1 resonance has been investigated by Lagrange [3] and Weierstrass. In the case of linear conservative system (purely imaginary eigenvalues, multiple resonance, simple elementary divisors). Lagrange concluded than the equilibrium is unstable. Weierstrass (1858) has pointed out Lagrange's mistake: the linear equations can be written in terms of normal variables and therefore the system is stable.

Later N.E. Kochin has investigated this problem for non-linear Hamiltonian equations in [6] . In the case of non-linear Hamiltonian system which has simple elementary divisors, the full decision of the problem is given in paper [13]: the equilibrium is stable as a rule. If non-linear Hamiltonian system has not simple elementary divisors, the equilibrium is stable as a rule. If non-linear Hamiltonian system has not simple elementary divisors, the equilibrium is stable in half of available cases and is unstable in other cases [13,7].

A theory of multiple resonance of non-Hamiltonian equations has been developed for reversible systems [8].

If non-Hamiltonian equations have the general form when elementary divisors are not

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simple, the equilibrium is unstable as a rule [4]. If the matrix of the linear part of the equations is diagonalized (simple elementary divisors), the problem of the construction of stability criteria is transcendental [5]. It means the surface that separates the set of asymptotically stable systems from the set of unstable systems in the real parameter space of the system is transcendental. However, this transcendence is not of universality since the separating surface has algebraic sections [9]. The last results were obtained by means of sign-definite function V that has signdefinite derivative $\dot{V}$.

Hereafter, we obtain the new algebraic criteria for asymptotic stability in this transcendental problem by means of the sign-constant function V which has sign-constant derivative $\dot{V}$.

1. Statement of the problem. Lyapunov function

Consider an autonomous system

$$\dot{\mathbf{x}} = \mathbf{X}(\mathbf{x}), \quad \mathbf{X}(0) = 0, \quad \mathbf{x} \in \mathbf{R}^4 \tag{1.1}$$

where $\mathbf{X}(\mathbf{x})$ is a smooth vector field such that the matrix $(\partial\mathbf{X}/\partial\mathbf{x})_0$ has purely imaginary eigenvalues $\lambda_1 = \lambda_2$. Let us assume that λ_1 has simple elementary divisors. The complex third-approximation normal system is

$$\begin{aligned}
\dot{z}_1 &= \lambda_1 z_1 + A_{11} z_1^2 \bar{z}_1 + A_{12} z_1 z_2 \bar{z}_2 + A_1 z_1 \bar{z}_1 z_2 + A_2 z_1^2 \bar{z}_2 + A_{31} z_2^2 \bar{z}_2 + A_4 z_2^2 \bar{z}_1 \\
\dot{z}_2 &= \lambda_1 z_2 + A_{21} z_1 \bar{z}_1 z_2 + A_{22} z_2^2 \bar{z}_2 + A_5 z_1^2 \bar{z}_1 + A_6 z_1 z_2 \bar{z}_2 + A_7 z_1^2 \bar{z}_2 + A_8 \bar{z}_1 z_2^2 \\
z_1 &= x_1 + i x_2, \quad z_2 = x_3 + i x_4, \quad A_{lm} = a_{lm} + i b_{lm}, \quad A_m = a_m + i b_m
\end{aligned} \tag{1.2}$$

Let us pass to the polar variables by means of formulas

$$z_j = \sqrt{r_j}\exp(i\theta_j), \quad \bar{z}_j = \sqrt{r_j}\exp(-i\theta_j)$$

Eqs (1.2) have the following form:

$$\dot{r}_j = R_j(r_1, r_2, \theta), \quad \dot{\theta} = \Omega(r_1, r_2, \theta) \quad (j = 1,2) \tag{1.3}$$

Here $\theta = (\theta_1 - \theta_2)$ is the resonance angle,

$$\begin{aligned}
\frac{1}{2}R_1 &= a_{11} r_1^2 + a_{12} r_1 r_2 + r_1\sqrt{r_1 r_2}\,[(a_1 + a_2)\cos\theta + (b_2 - b_1)\sin\theta] + \\
&\quad + r_2\sqrt{r_1 r_2}\,(a_3\cos\theta - b_3\sin\theta) + r_1 r_2 (a_4\cos 2\theta - b_4\sin 2\theta] \\
\frac{1}{2}R_2 &= a_{21} r_1 r_2 + a_{22} r_2^2 + r_2\sqrt{r_1 r_2}\,[(a_6 + a_8)\cos\theta + (b_6 - b_8)\sin\theta] + \\
&\quad + r_1\sqrt{r_1 r_2}\,(a_5\cos\theta + b_5\sin\theta) + r_1 r_2 (a_7\cos 2\theta + b_7\sin 2\theta]
\end{aligned} \tag{1.4}$$

$$\Omega = (b_{21} - b_{11} + b_7\cos 2\theta - a_7\sin 2\theta)r_1 + (b_{22} - b_{12} - a_4\sin 2\theta - b_4\cos 2\theta)r_2 +$$
$$+[(b_6 + b_8 - b_2 - b_1)\cos\theta + (a_8 - a_6 + a_2 - a_1)\sin\theta]\sqrt{r_1 r_2} +$$
$$+(b_5\cos\theta - a_5\sin\theta)r_1^{3/2}r_2^{-1/2} - (a_3\sin\theta + b_3\cos\theta)r_2^{3/2}r_1^{-1/2}$$

The Lyapunov function

$$V = D_{11}r_1^2 + 2D_{12}r_1r_2 + D_{22}r_2^2 + 2r_1\sqrt{r_1r_2}\,(D_1\cos\theta + D_2\sin\theta) +$$
$$2r_2\sqrt{r_1r_2}\,(D_3\cos\theta + D_4\sin\theta) + 2r_1r_2(D_5\cos 2\theta + D_6\sin 2\theta) \qquad (1.5)$$

(D_{ij} are an arbitrary constants) was constructed by means of T-extension technology [9,10]. Via this function, necessary and sufficient conditions of asymptotic stability were obtained in the paper [9] provided that V and it's derivative $\dot{V}$ are sign-definite functions.

Now consider more general case when this function (and it's derivative) can be sign-constant function. The derivative of V along the vector field of Eqs (1.3), (1.4) is

$$\dot{V} = r_2^3[\gamma_0(k) + \sum_{n=1}^{3}(\gamma_{n1}(k)\cos n\theta + \gamma_{n2}(k)\sin n\theta]$$

The coefficients γ_{ij} are the functions of variable $k = r_1/r_2$, $k\in(0,\infty)$:

$$\gamma_0(k) = G_0k^3 + G_1k^2 + G_2k + G_3, \quad \gamma_{1m}(k) = 2\sqrt{k}\,(B_{1m}k^2 + B_{2m}k + B_{3m})$$
$$\gamma_{2m}(k) = 2k(C_{1m}k + C_{2m}), \quad \gamma_{3m}(k) = 2k^{3/2}F_m \quad (m = 1,2)$$

The constant parameters G_j, B_{ij}, C_{ij}, F_m depend linearly on the arbitrary constants D_{ij}, D_j. Thus, for example

$$G_0 = 4a_{11}D_{11} + 2a_5D_1 + 2b_5D_2,\ G_3 = 4a_{22}D_{22} + 2a_3D_3 - 2b_3D_4$$

(for the other parameters, see [9]). The constants D_{ij} and D_j are chosen so that the coefficients of cos 2θ, sin 2θ, cos 3θ, sin 3θ vanish, i.e. we impose upon these numbers the conditions $C_{1m} = C_{2m} = F_m = 0$ (m = 1,2). In order to simplify the coefficients of cosθ and sinθ, we also require that $B_{2m} = 0$. Then

$$\mathbf{AD} = D_6\mathbf{R}\,,\ \mathbf{D} = (D_{11},D_{12},D_{22},D_1,....,D_5)^T \qquad (1.6)$$

where **A** and **R** are 8 by 8 and 8 by 1 matrices, respectively, whose elements are linear functions of the parameters. As the number D_6 appears as a factor in the right-hand side of Eqs (1.6), it does not play essential role, and we may assume it to be equal to unity. We assume that det A ≠ 0. Let $\mathbf{D} = \mathbf{D}^*$ be a family of solutions of Eqs (1.6), depending on the parameters of the problem.

Define the Lyapunov function be V^*, where V^* is the restriction of V to these family.

2. Criteria for V^* and $\dot{V}^*$ to be sign-constant

Clearly

$$\dot{V}^* = r_2^3 g_1(k,\theta)$$

where

$$g_1(k,\theta) = \gamma_0^*(k) + \gamma_{11}^*(k)\cos\theta + \gamma_{12}^*(k)\sin\theta\,, \quad k = r_1/r_2$$

$$\gamma_0^*(k) = G_0^* k^3 + G_1^* k^2 + G_2^* k + G_3^*\,, \quad \gamma_{1m}^*(k) = 2\sqrt{k}(B_{1m}^* k^2 + B_{3m}^*) \quad (m = 1,2)$$

The function $\dot{V}^*$ is sign-constant in the region $r_1 > 0$, $r_2 > 0$, $0 \le \theta < 2\pi$ if and only if $(\gamma_0^*)^2 \ge (\gamma_{11}^*)^2 + (\gamma_{12}^*)^2$ for any $k > 0$. It is evident that $\dot{V}^*$ is sign-constant in the cone $r_1 \ge 0$, $r_2 \ge 0$ since $\dot{V}^*$ is continued function in the planes $r_1 = 0$, $r_2 = 0$.

Hence, it follows that $\dot{V}^*$ is sign-constant in the cone $r_1 \ge 0$, $r_2 \ge 0$, $0 \le \theta < 2\pi$ if and only if the equation

$$(\gamma_0^*)^2 - (\gamma_{11}^*)^2 - (\gamma_{12}^*)^2 = 0 \tag{2.1}$$

has not positive roots $k = k_j$ except multiple roots.

We now get the criteria for V^* to be sing-constant in the cone $r_1 \ge 0$, $r_2 \ge 0$, $0 \le \theta < 2\pi$. Let us convert V^* to the form

$$V^* = r_2^2 g_2(k,\theta)$$

where

$$g_2(k,\theta) = \sigma_0(k) + \sigma_1(k)\cos\theta + \mu_1(k)\sin\theta + \sigma_2(k)\cos 2\theta + \mu_2(k)\sin 2\theta \tag{2.2}$$

$$\begin{gathered} \sigma_0(k) = D_{11}^* k^2 + 2D_{12}^* k + D_{22}^*\,, \quad \sigma_1(k) = 2\sqrt{k}(D_1^* k + D_3^*)\,, \\ \sigma_2(k) = 2D_5^* k\,, \quad \mu_1(k) = 2\sqrt{k}(D_2^* k + D_4^*)\,, \quad \mu_2(k) = 2k\,, \quad k = r_1/r_2 \end{gathered} \tag{2.3}$$

Lemma 1 [11]. The trigonometric polynomial

$$g(0) = \sigma_0 + \sigma_1\cos\theta + \mu_1\sin\theta + .. + \sigma_n\cos n\theta + \mu_n\sin n\theta$$

$$\sigma_j, \mu_j = const$$

is the positive sign-constant function of θ $(0 \le \theta < 2\pi)$ if and only if there exist the complex numbers $x_0, x_1, \ldots x_n$ satisfying equations

$$\begin{cases} \sigma_0 = |x_0|^2 + |x_1|^2 + \ldots + |x_n|^2 \\ \frac{1}{2}(\sigma_k + i\mu_k) = x_0\bar{x}_k + x_1\bar{x}_{k+1} + \ldots + x_{n-k}\bar{x}_n \\ k = 1, \ldots, n \end{cases} \tag{2.4}$$

The proof of this lemma is based on the transformation of polynomial $g(\theta)0 \ge 0$ to the form

$$g(\theta) = |x_0 + x_1 z + \ldots + x_n z^n|^2 \tag{2.5}$$

where $z = \exp(i\theta)$. If we now transform $g(\theta)$ to the standard algebraic form

$$g(0) = z^{-n}G(z),\ \ G(z) = u_0 + u_1 z + ... + u_{2n} z^{2n},\ \ G(z) = z^{2n}\overline{G}(z^{-1})$$

and set the coefficients of polynomial (2.5) equal to the corresponding coefficients of polynomial $z^{-n}G(z)$, we then have equalities (2.4).

Let us consider the case $n = 2$. If $4|x_0|^4 \neq \sigma_2^2 + \mu_2^2$, the last two equations of the system (2.4) define x_1, x_2 as the univalent functions of x_0. If we substitute these functions into the first equation of system (2.4), we have

$$\sum_{k=0}^{6} L_{6-k}\xi^k = 0,\ \ (\xi = |x_0|^2) \tag{2.6}$$

where

$$\begin{gathered} L_0 = 64,\ \ L_1 = -64\sigma_0,\ \ L_2 = 16(\sigma_1^2 + \mu_1^2 - \sigma_2^2 - \mu_2^2) \\ L_3 = 32\sigma_0(\sigma_2^2 + \mu_2^2) - 16[\sigma_2(\sigma_1^2 - \mu_1^2) + 2\mu_1\mu_2\sigma_1] \\ L_4 = 4(\sigma_2^2 - \mu_2^2)(\sigma_1^2 + \mu_1^2 - \sigma_2^2 - \mu_2^2),\ \ L_5 = -4\sigma_0(\sigma_2^2 + \mu_2^2)^2,\ \ L_6 = (\sigma_2^2 + \mu_2^2)^3 \end{gathered} \tag{2.7}$$

Therefore, system (2.4) is solvable for $x_1, x_2\,(n = 2)$ if and only if the algebraic equation (2.6) has at least one positive root $\xi \neq \sqrt{\sigma_2^2 + \mu_2^2}/2$. So, the trigonometric polynomial $g(\theta)$ is the positive sign-constant function of θ in this case only.

Obviously, the criteria of $g(\theta)$ to be the negative sign-constant function has the similar form with a glance to the change $\sigma_j, \mu_j \to -\sigma_j, \mu_j$. Coefficients L_{2k} preserve values, L_{2k+1} change sign to opposite one.

Let us consider the trigonometric polynomial, which is situated in the right hand side of expression (2.2). His coefficients are the functions of variable k. By formulas (2.2), (2.3), we have $\operatorname{sign} V^* = \operatorname{sign} D_{11}^*$ if V^* is sign-constant for any $k \in (0,\infty)$. The next statement follows from lemma l at once.

Corollary. Let $D_{11}^* \neq 0$. Function V^* is sign-constant in the cone $r_1 \geq 0$, $r_2 \geq 0$, $0 \leq \theta < 2\pi$ if and only if for any $k \in (0,\infty)$ equation

$$\sum_{l=0}^{6} (\operatorname{sign} D_{11}^*)^l L_{6-k}(k)\xi^l = 0 \tag{2.8}$$

has one positive root $\xi(k) \neq \sqrt{\sigma_2^2(k) + \mu_2^2(k)}/2$ at least.

3. An investigation of the manifolds $V^* = 0$ and $\dot{V}^* = 0$

To research the stability of the equilibrium by means of sign-constant function V^* which has sign-constant derivative we must present additional demands to functions V^* and $\dot{V}^*$. Indeed, from the paper [1], the manifold $M_0 = \{\mathbf{x}: V^* = 0,\ 0 < \|\mathbf{x}\| \leq h\}$ (h is the positive number) can't hold the whole negative semitrajectories, moreover manifold $M \backslash M_0$ can't hold the whole paths where $M = \{\mathbf{x} : \dot{V}(x) = 0, 0 < \|\mathbf{x}\| \leq h\}$.

Let us get the conditions satisfying these demands. It is evident, the sign-constant function $\dot{V}^*(V^*)$ will be equals to zero for $\theta = \theta^*$ if and only if θ^* is the multiple root of the trigonometric polynomial $g_1(k,\theta)$ ($g_2(k,\theta)$).

From the beginning we shall consider the function $\dot{V}^*$. As described above, $\theta = \theta^*$ is the root of sign-constant polynomial $g_1(k,\theta)$ if and only if equation (2.1) has the multiple positive root $k = k^*$. Hence, it follows sign-constant function $\dot{V}^*$ can be equals to zero only along the rays

$$r_1 = k^* r_2 \, \theta=\theta^* \tag{3.1}$$

Let us show in general case there does not exist the final interval of time (t_1,t_2), $t_2 > t_1$ such that the phase point belong to one of the rays (3.1) when $t \in (t_1,t_2)$.

Indeed, if we substitute equalities (3.1) to Eqs (1.3), two trigonometric equations for θ were obtained. By means of exchange $x = \exp(i\theta^*)$, received equations can be transformed to the follow form:

$$\begin{aligned} f_1 &\equiv v_0 + v_1 z + v_2 z^2 + \overline{v}_1 z^3 + \overline{v}_0 z^4 = 0 \\ f_2 &\equiv w_0 + w_1 z + w_2 z^2 + \overline{w}_1 z^3 + \overline{w}_0 z^4 = 0 \end{aligned} \tag{3.2}$$

Here

$$\begin{aligned} &v_0 = \frac{1}{2}[(a_4 - k^* a_7) - i(b_4 + k^* b_7)], \quad v_2 = (a_{11} - a_{21})k^* + (a_{12} - a_{22}) \\ &v_1 = \frac{1}{2\sqrt{k^*}}\{[a_3 + (a_1 + a_2 - a_6 - a_8)k^* - a_5 k^{*2}] - i[b_3 + (b_6 + b_1 - b_8 - b_2)k^* + b_5 k^{*2}]\} \end{aligned} \tag{3.3}$$

k^* is the multiple positive root of equation (2.1). Coefficients w_0, w_1, w_2 have the similar form with a glance to the substitution $b_j \to a_j$, $a_j \to -b_1$, $a_{lm} \to -b_{lm}$.

If the resultant $R_1(f_1,f_2)$ of polynomials f_1,f_2 does not equal to zero, the systems (3.2) is unsolvable for z. Therefore there does not exist continuous section of phase path which belongs to one of the rays (3.1).

So, if $R_1(f_1,f_2) \neq 0$ when k^* passes through the set of positive multiple roots of equation (2.1), the manifold $\dot{V}^* = 0$ can't hold the whole paths.

Let us consider the function V^*. By means of exchange $z = \exp(i\theta)$ for polynomial $g_2(k,\theta)$ we have

$$g_2(k,\theta) = x^{-2} G(k,z) ,$$

where

$$G(k,z) = u_0(k) + u_1(k)z + u_2(k)z^2 + \overline{u}_1(k)z^3 + \overline{u}_0(k)z^4$$

$$u_0(k) = \frac{1}{2}(\sigma_2(k) + i\mu_2(k)), \quad u_1(k) = \frac{1}{2}(\sigma_1(k) + i\mu_1(k)), \quad u_2(k) = \sigma_0(k)$$

Every multiple root $\zeta^0 = \exp(i\theta^0)$ of polynomial $G(k,z)$ has the corresponding multiple root θ^0 of $g_2(k,\theta)$ herewith their multiplicities are equal one another. Thus, function V^* equals to zero if and only if

$$F(G) = 0 , \tag{3.4}$$

where $F(G)$ is the discriminant of polynomial $G(k,z)$. Obviously, $F(G)$ is the function of

k. Hence, it follows that the sign-constant function V^* can be equal to zero only along the rays

$$r_1 = k^0 r_2\,,\ \theta = \theta^0$$

where k^0 is the positive root of (3.4).

Let $R_2(f_1,f_2)$ be the resultant of polynomials (3.2), (3.3) provided that $k^* \to k^0$. As described above, if $R_2(f_1,f_2) \neq 0$ when k^0 passes through the set of positive roots of equation (3.4), the manifold $V^* = 0$ can't hold the some kind of continuous section of any phase path.

4. AN ALGEBRAIC CRITERIA FOR ASYMPTOTIC STABILITY

Let us get the new criteria for asymptotic stability by means of sign-constant function V^*. We take advantage of follow theorem.

Theorem 1 [12]. Let the differential equations of perturbed motion have the continuous function $V(\mathbf{x})$, $V(\mathbf{0}) = 0$ such that

a) $V(\mathbf{x}) \geq 0\,,\ \dot{V}(\mathbf{x}) \leq 0\,,\ \|\mathbf{x}\| \leq h$

b) Set $M_0 = \{\mathbf{x} : V(\mathbf{x}) = 0\,,\ 0 < \|\mathbf{x}\| < h\}$ can't hold the negative semitrajectories

c) Set $M \setminus M_0\,,\ M = \{\mathbf{x} : \dot{V}(\mathbf{x}) = 0\,,\ 0 < \|\mathbf{x}\| < h\}$ can't hold whole trajectories

Then the unperturbed equilibrium $\mathbf{x} = \mathbf{0}$ is asymptotic stable.

Let A be the matrix of linear system (1.6), and D^*_{ij}, D^*_j are the parameters of V that satisfy Eqs (1.6), G^*_0, G^*_3 are the corresponding values of the coefficients in the derivative $\dot{V}^*$, $\sigma_j(k)$, $\mu_j(k)$, $L_j(k)$ are calculated by (2.3), (2.7); $R_1(f_1,f_2)$, $R_2(f_1,f_2)$ are the resultants of polynomials (3.2), (3.3) when $k = k^*$ or $k = k^0$ correspondingly.

Theorem 2. Let $\det A \neq 0, (G^*_0, G^*_3, D^*_{11}, D^*_{22}) \neq 0$ and equation (2.1) has not positive roots except multiple roots, $G^*_0 D^*_{11} < 0; R_1(f_1,f_2) \neq 0,\ R_2(f_1,f_2) \neq 0$ when values k^* and k^0 pass through the set of positive roots of Eqs (2.1) and (3.4) correspondingly. Then the equilibrium position of the model system (1.2) is asymptotic stable if the equation (2.8) has at least one positive root $\xi(k) \neq (1/2)\sqrt{\sigma_2^2(k) + \mu_2^2(k)}$ for any $k > 0$; otherwise, if equation (2.8) has not such kinds of roots for some $k > 0$, the model system (1.2) is unstable.

If the equations (2.1), (3.4) have no multiple roots, the asymptotic stability and instability are retained by the full system (1.1).

Proof. From the assumptions of the theorem, function $\dot{V}^*$ is sign-constant in the cone $r_1 \geq 0,\ r_2 \geq 0,\ 0 \leq \theta < 2\pi$ herewith $\operatorname{sign}\dot{V}^* = \operatorname{sign} G^*_0$. If the equation (2.8) has the positive rots for any $k \in (0,\infty)$, function V^* is sign-constant also, $\operatorname{sign}\dot{V}^* = \operatorname{sign} D^*_{11}$. Manifold $M_0 = \{r_1, r_2, \theta\!: V^* = 0\}$ can't hold negative semitrajectories, $V^*\dot{V}^* \leq 0$. Thus, function V^* satisfies the conditions of Theorem 1. Therefor, the equilibrium, of model system (1.2) is asymptotic stable. If equation (2.8) has no positive roots $\xi(k)$ for some $k > 0$, then V^* changes the sign, therefore the conditions of Krasovskii instability theorem are satisfied.

If Eqs (2.1) has no multiple roots, $\dot{V}^*$ is sign-definite function by virtue of the full system (1.1). Indeed, the higher-order terms dropped when deriving model equations (1.2) do not have an affect on the sign of $\dot{V}^*$, since $\dot{V}^*$ and the dropped higher-order terms are homogeneous polynomials in z_j, $\bar{z}_j$. As the equation (3.4) has no multiple roots, V^* is sign-definite function. Therefore, by means of Lyapunov's theorems, the asymptotic stability and instability of model system (1.2) are retained by the full system (1.1). This completes the proof.

Note these new results supplement the investigations [9] for the case when Eqs (2.2) and (3.4) have the multiple roots. In general case, the appearance of multiple roots does not destroy the asymptotic stability of model system (1.2) at least.

However, in degenerate case, when manifold $\dot{V}^* = 0$ contains whole path the appearance of multiple roots can destroys the asymptotic stability.

Indeed, if $R_1(f_1,f_2) = 0$, Eqs (1.3), (1.4) have particular solution $r_1 = k^* r_2$, $\theta = \theta^*$ on the manifold $\dot{V}^* = 0$. Let $R_2(k^*,1, \theta^*) > 0$, then as follows from the equality

$$r_2 = r_2^2 R_2(k^*,1,\theta^*)$$

and from the paper [2] the system (1.1) is unstable.

REFERENCES

1. Bulgakov N.G., Kalitin B.S. *The generalization of theorems of second Lyapunov's method*. 1. Theory. Izv. Akad. Nauk Belr SSR, Ser. Fiz.-Mat. Nauk, 3,(1978), 32-36
2. Furta S.D. *On the asymptotic solutions of systems of differential equations in the case of purely imaginary eigenvalues*. Differents. Uravn., 26, 8, (19990), 1348-1351
3. J.-L. Lagrange. *Mécanique analitique*, Paris, 1788
4. Khazin L.G., Shnol'E.E. *Stability of critical equilibrium positions*. Scientific Center for Biological Research, Akad. Nauk, SSSR, Pushchino, 1985
5. Khazin L.G., Shnol' E.E. *The simplest cases of algebraic unsolvability in asymptotic stability problems.* Dokl. Akad. Nauk SSSR, 240, 6, (1978), 1309-1311
6. Kochin N.E., Kibel L.A., Roze N.V. *Theoretical hydromechanics*. Moscow. 1955. (in Russian)
7. Kovalev A.M., Chudnenko A.N. *On the stability of equilibrium positions of a two-dimensional Hamiltonian system in the case of equal frequencies* Dokl. Akad. Nauk UkrSSR, Ser. A 11, (1977), 1010-1013
8. Krasil'nikov P.S., Tkhai V.N. *Reversible systems. Stability at 1:1 resonance*. Prikl. Mat. Mekh. 56,4, (1992) 570-579
9. Krasil'nikov P.S. *Algebraic criteria for asymptotic stability at 1:1 resonance.* Prikl. Mat. Mekh., 57,4 (1993), 5-11
10. Krasil'nikov P.S. *Functional extensions of a solution germ space of first order differential equation and their applications.* Nonlinear Analysis, Theory, Methods & Applications, 28,2 (1997), 359-375
11. Krasil'nikov P.S. *Generalized space of germs of smooth solutions of a first-order equation and their relationship to Lyapunov's direct method*. Izv. Vuzov, Mat., 5, (1990), 47-53
12. Polya G., Szego G. *Aufgaben und lehrsatze aus der analysis*. Zweiter band. Springer-Verlag Berlin • Göttingen • Heidelberg • New York, 1964
13. Sokol'skii A.G. *On the stability of an autonomous Hamiltonian system with two degrees of freedom in the case of equal frequencies*. Pricl. Mat. Mekh, 38,5,(1974), 791-799

ALGEBARSKI KRITERIJUM ZA ASIMPTOTSKU STABILNOST PRI REZONANCIJI 1:1 ZA SLUČAJ KONSTANTNOG ZNAKA LJAPUNOV-LJEVE FUNKCIJE

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Istraživana je asimptotska stabilnost sistema sa dva stepena slobode za kritični slučaj dve čisto imaginarne sopstvene vrednosti pri 1:1 rezonanciji. Pretpostavljeno je da sopstvene vrednosti imaju proste elementarne delioce. Pomoću Ляпунов-ljeve funkcije konstantnog znaka sa konstantnim znakom izvoda, dobijen je novi algebarski kriterijum asimptotske stabilnosti.