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DISCRETE-TIME VARIABLE STRUCTURE CONTROLLER SYNTHESIS FOR OBJECTS WITH FINITE ZEROS

UDC: 62-52

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Abstract. *The paper deals with the control of n -th order objects with finite zeros in the transfer function. Control signal is formed on the basis of the discrete-time values of the error signal and its derivatives using variable structure control algorithm. The structure commutation law process is realized on the base of smaller sample period, which is neglected. Only regulator type of system will be analyzed. For the considered discrete-time variable structure systems mathematical model is given and the general quasi-sliding mode existence conditions on the switching hyperplane are derived.*

1. INTRODUCTION

The variable structure system (VSS) is a special type of nonlinear systems characterized by a discontinuous control action which changes the system structure upon state reaching a set of switching surfaces. The VSS theory was developed in Russia by S. Emel'yanov [2], V. Utkin [3] and their co-workers. From nearly fifteen years ago the VSS theory was widely applied in the West to the design of practical control systems. The main advantage of VSS is its insensitivity of plant parameter variations and external disturbances. Up to 1980. the theory of analogous VSS was studied only. Microprocessor based control systems, as it is well known, are widely used in a great number of applications as a flexible, reliable and not expensive solution. However, the control design implementation by digital computers requires a certain sampling interval which may bring chattering along the switching surface and cause instability. The discrete-time VSS theory is in a studding phase. Up to now a few published results, in English language, which deals with discrete-time VSS are known: Dote, et. al. [4], Milosavljevic [5, 6, 7], Furuta [8], Chan [9], Sira-Ramirez [10], Bartolini [11], Wang and Wu [12], Gao [13].

There were no attempts in the stated papers in the field of discrete-time VSS to analyze the possibilities of implementing sliding modes (quasi-sliding modes), when the object has finite zeros. The problem of implementing sliding modes in analogue VSS,

when the object has finite zeros, has been discussed in several papers, the results of which have been summed up in monographs [2], [3]. It was pointed out there that due to the differentiable features of the object the sliding mode could not be implemented by the control, suffering from break, which is characteristic of VSS. Because of that, three methods of implementing VSS with sliding operating modes were proposed:

(i) to decompose the starting system to the noncontrollable and controllable part by introducing the output variable differentials so that the sliding mode can be organized in the controllable subspace [3];

(ii) to use method of model reference control [11];

(iii) the control break signal to be previously passed through the first order low-pass filters cascade; if the object parameters are variable, then the cascade of these filters is also encompassed by the commutation feedback [1, 2] (Fig.1.).

An attempt will be made in this paper to unite the approaches of obtaining conditions of the quasi-sliding mode existence in the discrete-time VSS without finite zeros from [5, 6, 7] and introducing the commutation low-pass filters cascade from [1, 2]. In principle, the impossibility of using digital low-pass filters cascade instead of those analogous should then be pointed out. The reason lies in the fact that, due to the continuous object, the coupling between the filter output and the object input would be realized by means of a D/A converter. A D/A converter, as we know, gives a signal at the output with the first order breaks, which would, considering differentiable features of the object, result in problems which in VSS are resolved by introducing a commutation filter. Thus, a system, with analogous commutation filters and a variable structure digital regulator will be discussed as shown in Fig. 1.

A mathematical system model in analogous-discrete and fully discrete forms is given in the second part of the paper, while conditions of the quasi-sliding mode existence on the selected sliding hyperplane are determined. Then, an illustrative example is given. Detailed mathematical derivations to obtain a discrete mathematical model of the system discussed and derivations of the quasi-sliding mode existence are given in the appendix to this paper.

2. PROBLEM STATEMENT

The system block diagram to be analyzed is shown in Fig. 1. It is supposed that all necessary information of the plant are accessible for measurement. The problem of measuring required state coordinates is a general one, which is also present in the system without finite zeros. Generally, it can be practically resolved by using an observer. Because of that, we will further suppose that the object is controllable and fully observable. The variable structure system formatter is the system structure commutator which is digital. Dual rate sampling periods assumed: large (T), in the process of the control signal formation and small (T_s), in the process of obtaining switching conditions. It is assumed that $T_s \ll T$ and the effects of small sampling-time (T_s) will be neglected for further analysis. Because of phase trajectories bricking as a consequence of differentiability of plant, we should use the method of eliminating finite zeros influence in the object transfer function by introducing additional commutation filters of adequate time constants proposed by Kostylova [1].

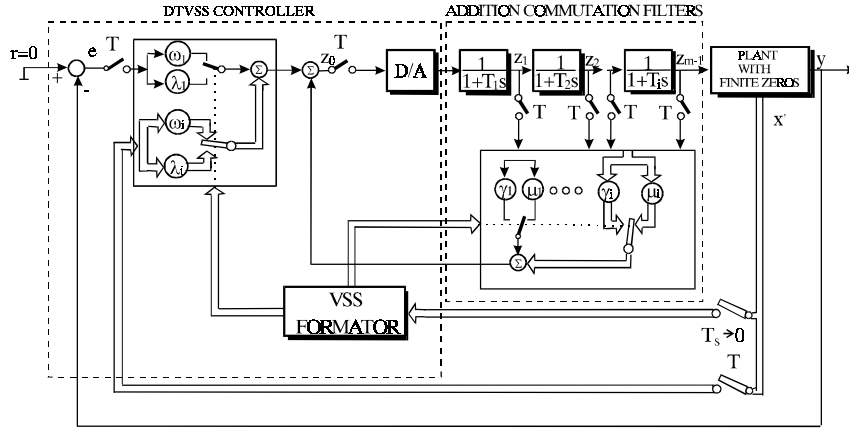


Fig.1. Variable structure control system block diagram

The system type regulator ($r = const$) is analysed in this paper. Let us adopt $r = 0$. Mathematical model of the plant (Fig.1.) was given in the form:

$$\begin{aligned} \dot{\bar{\mathbf{x}}}(t) &= \bar{\mathbf{A}}\bar{\mathbf{x}}(t) + \bar{\mathbf{b}}u(t) \\ y(t) &= \bar{\mathbf{d}}\bar{\mathbf{x}}(t) \end{aligned} \tag{1}$$

where: $\bar{\mathbf{x}}(t) \in R^n$ - the object state vector in the phase space,
 $y(t) \in R$ - the object (system) output variable,
 $u(t) \in R$ - the controll signal,

$$\begin{aligned} \bar{\mathbf{A}} &= \begin{bmatrix} \mathbf{0}_{1 \times (n-1)} & \mathbf{I}_{(n-1) \times (n-1)} \\ -\bar{a}_1 & -\bar{\mathbf{a}}_{1 \times (n-1)} \end{bmatrix}, \bar{\mathbf{a}} = [-\bar{a}_2 \quad -\bar{a}_3 \quad \dots \quad -\bar{a}_n] \\ \bar{\mathbf{b}} &= \begin{bmatrix} \mathbf{0}_{(n-1) \times 1} \\ 1 \end{bmatrix}, \bar{\mathbf{d}} = [\bar{d}_1 \quad \bar{d}_2 \quad \dots \quad \bar{d}_m] \end{aligned}$$

$\bar{a}_i, i = 1, \dots, n; \bar{d}_j, j = 1, \dots, m; m \leq n$ - the objects parameters.

Let us adopt as new the state coordinates of the system in Fig.1. :

$x_1 = e$ - the system error

$x_i = e^{(i)}$ - the i-th derivative of the system error

which can be expressed by the following relations :

$$\begin{aligned} x_1 &= e = r - y = -\bar{\mathbf{d}}\bar{\mathbf{x}} \\ x_i = e^{(i)} &= -y^{(i)} = -\bar{\mathbf{d}}\bar{\mathbf{A}}^{i-1}\bar{\mathbf{x}} - \sum_{j=1}^{n-1} \bar{\mathbf{d}}\bar{\mathbf{A}}^{m-j-1}\bar{\mathbf{b}}u^{(j-1)}; \quad i = 1, \dots, n \end{aligned}$$

or

$$\dot{\mathbf{x}} = -\mathbf{Q}\bar{\mathbf{x}} - \mathbf{E}u \tag{2}$$

where:

$$\mathbf{x} = [x_1 \quad x_2 \quad \dots \quad x_n]^T, \quad \mathbf{u} = [u^{(1)} \quad u^{(2)} \quad \dots \quad u^{(m)}]$$

$$\mathbf{Q} = \begin{bmatrix} \bar{\mathbf{d}} \\ \bar{\mathbf{dA}} \\ \dots \\ \bar{\mathbf{dA}}^{n-1} \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \bar{\mathbf{dB}} & 0 & \dots & 0 \\ \bar{\mathbf{dAb}} & \bar{\mathbf{dB}} & \dots & \dots \\ \bar{\mathbf{dA}}^{m-2}\bar{\mathbf{b}} & \bar{\mathbf{dA}}^{m-3}\bar{\mathbf{b}} & \dots & \bar{\mathbf{dB}} \end{bmatrix}$$

Taking into consideration relations (2), expressions for \mathbf{x} and \dot{x}_n assume the forms :

$$\bar{\mathbf{x}} = -\mathbf{Q}^{-1} \mathbf{x} - \mathbf{EQ}^{-1} \mathbf{u}$$

$$\dot{x}_n = \bar{\mathbf{dA}}^n \mathbf{Q}^{-1} \mathbf{x} + \bar{\mathbf{dA}}^{n-1} \mathbf{EQ}^{-1} \mathbf{u} - \sum_{j=1}^{m-1} \bar{\mathbf{dA}}^{m-j-1} u^{(j)}$$

The mathematical model of the system in the new state coordinates system has the following form :

$$\begin{aligned} \dot{x}_i &= x_{i+1} \quad i = 1, 2, \dots, n-1 \\ \dot{x}_n &= \bar{\mathbf{dA}}^n \mathbf{Q}^{-1} \mathbf{x} + \bar{\mathbf{dA}}^{n-1} \mathbf{EQ}^{-1} \mathbf{u} - \sum_{j=1}^{m-1} \bar{\mathbf{dA}}^{m-j-1} u^{(j)} \\ \Leftrightarrow \dot{x}_i &= x_{i+1} \quad i = 1, 2, \dots, n-1 \\ \dot{x}_n &= -\sum_{i=1}^n a_i x_i - \sum_{j=1}^m b_j u^{(j-1)}; \quad a_i = \bar{a}_i; b_j = \bar{b}_j \end{aligned} \quad (3)$$

Based on relations (1) i (3), it can be concluded that the transformation matrix for transition from the mathematical model of the plant to the mathematical model of the system in the new state coordinate system is the unit matrix. From the practical point of view it means that the system error signal and its differential are not accessible for measuring purposes, the corresponding object state coordinates can be applied. If the required object state coordinates are not accessible for immediate measuring, some of the known methods for the object state reconstruction (observer) can be applied.

We should use the method of eliminating finite zeros influence by introducing additional analogous filters (aperiodical elements of the first order) of adequate time constants, proposed by Kostyleva [1]. Mathematical model of the given system with additional filters (Fig.1) was given in the next form [1, 2]¹:

¹ To complete the paper, a method of obtaining this mathematical model, taken over from [2], is given in Appendix 1.

$$\begin{aligned} \dot{x}_i &= x_{i+1} \quad i = 1, 2, \dots, n-1; \\ \dot{x}_n &= -\sum_{i=1}^n a_i x_i - \sum_{j=1}^{m-2} \left(\sum_{i=j}^{m-1} b_{i+1} A_{j,i} \right) z_{m-1-j} - \\ &\quad \sum_{i=1}^{m-1} (b_{i+1} A_{0,i} + b_1) z_{m-1} - b_m A_{m-1,m-1} z_0; \end{aligned} \quad (4)$$

$$\begin{aligned} A_{j,i+1} &= A_{j-1,i} \frac{1}{T_{m-j}} - A_{j,i} \frac{1}{T_{m-j-1}}; \\ A_{0,1} &= -\frac{1}{T_{m-1}}, \quad A_{1,1} = \frac{1}{T_{m-1}}, \quad A_{-1,i} = 0, \quad A_{i+1,i} = 0; \end{aligned} \quad (5)$$

$$z_0 = \sum_{i=1}^{n-1} \Psi_i x_i(kT) + \sum_{i=1}^{m-1} \Phi_i z_i(kT); \quad (6)$$

$$\Psi_i = \begin{cases} \omega_i & \text{for } x_i(kT_S)g(kT_S) > 0 \\ \lambda_i & \text{for } x_i(kT_S)g(kT_S) \leq 0 \end{cases}, \quad T_S = \frac{T}{d}, T \gg T_S \rightarrow 0, d \in \mathbf{N}; \quad (7)$$

$$\Phi_i = \begin{cases} \gamma_i & \text{for } z_i(kT_S)g(kT_S) > 0 \\ \mu_i & \text{for } z_i(kT_S)g(kT_S) \leq 0 \end{cases} \quad (8)$$

$$g(kT_S) = \sum_{i=1}^n c_i x_i(kT_S), c_i = \text{const} > 0, i = 1, 2, \dots, n-1, c_n = 1; \quad (9)$$

where :

$$\begin{aligned} a_i \min \leq a_i \leq a_i \max, \quad i = 1, 2, \dots, n; \\ b_j \min \leq b_j \leq b_j \max, \quad j = 1, 2, \dots, m; \end{aligned} \quad \text{are objects parameters.}$$

Discrete-time VSS controller parameters $c_i, \omega_i, \lambda_i, i = 1, 2, \dots, n-1$ and $\gamma_j, \mu_j, j = 1, 2, \dots, m$ - should be determined so that the zigzag (quasi-sliding) movement mode will occur in the system.

3. DETERMINING THE DISCRETE-TIME VSS CONTROLLER PARAMETERS

The discrete-time mathematical model of the given system may be obtained in this form :

$$\mathbf{x}[(k+1)T] = \mathbf{H}(a, b, T)\mathbf{x}(kT) + \mathbf{L}(a, b, T)\mathbf{z}(kT), \quad (10)$$

where :

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{1,p} & h_{1,n} \\ \mathbf{H}_{i,p} & \mathbf{H}_{i,n} \end{bmatrix}_{n,n}, \quad \begin{matrix} p = 1, 2, \dots, n-1, \\ i = 2, 3, \dots, n; \end{matrix} \quad (11)$$

$$\mathbf{L} = \begin{bmatrix} \mathbf{L}_{i,q}^1 & \mathbf{L}_{i,m-1}^2 & \mathbf{L}_{i,j}^3 \\ \mathbf{L}_{m-1,q}^1 & \mathbf{L}_{m-1,m-1}^2 & \mathbf{L}_{m-1,j}^3 \\ \mathbf{L}_{j,q}^1 & \mathbf{L}_{j,m-1}^2 & \mathbf{L}_{j,j}^3 \end{bmatrix}_{n,n}, \quad \begin{matrix} i = 1, 2, \dots, m-2 \\ j = m, m+1, \dots, n; \\ q = 1, 2, \dots, m-2 \end{matrix} \quad (12)$$

The elements of matrices H and L are given in Appendix 2.

Sufficient conditions of the zig-zag mode existence on the hyperplane $g = \mathbf{c}^T \mathbf{x} = 0$ were derived in [5,6,7] in the form:

$$\lim_{g(kT) \rightarrow 0^+} \Delta g(kT) \leq 0, \quad \lim_{g(kT) \rightarrow 0^-} \Delta g(kT) \geq 0; \quad (13)$$

$$g(kT) = \sum_{i=1}^n c_i x_i(kT) = \mathbf{c}^T \mathbf{x}(kT), \quad k = 0, 1, 2, \dots, n; \quad (14)$$

$$\Delta g(kT) = g((k+1)T) - g(kT). \quad (15)$$

Taking into consideration relations (10) and (14), expression (15) assumes the form of (16):

$$\begin{aligned} \Delta g(kT) &= \mathbf{c}^T [\mathbf{H} - \mathbf{I}] \mathbf{x}(kT) + \mathbf{c}^T \mathbf{Lz}(kT) \\ &= \mathbf{c} \mathbf{D} \mathbf{x}(kT) + \mathbf{c} \mathbf{Lz}(kT), \mathbf{I} - \text{unity matrix}; \end{aligned} \quad (16)$$

where :

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}_{1,p} & \mathbf{d}_{1,n} \\ \mathbf{D}_{i,p} & \mathbf{d}_{i,n} \end{bmatrix}_{n,n}, \quad (17)$$

$$\mathbf{d}_{1,p} = [h_{1,1} - 1, h_{1,p}]_{1,n-1}; \quad (18)$$

$$d_{1,n} = h_{1,n}; \quad (19)$$

$$\mathbf{d}_{i,n} = [h_{i,n}, h_{n,n} - 1]_{n-1,1}; \quad (20)$$

$$\mathbf{D}_{i,p} = [d_{i,p}]_{n-1,n-1}, d_{ip} = \begin{cases} h_{i,p}; & i \neq p, p = 1, 2, \dots, n-1 \\ h_{i,p} - 1; & i = p, p = 2, 3, \dots, n-1; \end{cases} \quad (21)$$

Starting from the conditions (13), taking in account (16) and (17) of the zig-zag movement mode existence for the system analyzed, the following relations are obtained:

$$[c_1 \mathbf{d}_{1,p} - c_1 \mathbf{c}_p^T d_{1,n} + \mathbf{c}_j^T \mathbf{D}_{i,p} - \mathbf{c}_j^T \mathbf{d}_{i,n} \mathbf{c}_p^T] \mathbf{x}_p(kT) \begin{cases} < 0 \text{ for } g > 0; \\ > 0 \text{ for } g < 0; \end{cases} \quad (22)$$

$$\left(\sum_{i=1}^n c_i L_{i,q}^1 \right) z_q(kT) + \left(\sum_{i=1}^n c_i L_{i,m-1}^1 \right) z_{m-1}(kT) \begin{cases} < 0 \text{ for } g > 0; \\ > 0 \text{ for } g < 0; \end{cases} \quad (23)$$

where: $\mathbf{c}_p^T = [c_1, c_2, \dots, c_{n-1}]$, $\mathbf{c}_j^T = [c_2, c_3, \dots, c_n]$, $q = 1, 2, \dots, m-2$.

Replacing relations (17) - (21) and (11), expression (23) assumes the following form:

$$\begin{aligned}
 & [(A\Psi_p (\sum_{i=1}^n c_i w^{(i-2)}) + \sum_{i=p+1}^n (\sum_{q=1}^p (c_q a_i - c_i a_q) w^{(m+i-p+q-3)} - \\
 & c_p c_{i-1} w^{(m+i-3)}) - \sum_{i=1}^{p-1} (c_p c_i w^{(m+i-2)} - c_i w^{(m+n-p+i-2)})] + \\
 & (\sum_{i=p+1}^n (\sum_{q=1}^p (c_q a_i - c_i a_q) w^{(i-p+q-2)} - c_p c_{i-1} w^{(i-2)}) - \\
 & \sum_{i=1}^{p-1} c_p c_{i-1} w^{(i-1)} + \sum_{i=1}^n c_i w^{(n-p)}) B] x(kT) \begin{cases} <0 \text{ for } q > 0 \\ >0 \text{ for } q < 0 \end{cases} \\
 & p = 1, 2, \dots, n-1.
 \end{aligned} \tag{24}$$

Taking into consideration relation (7), expression (24) can be written in the form:

$$\begin{aligned}
 \omega_p \geq \max_{a_i, \dots, a_n, b_m} & \frac{1}{A(c_1 j + \sum_{i=2}^n c_i w^{(i-2)})} \{ [\sum_{i=p+1}^n (\sum_{q=1}^p (c_q a_i - c_i a_q) w^{(m+i-p+q-3)} - \\
 & c_p c_{i-1} w^{(m+i-3)}) - \sum_{i=1}^{p-1} (c_p c_i w^{(m+i-2)} - c_i w^{(n+m-p+i-2)})] + \\
 & [\sum_{i=p+1}^n (\sum_{q=1}^p (c_q a_i - c_i a_q) w^{(i-p+q-2)} - c_p c_{i-1} w^{(i-2)}) - \sum_{i=1}^{p-1} c_p c_i w^{(i-1)} + \\
 & \sum_{i=1}^n c_i w^{(p-2)}] B \}, p = 1, 2, \dots, n-1; \\
 \lambda_p \leq \min_{a_i, \dots, a_n, b} & \frac{-1}{A(c_1 j + \sum_{i=2}^n c_i w^{(i-2)})} \{ [\sum_{i=p+1}^n (\sum_{q=1}^p (c_q a_i - c_i a_q) w^{(m+i-p+q-2)} - c_p c_{i-1} w^{(m+i-3)}) \\
 & - \sum_{i=1}^{p-1} (c_p c_i w^{(m+i-2)} - c_i w^{(n+m-p+i-2)})] + [\sum_{i=p+1}^n (\sum_{q=1}^p (c_q a_i - c_i a_q) w^{(i-p+q-2)} - c_p c_{i-1} w^{(i-2)}) \\
 & - \sum_{i=1}^{p-1} c_p c_i w^{(i-1)} + \sum_{i=1}^n c_i w^{(p-2)}] B \}, p = 1, 2, \dots, n-1.
 \end{aligned}$$

Replacing (12), expression (23) assumes form :

$$\begin{aligned}
 & [\sum_{i=1}^n c_i (B_{i,k} + \Phi_q E_{i,0})] z_q(kT) + \\
 & [\sum_{i=1}^n c_i (C_{i,m-1} + \Phi_{m-1} E_{i,0})] z_{m-1}(kT) \begin{cases} <0 \text{ for } g > 0 \\ >0 \text{ for } g < 0 \end{cases}
 \end{aligned} \tag{25}$$

Taking into consideration relation (8), expression (25) can be written in the form :

$$\gamma_q \geq \max_{b_1, \dots, b_m} \left(\frac{\sum_{i=1}^n c_i B_{i,q}}{\sum_{i=1}^n c_i E_{i,0}} \right) = \max_{b_1, \dots, b_m} \left(\frac{\sum_{i=1}^n c_i [D_{z_q}^1 - (\sum_{i=q}^{m-1} b_{i+1} A_{q,i}) (w^{(i+m-3)} + D_{z_q}^2)]}{\sum_{i=1}^n c_i [D_{z_0}^1 - b_m A_{m-1, m-1} (w^{(i+m-3)} + D_{z_0}^2)]} \right), q = 1, 2, \dots, m-2;$$

$$\mu_q \leq \min_{b_1, \dots, b_m} \left(\frac{\sum_{i=1}^n c_i B_{i,q}}{\sum_{i=1}^n c_i E_{i,0}} \right) = \min_{b_1, \dots, b_m} \left(\frac{\sum_{i=1}^n c_i [D_{z_q}^1 - (\sum_{i=q}^{m-1} b_{i+1} A_{q,i}) (w^{(i+m-3)} + D_{z_q}^2)]}{\sum_{i=1}^n c_i [D_{z_0}^1 - b_m A_{m-1, m-1} (w^{(i+m-3)} + D_{z_0}^2)]} \right), q = 1, 2, \dots, m-2;$$

$$\begin{aligned} \gamma_{m-1} &\geq \max_{b_1, \dots, b_m} \left(\frac{\sum_{i=1}^n c_i C_{i, m-1}}{\sum_{i=1}^n c_i E_{i,0}} \right) = \\ &\max_{b_1, \dots, b_m} \left(\frac{\sum_{i=1}^n c_i [-(b_1 + \sum_{j=1}^{m-1} b_{j+1} A_{0,j}) (w^{(i+m-3)} + D_{z_{m-1}}^1)]}{\sum_{i=1}^n c_i [D_{z_0}^1 - b_m A_{m-1, m-1} (w^{(i+m-3)} + D_{z_0}^2)]} \right); \end{aligned}$$

$$\begin{aligned} \mu_{m-1} &\leq \max_{b_1, \dots, b_m} \left(\frac{\sum_{i=1}^n c_i C_{i, m-1}}{\sum_{i=1}^n c_i E_{i,0}} \right) = \\ &\max_{b_1, \dots, b_m} \left(\frac{\sum_{i=1}^n c_i [-(b_1 + \sum_{j=1}^{m-1} b_{j+1} A_{0,j}) (w^{(i+m-3)} + D_{z_{m-1}}^1)]}{\sum_{i=1}^n c_i [D_{z_0}^1 - b_m A_{m-1, m-1} (w^{(i+m-3)} + D_{z_0}^2)]} \right); \end{aligned}$$

4. ILLUSTRATIVE EXAMPLE

For the purpose of verifying the relations obtained for the discrete-time VSS regulator synthesis, a digital regulation of a dc motor rotor current was simulated. A dc motor of the following characteristics was taken as an example :

$$K_y = 2.12 \text{ Vs/rad}, T_{meh} = 23\text{ms}, 0.575 \text{ kgm}^2 \leq J \leq 2.3 \text{ kgm}^2;$$

$$K_t = 2.12 \text{ Nm/A}, T_{el} = 25.5\text{ms}, 0.05 \text{ Nms/rad} \leq F \leq 0.1 \text{ Nms/rad}.$$

Starting from the basic relations for the dc motor, the following transfer function is obtained :

$$W(s) = \frac{I_R(s)}{U_R(s)} = \frac{(T_{meh} / R)s + (T_{meh} / R)(F / J)}{T_{meh} T_{el} s^2 + (T_{meh} + T_{meh} T_{el} F / J)s + 1} = \frac{1.15s + 0.1}{0.002645s^2 + 0.11523s + 4.5}$$

If discretization of the sampling period $T = 5 \text{ ms}$ was introduced and the influence of finite zero is eliminated by introducing time constant of filter $T_1 = 11.5 \text{ s}$, the following parameter values can be selected for the implementation of the discrete-time VSS controller :

$$\omega_1 = 100, \lambda_1 = -100, \gamma_1 = 0.02, \mu_1 = -0.02 \text{ and } c = 40,$$

so as to satisfy relations (22) and (23). The simulation results are presented in the form of the control signal diagram (Fig. 4.) as well as the output variable diagram (Fig. 2. and Fig. 3.). They show that the implemented discrete-time VSS controller makes the high quality process regulation possible which is reflected in: monotonous aperiodic regulation process, high speed performance and system movement invariance at the final phase of regulation from the object parameters change over the wide range.

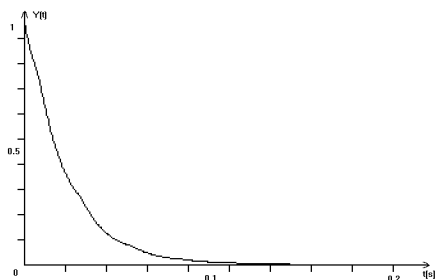


Fig. 2. Output variable diagram for $J = 1.15 \text{ kgm}^2$ and $F = 0.1 \text{ Nms/rad}$.

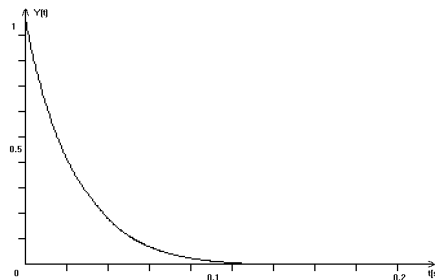


Fig. 3. Output variable diagram for $J = 2.3 \text{ kgm}^2$ and $F = 0.05 \text{ Nms/rad}$.

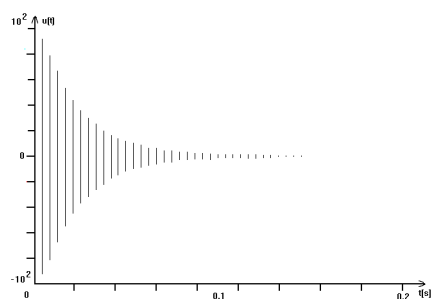


Fig. 4. Control signal

5. CONCLUSION

The conditions of the zigzag movement mode in the discrete-time VSS, when regulating the objects with finite zeros, were analyzed in this paper. For the considered discrete-time VSS, mathematical model is given and the general quasi-sliding mode existence conditions on the switching hyperplane are derived.

A variable structure system regulator synthesis procedure was presented in an illustrative example. Based on a test, a conclusion was drawn that the proposed controlling mode is possible and that it provides good results as well as that the discrete-time VSS characteristics are better than those of the corresponding conventional linear system.

6. APPENDIX

Appendix 1 [2]

The mathematical model of the given system, Fig. 1., may be given in the following initial form:

$$\begin{aligned} \dot{x}_i &= x_{i+1}, \quad i = 1, \dots, n-1, \\ \dot{x}_n &= -\sum_{i=1}^n a_i x_i - P\left(\frac{d}{dt}\right)u, \quad P\left(\frac{d}{dt}\right) = \sum_{i=0}^{m-1} b_{i+1} \frac{d^i}{dt^i}, \\ \dot{z}_i &= \frac{1}{T_i}(z_{i-1} - z_i), \quad i = 1, \dots, m-1; \\ z_0 &= v; \quad z_{m-1} = u. \end{aligned} \quad (26)$$

After simple transformations, \dot{x}_n can be written in the form :

$$\dot{x}_n = -\sum_{i=1}^n a_i x_i - \sum_{j=1}^{m-2} \left(\sum_{i=j}^{m-1} b_{i+1} A_{j,i} \right) z_{m-1-j} - \left(\sum_{i=1}^{m-1} b_{i+1} A_{0,i} + b_1 \right) u - b_m A_{m-1,m-1} v \quad (27)$$

where:

$$\begin{aligned} A_{j,i+1} &= A_{j-1,i} \frac{1}{T_{m-j}} - A_{j,i} \frac{1}{T_{m-j-1}}, \\ A_{-1,i} &= 0, \quad A_{i+1,i} = 0, \quad A_{0,1} = -\frac{1}{T_{m-1}}, \quad A_{1,1} = \frac{1}{T_{m-1}}. \end{aligned}$$

Let us prove (27). On the basis of (26) \dot{x}_n is the function of the coordinate $z_{m-1} = u$ and $(m-1)$ of its derivations. Let us express all $z_{m-1}^{(i)}$ by means of z_1, \dots, z_{m-1} .

Let

$$z_{m-1}^{(i)} = \sum_{j=0}^i A_{j,i} z_{m-1-j} \quad (28)$$

Then

$$z_{m-1}^{(i+1)} = \sum_{j=0}^i A_{j,i} \dot{z}_{m-1-j} \quad (29)$$

Taking into consideration the next to last relation in (26),

$$z_{m-1}^{(i+1)} = \sum_{j=0}^i A_{j,i} \frac{1}{T_{m-1-j}} (z_{m-2-j} - z_{m-1-j}) \quad (30)$$

is obtained from relation (29).

The following grouping of addends can be done in the expression (30) :

$$\begin{aligned} z_{m-1}^{(i+1)} &= (A_{-1,i} \frac{1}{T_m} - A_{0,i} \frac{1}{T_{m-1}}) z_{m-1} + (A_{0,i} \frac{1}{T_{m-1}} - A_{1,i} \frac{1}{T_{m-2}}) z_{m-2} + \dots \\ & (A_{i,i} \frac{1}{T_{m-i-1}} - A_{i+1,i} \frac{1}{T_{m-i-2}}) z_{m-i-2} = \sum_{j=0}^{i+1} (A_{j-1,i} \frac{1}{T_{m-j}} - A_{j,i} \frac{1}{T_{m-j-1}}) z_{m-j-1} \end{aligned} \quad (31)$$

where : $A_{-1,i} = 0, A_{i+1,i} = 0$.

Let

$$z_{m-1}^{(i+1)} = \sum_{j=0}^{i+1} A_{j,i+1} z_{m-1-j} \tag{32}$$

Then, based on the relation (28), (31) and (32) :

$$A_{j,i+1} = A_{j-1,i} \frac{1}{T_{m-j}} - A_{j,i} \frac{1}{T_{m-j-1}};$$

$$A_{-1,i} = 0, A_{i+1,i} = 0, A_{0,1} = -\frac{1}{T_{m-1}}, A_{1,1} = \frac{1}{T_{m-1}};$$

Taking into consideration the relation (26), the expression for \dot{x}_n becomes :

$$\begin{aligned} \dot{x}_n &= -\sum_{i=1}^n a_i x_i - \sum_{i=0}^{m-1} b_{i+1} \sum_{j=0}^i A_{j,i} z_{m-1-j} = -\sum_{i=1}^n a_i x_i - \sum_{i=0}^{m-1} b_{i+1} \sum_{j=0}^i A_{j,i} z_{m-1-j} - b_1 z_{m-1} \\ &= -\sum_{i=1}^n a_i x_i - b_2 (A_{0,1} z_{m-1} + A_{1,1} z_{m-2}) - b_3 (A_{0,2} z_{m-1} + A_{1,2} z_{m-2} + A_{2,2} z_{m-3}) \\ &\quad - b_m (A_{0,m-1} z_{m-1} + A_{1,m-1} z_{m-2} + \dots + A_{m-1,m-2} z_1 + A_{m-1,m-1} z_0) - b_1 z_{m-1} \end{aligned} \tag{33}$$

The following grouping of addends can be done in the expression (33) :

$$\begin{aligned} \dot{x}_n &= -\sum_{i=1}^n a_i x_i - (b_2 A_{0,1} + b_3 A_{0,2} + \dots + b_{m-1} A_{0,m-2} + b_m A_{0,m-1}) z_{m-1} \\ &\quad - (b_2 A_{1,1} + b_3 A_{1,2} + \dots + b_{m-1} A_{1,m-2} + b_m A_{1,m-2}) z_1 - b_m A_{m-1,m-1} z_0 \end{aligned}$$

Appendix 2

The mathematical induction method was used to obtain the **H** and the **L** matrix elements values (11) and (12), respectively. Successive relations for the **H** matrix and **L** matrix of the complex form for the II and III order system were derived and the **H** matrix and the **L** matrix forming rule was established for the n-th order system.

Designating with :

$$w = w(T) = w^{(0)}(T)$$

- normal pulse response of the plant with the additional filters,

$$j = j(T) = w^{(-1)}(T)$$

- normal step response of the plant with the additional filters,

$w^{(i)}(T)$ - i-th derivative of the normal pulse response of the plant with the additional filters,
matrix **H** and matrix **L** have the form :

- for the II order system (Fig. 5)

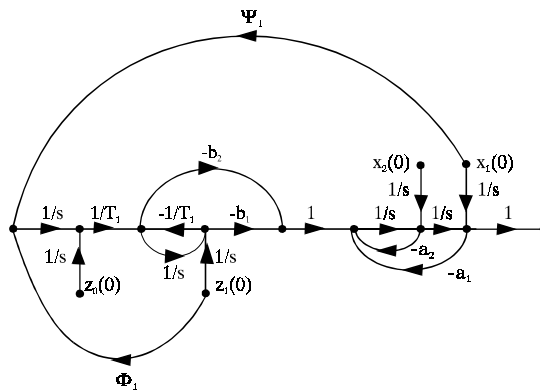


Fig. 5. Signal flow graph for the II order system

$$A_{0,1} = -\frac{1}{T_1}, \quad A_{1,1} = \frac{1}{T_1};$$

$$A = -\frac{b_1}{T_1}w - b_2 A_{1,1}w^{(1)}, \quad B = \frac{w}{T_1};$$

$$\mathbf{H} = \begin{bmatrix} A\Psi_1 j + a_2 w^{(1)} + w^{(2)} + B(a_2 w + w^{(1)}) & w^{(1)} + wB \\ A\Psi_1 w - a_1 w^{(1)} - Ba_1 w & w^{(2)} + w^{(1)}B \end{bmatrix};$$

$$\mathbf{L} = \begin{bmatrix} -(b_1 + b_2 A_{0,1})w + \Phi_1(-\frac{b_1}{T_1}j - b_2 A_{1,1}w) & 0 \\ -(b_1 + b_2 A_{0,1})w^{(1)} + \Phi_1(-\frac{b_1}{T_1}w - b_2 A_{1,1}w^{(1)}) & 0 \end{bmatrix};$$

- for the III order system (Fig. 6.)

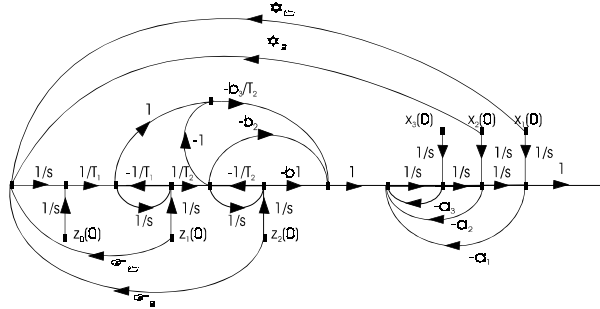


Fig. 6. Signal flow graph for the III order system

$$A_{0,1} = -\frac{1}{T_2}, \quad A_{0,2} = \frac{1}{T_2^2}, \quad A_{1,1} = \frac{1}{T_2}, \quad A_{1,2} = -\frac{1}{T_2^2} - \frac{1}{T_1 T_2}, \quad A_{2,2} = \frac{1}{T_1 T_2};$$

$$\mathbf{H} = \begin{bmatrix} A\Psi_1 j + \sum_{i=2}^4 a_i w^{(i)} + B(\sum_{i=2}^3 a_i w^{(i-2)} + w^{(2)}) & A\Psi_2 j + \sum_{i=3}^4 a_i w^{(i-1)} + B(a_3 w + w^{(1)}) & w^{(2)} + wB \\ A\Psi_1 w - a_1 w^{(2)} - Ba_1 w & A\Psi_2 w + \sum_{i=3}^4 a_i w^{(i)} + B(a_3 w^{(1)} + w^{(1)}) & w^{(3)} + w^{(1)}B \\ A\Psi_1 w^{(1)} - a_1 w^{(3)} - Ba_1 w^{(1)} & A\Psi_2 w^{(1)} - \sum_{i=1}^2 a_i w^{(i+1)} - B \sum_{i=1}^2 a_i w^{(i-1)} & w^{(4)} + w^{(2)}B \end{bmatrix};$$

$$C = b_1 A_{0,1} w - (b_2 A_{1,1} + b_3 A_{1,2}) w^{(1)} + b_3 w;$$

$$D = -(b_1 + b_2 A_{0,1} + b_3 A_{0,2})(w^{(1)} + \frac{w}{T_1});$$

$$E = \frac{b_1}{T_1} A_{0,1} j + (b_2 A_{0,1} + b_3 A_{0,2}) \frac{w}{T_1} - b_3 A_{2,2} (w^{(1)} + \frac{w}{T_2});$$

$$\mathbf{L} = \begin{bmatrix} C + \Phi_1 E & D + \Phi_2 E & 0 \\ C^{(1)} + \Phi_1 E^{(1)} & D^{(1)} + \Phi_2 E^{(1)} & 0 \\ C^{(2)} + \Phi_1 E^{(2)} & D^{(2)} + \Phi_2 E^{(2)} & 0 \end{bmatrix};$$

and for the n-th order system, the \mathbf{H} matrix elements values (11) of the general form are :

$$h_{1,p} = [\dots A \Psi_p j + \sum_{i=p+1}^{n+1} a_i w^{(m+i-p-2)} + B(\sum_{i=p+1}^n a_i w^{(i-p-1)} + w^{(n-p)}) \dots];$$

$$h_{1,n} = w(w^{(m-1)} + B);$$

$$H_{i,p} = \begin{bmatrix} [h_{q,p}]_{p-1,p} \\ [h_{z,p}]_{n-p,p} \end{bmatrix}_{n-1,n-1} \quad \begin{matrix} p = 1, 2, \dots, n-1 \\ q = 2, 3, \dots, p \\ z = p+1, p+2, \dots, n \end{matrix};$$

$$h_{q,p} = [\dots A \Psi_p w^{(q-2)} + \sum_{i=p+1}^{n+1} a_i w^{(m+i-p+q-3)} + B(\sum_{i=p+1}^n a_i w^{(i-p+q-2)} + w^{(n-p)}) \dots];$$

$$h_{z,p} = [\dots A \Psi_p w^{(z-2)} - \sum_{i=1}^p a_i w^{(m+i-p+z-3)} - B \sum_{i=1}^p a_i w^{(i-p+z-2)} \dots];$$

$$H_{i,n} = [\dots w^{(i-1)}(w^{(m-1)} + B) \dots];$$

$$A = -b_1 \frac{1}{\prod_{i=1}^{m-1} T_i} w - \sum_{i=1}^{m-1} b_{i+1} A_{0,i} \frac{1}{\prod_{j=1}^{m-2} T_j} w^{(1)} - \sum_{l=2}^{m-2} \left\{ \left(\sum_{i=l}^{m-1} b_{i+1} \sum_{j=l}^i A_{j,i} \right) \frac{1}{\prod_{k=1}^{m-l-1} T_k} \right.$$

$$\left. [w^{(l)} + \sum_{r=2}^l \left(\sum_{d_1=1}^{l-r+1} \sum_{d_2=d_1+1}^{l-r} \dots \sum_{d_{r-1}=d_{r-2}+1}^{l-1} \prod_{q=1}^{r-1} T_{m-d_q} \right) \frac{w^{(l-r+1)}}{r-1} \right] \} -$$

$$-b_m A_{m-1,m-1} [w^{(m-1)} + \sum_{r=2}^{m-1} \left(\sum_{d_1=1}^{m-r} \sum_{d_2=d_1+1}^{m-r+1} \dots \sum_{d_{r-1}=d_{r-2}+1}^{m-2} \prod_{q=1}^{r-1} T_{m-d_q} \right) \frac{w^{(m-r)}}{r-1}];$$

$$B = \sum_{r=1}^{m-1} \left(\sum_{d_1=1}^{m-r} \sum_{d_2=d_1+1}^{m-r+1} \dots \sum_{d_r=d_{r-2}+1}^{m-1} \prod_{q=1}^r T_{d_q} \right) \frac{w^{(m-r-1)}}{r}, \quad d_0 = 0;$$

and the \mathbf{L} matrix elements values (12) of the general form are :

$$L_{s,q}^1 = B_{s,q} + \Phi_q E_{s,0};$$

$$L_{s,m-1}^2 = C_{s,m-1} + \Phi_{m-1} E_{s,0}, \quad s = 1, 2, \dots, n;$$

$$L_{s,j}^3 = 0;$$

$$E_{s,0} = D_{z_0}^1 - b_m A_{m-1,m-1} (w^{(s+m-3)} + D_{z_0}^2);$$

$$\begin{aligned}
B_{s,q} &= D_{z_q}^1 - \left(\sum_{i=q}^{m-1} b_{i+1} A_{q,i} \right) (w^{(s+m-3)} + D_{z_k}^2); \\
C_{s,m-1} &= -(b_1 + \sum_{i=1}^{m-1} b_{i+1} A_{0,i}) (w^{(s+m-3)} + D_{z_{m-1}}^1); \\
D_{z_0}^1 &= b_1 \frac{1}{\prod_{i=1}^{m-1} T_i} w^{(s-2)} + \sum_{i=1}^{m-1} b_{i+1} A_{0,i} \frac{1}{\prod_{j=1}^{m-2} T_j} w^{(s-1)} - \sum_{l=2}^{m-2} \left\{ \left(\sum_{i=l}^{m-1} b_{i+1} \sum_{j=l}^i A_{j,i} \right) \frac{1}{\prod_{k=1}^{m-l-1} T_k} \right. \\
&\quad \left. [w^{(l+s-2)} + \sum_{p=2}^l \left(\sum_{d_1=1}^{l-p} \sum_{d_2=d_1+1}^{l-p+1} \dots \sum_{d_{p-1}=d_{p-2}+1}^{l-1} \frac{w^{(l+s-p-1)}}{\prod_{q=1}^{p-1} T_{m-d_q}} \right)] \right\}; \\
D_{z_0}^1 &= b_1 \frac{1}{\prod_{i=1}^{m-1} T_i} w^{(s-2)} + \sum_{i=1}^{m-1} b_{i+1} A_{0,i} \frac{1}{\prod_{j=1}^{m-2} T_j} w^{(s-1)} - \sum_{l=2}^{m-2} \left\{ \left(\sum_{i=l}^{m-1} b_{i+1} \sum_{j=l}^i A_{j,i} \right) \frac{1}{\prod_{k=1}^{m-l-1} T_k} \right. \\
&\quad \left. [w^{(l+s-2)} + \sum_{p=2}^l \left(\sum_{d_1=1}^{l-p} \sum_{d_2=d_1+1}^{l-p+1} \dots \sum_{d_{p-1}=d_{p-2}+1}^{l-1} \frac{w^{(l+s-p-1)}}{\prod_{q=1}^{p-1} T_{m-d_q}} \right)] \right\}; \\
D_{z_0}^2 &= \sum_{p=2}^{m-1} \left(\sum_{d_1=1}^{m-p} \sum_{d_2=d_1+1}^{m-p+1} \dots \sum_{d_{p-1}=d_{p-2}+1}^{m-1} \frac{w^{(m+s-p-2)}}{\prod_{q=1}^{p-1} T_{m-d_q}} \right); \\
D_{z_q}^1 &= D_{z_q}^{11} + D_{z_q}^{12} + D_{z_q}^{13}; \\
D_{z_q}^{11} &= b_1 A_{0,1} \frac{1}{\prod_{i=q+1}^{m-2} T_i} [w^{(s+q-2)} + \sum_{i=1}^{q-1} \left(\sum_{d_1=1}^{q-i+1} \sum_{d_2=d_1+1}^{q-i} \dots \sum_{d_i=d_{i-1}+1}^{q-1} \frac{w^{(q+s-i-2)}}{\prod_{j=1}^i T_{d_j}} \right) \\
&\quad + \sum_{i=1}^{m-1} b_{i+1} A_{0,i} \frac{1}{\prod_{i=q+1}^{m-2} T_i} [w^{(s+q-1)} + \sum_{i=1}^{q-1} \left(\sum_{d_1=1}^{q-i+1} \sum_{d_2=d_1+1}^{q-i+2} \dots \sum_{d_i=d_{i-1}+1}^{q-1} \frac{w^{(q+s-i-2)}}{\prod_{j=1}^i T_{d_j}} \right)]; \\
D_{z_q}^{11} &= b_1 A_{0,1} \frac{1}{\prod_{i=q+1}^{m-2} T_i} [w^{(s+q-2)} + \sum_{i=1}^{q-1} \left(\sum_{d_1=1}^{q-i+1} \sum_{d_2=d_1+1}^{q-i} \dots \sum_{d_i=d_{i-1}+1}^{q-1} \frac{w^{(q+s-i-2)}}{\prod_{j=1}^i T_{d_j}} \right) + \\
&\quad + \sum_{i=1}^{m-1} b_{i+1} A_{0,i} \frac{1}{\prod_{i=q+1}^{m-2} T_i} [w^{(s+q-1)} + \sum_{i=1}^{q-1} \left(\sum_{d_1=1}^{q-i+1} \sum_{d_2=d_1+1}^{q-i+2} \dots \sum_{d_i=d_{i-1}+1}^{q-1} \frac{w^{(q+s-i-2)}}{\prod_{j=1}^i T_{d_j}} \right)];
\end{aligned}$$

$$\begin{aligned}
D_{z_q}^{12} = & - \sum_{l=2}^{m-q-2} \left(\sum_{i=l}^{m-1} b_{i+1} \sum_{j=l}^i A_{j,i} \right) \frac{1}{\prod_{r=k+1}^{m-l-1} T_r} \{ w^{(s-l+1)} + \sum_{p=1}^{q-1} \left(\sum_{d_1=1}^{q-p+1} \sum_{d_2=d_1+1}^{q-p+2} \dots \sum_{d_p=d_{p-1}+1}^{q-1} \frac{w^{(s+l+1-p)}}{\prod_{i=1}^p T_{d_i}} \right) + \\
& + \sum_{p=1}^{q-1} \left[\sum_{r=1}^{q-1} \left(\sum_{d_1=1}^{q-r+1} \sum_{d_2=d_1+1}^{q-r+2} \dots \sum_{d_r=d_{r-1}+1}^{q-1} \left(\sum_{e_1=1}^{q-p+1} \sum_{e_2=e_1+1}^{q-p+2} \dots \sum_{e_p=e_{p-1}+1}^{q-1} \frac{w^{(s+l-p-r+1)}}{\prod_{i=1}^r T_{d_i} \prod_{j=1}^p T_{m-e_j}} \right) \right) \right. \\
& \left. + \sum_{f_1=1}^{q-p+1} \sum_{f_2=f_1+1}^{q-p+2} \dots \sum_{f_p=f_{p-1}+1}^{k-1} \frac{w^{(s+l-p+1)}}{\prod_{r=1}^p T_{m-f_r}} \right] \}; \\
D_{z_q}^{13} = & - \left(\sum_{i=m-q}^{m-1} b_{i+1} \sum_{j=m-k}^i A_{j,i} \right) \{ w^{(s+m-4)} + \sum_{i=1}^{q-1} \left(\sum_{d_1=1}^{q-i+1} \sum_{d_2=d_1+1}^{q-i+2} \dots \sum_{d_i=d_{i-1}+1}^{q-1} \frac{w^{(s+m-q-i+1)}}{T_{q+1} \prod_{j=1}^i T_{d_j}} \right) + \\
& + \sum_{p=2}^{m-q} \left[\sum_{r=1}^{q-1} \left(\sum_{d_1=1}^{q-r+1} \sum_{d_2=d_1+1}^{q-r+2} \dots \sum_{d_r=d_{r-1}+1}^{q-1} \left(\sum_{e_1=1}^{m-q-p} \sum_{e_2=e_1+1}^{m-q-p+1} \dots \sum_{e_{p-1}=e_{p-2}+1}^{k-1} \frac{w^{(s+m-p-r+3)}}{T_q \prod_{i=1}^r T_{d_i} \prod_{j=1}^{p-1} T_{m-e_j}} \right) \right) \right] \}; \\
D_{z_q}^2 = & \sum_{i=1}^{q-1} \left(\sum_{d_1=1}^{q-i+1} \sum_{d_2=d_1+1}^{q-i+2} \dots \sum_{d_i=d_{i-1}+1}^{q-1} \frac{w^{(s+m-q-i)}}{\prod_{j=1}^i T_{d_j}} \right) + \\
& + \sum_{p=2}^{m-q-1} \left[\sum_{r=1}^{q-1} \left(\sum_{d_1=1}^{q-r+1} \sum_{d_2=d_1+1}^{q-r+2} \dots \sum_{d_q=d_{r-1}+1}^{q-1} \left(\sum_{e_1=1}^{q-p+1} \sum_{e_2=e_1+1}^{q-p+2} \dots \sum_{e_{p-1}=e_{p-2}+1}^{q-1} \frac{w^{(s+m-p-r-2)}}{\prod_{i=1}^r T_{d_i} \prod_{j=1}^{p-1} T_{m-e_j}} \right) \right) \right. \\
& \left. + \sum_{f_1=1}^{q-p+1} \sum_{f_2=f_1+1}^{q-p+2} \dots \sum_{f_{p-1}=f_{p-2}+1}^{q-1} \frac{w^{(s+m-p-2)}}{\prod_{j=1}^{p-1} T_{m-e_j}} \right] \}; \\
D_{z_{m-1}}^1 = & \sum_{p=1}^{m-2} \left(\sum_{d_1=1}^{m-p-1} \sum_{d_2=d_1+1}^{m-p} \dots \sum_{d_i=d_{i-1}+1}^{m-2} \frac{w^{(s+m-p-3)}}{\prod_{j=1}^p T_{d_j}} \right); d_0 = 0, e_0 = 0, f_0 = 0.
\end{aligned}$$

REFERENCES

1. E. Kostyleva: *Variable structure automatic control systems for plants with finite zeros*, Automatic systems theory and application, Nauka publ., pp. 74 - 78, 1964 (in Russian)
2. V. Emel'yanov, et. al.: *Variable Structure Control Systems Theory*, Nauka publ., 1970 (in Russian)
3. I. Utkin: *Sliding Modes in Optimization and Control*, Nauka publ., 1981 (in Russian)
4. Dote, et. al.: *Microprocessor-based force control for manipulator using variable structure with sliding*

- mode*, Proc. IFAC Conference: Control in Power Electronics and Electrical Drives, Lausanne, Switzerland, 1983, pp. 145-149
5. Č. Milosavljević: *General conditions for the existence of a quasi-sliding mode on the switching hyperplane in discrete variable structure systems*, Automat. & Remote Contr, 46 (1985), 307-314.
 6. Č. Milosavljević: *On one class of discrete-time variable structure control systems*, International Journal of Control and Computers, 46 (1988), no. 13, 56 - 60.
 7. Č. Milosavljević, B. Peruničić: *General discrete-time quasi-sliding mode existence conditions*, IEEE Workshop on variable structure and Lyapunov control of uncertain dynamical systems, (preprint) 7. - 9. September 1992, Sheffield
 8. Furuta: *Sliding mode control of a discrete system*, Syst. and Contr. Letters, 14 (1990), 145-152, North-Holland
 9. Y. Chan: *Servo-systems with discrete-variable structure control*, Syst. and Contr. Letters, 17 (1991) 321-325, North-Holland
 10. Sira-Ramirez: *Non-linear variable structure systems in quasi-sliding mode*, Int. J. Control, 54 (1991), no 5, 1171-1187
 11. Bartolini, T. Tolezzi: *The VSS approach to the model reference control of nonminimum phase linear plants*, IEEE trans. Autom. Control, 33 (1988), no 9, 859-863.
 12. J. Wang and G. H. Wu: *Variable structure control design on discrete-time systems from another viewpoint*, Control Theory and Advanced Technology, 8 (1992), no. 1, 1-16
 13. Gao: *Discrete-time variable structure systems*, IEEE Trans. Ind. Electronics, 42 (1995), no. 2, 117-122.

SINTEZA DISKRETNOG REGULATORA PROMENLJIVE STRUKTURE ZA OBJEKTE UPRAVLJANJA SA KONAČNIM NULAMA U FUNKCIJI PRENOSA

Olivera Iskrenović-Momčilović

Rad razmatra upravljanje objektima n -tog reda sa konačnim nulama u funkciji prenosa. Upravljački signal se formira na osnovu diskretnih vrednosti signala greške i njegovih izvoda korišćenjem algoritma upravljanja promenljivom strukturom. Zakonitost promene strukture je realizovana na bazi malog perioda odabiranja, koji se zanemaruje. Analizira se samo sistem tipa regulatora. Dat je matematički model za odgovarajući diskretni sistem promenljive strukture i izvedeni su opšti uslovi egzistencije kvazi-kliznog režima na kliznoj hiperpovršini.