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APPLICATION OF PSEUDO-DERIVATIVE FEEDBACK IN INDUSTRIAL ROBOTS CONTROLLERS

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Abstract. *Concept of pseudo derivative feedback control, introduced by R. M. Phelan, is applied for efficient and practical control of industrial robots. It is shown in the paper that this way of control is very simple and easily applicable, since the IR d.c. motors contain tachometers. The obtained results show that even with the significantly smaller degree of the loading torque reduction, the fast aperiodic response of the system is ensured.*

1. THEORETICAL BACKGROUND

Although the problem of robot control driven by d.c. motors was a subject of extensive studies, [1]–[5], current industrial practice for the control of robotic manipulators is application of independent servo loops for each joint. This approach, because of its simplicity and easy application of digital control, will still be prevailing in industrial application. Also, some improvements are still possible in this approach to control of industrial robots.

The procedure of determination of the transfer function for the d.c. motor controlled by the rotor's current is known from the literature, so here is its final form:

$$G(s)=\frac{\theta_m(s)}{U(s)}=\frac{K_i}{s[(Ls+R)(J_{ef}s+B_{ef})+K_bK_i]} \tag{1}$$

where:

L - is the rotor winding inductance; R - is the rotor winding resistance;
K_b - is the coefficient of the back electro-motor force;
K_i - is the torque constant.

The block - diagram of the position servo-mechanism for control of the IR segment's position by application of the d.c. motors controlled by rotor's current, with taking into account the effect of the disturbance torque M_p is shown in Figure 1.

Here θ_d denotes the desired value of the segment's angular position, and θ_l denotes the real angular position. The coefficient K_θ [V/rad] represents the potentiometer's conversion constant. $G_C(s)$ denotes the controller's transfer function.

Considering that the motors that are used for the IR segment's drive have relatively large power, the inductance of the rotor's winding can be considered as negligibly small with respect to the value of the effective moment of inertia J_{ef}. Then, the motor transfer function gets the form:

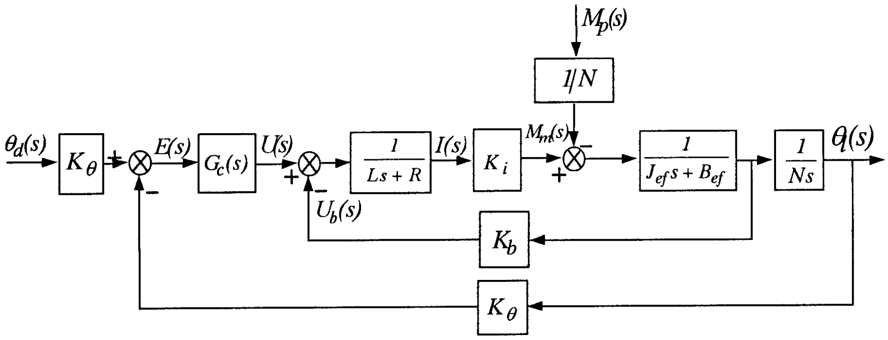

Fig. 1. Block diagram of the servo-mechanism for control of the IR segment's position.

$$G(s) = \frac{\theta_m(s)}{U(s)} = \frac{K_i}{s(RJ_{ef}s + RB_{ef} + K_b K_i)} = \frac{K}{s(Ts+1)} \tag{2}$$

where the gain and the time constant are

$$K = \frac{K_i}{RB_{ef} + K_b K_i}, \quad T = \frac{RJ_{ef}}{RB_{ef} + K_b K_i} \tag{3}$$

In order to realize the influence of the effective moment of inertia J_{ef} and effective coefficient of the viscous friction B_{ef} on static and dynamic characteristics of the system, the block diagram of Figure 3 can be simplified as shown in Figure 2.

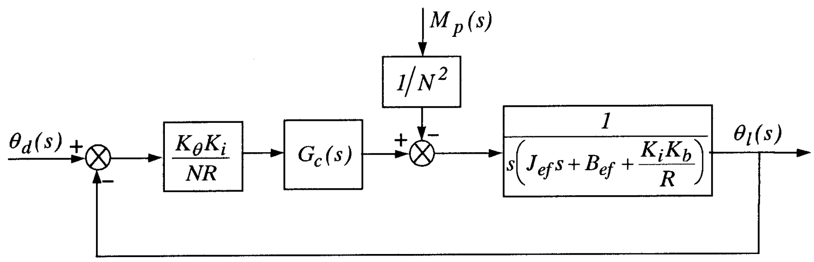

Fig. 2. The simplified block diagram of the servo-mechanism for control of the IR segment's position

2. Specifics of Industrial Robots' Dynamics

Industrial robots' control incorporates some specific characteristics that can be noticed by analysis of dynamic equations of motion. The general form of those equations is:

$$H(\theta)\ddot{\theta} + C(\theta,\dot{\theta}) + G(\theta) = M \tag{4}$$

where:

- H(θ) - is the n x n matrix of inertial coefficients (n is the number of IR degrees of freedom);
- $C(\theta,\dot{\theta})$ - is the n x 1 vector of Coriolis and centrifugal forces;
- $G(\theta)$ - is the n x 1 vector that represents the action of gravitational forces;
- M - is the n x 1 vector of driving torque of motors, reduced to shafts of the driven segments.

It can be seen that the IR's dynamic equations of motion are nonlinear and coupled. It is obvious that control, that would take into account all dynamic characteristics of IR, ought to be extremely complex. The methods of control, to be considered here, belong to the group of linear control systems. Strictly speaking, the application of linear control models is valid only if the considered system is mathematically modeled by linear differential equations. Thus, the application of linear control systems, for the case of IR control, basically represents the approximate method considering the noticed nonlinearity of dynamic equations of motion.

In order to define the necessary approximations, we shall consider the dynamic equation of the i-th segment:

$$\sum_{j=1}^{n} H_{ij}\ddot{\theta}_j + \sum_{j=1}^{n}\sum_{k=1}^{n} C_{ijk}\dot{\theta}_j\dot{\theta}_k + G_i = M_i \tag{5}$$

where:

$H_{ij} = H_{ij}(\theta_{k+1},...,\theta_n)$; $k = \min(i,j)$ are the terms of the inertial coefficients matrix, and

$C_{ijk} = \frac{1}{2}\left(\frac{\partial H_{ij}}{\partial\theta_k} + \frac{\partial H_{ki}}{\partial\theta_j} + \frac{\partial H_{jk}}{\partial\theta_i}\right)$ are the Christoffel's symbols of the first kind.

In this model, presented in Figure 4, in the effective moment of inertia J_{ef} is contained the term J_i, which actually represents the term $H_{ii} = H_{ii}(\theta_{i+1},...,\theta_n) \neq const$. This is why J_{ef} varies, during motion of robot, between some minimum and maximum values, that are functions of the robot's configuration and instantaneous position of its segments. The variation of the effective moment of inertia during IR motion must be taken into account in design of the control system.

All other terms present in equation (5), that represent the mutual coupling of the IR segments' motion, are going to be treated as the disturbance. This statement enables to reduce one basically MIMO (Multi Input Multi Output) system of automatic control to the problem of n independent controls in n joints. It should be mentioned that, due to simplicity of the control structures, which such an approach allows for, it is still the most frequently applied in IR that are currently in operation [1].

However, performances of an IR controlled by some of the simpler control structures (like for instance the PID control law), are not easy to define. Considering that a structure was not chosen that would enable decoupling of the system, the motion of each segment is also affected by other segments. These interactions lead to errors in control, but they can be suppressed, to a certain degree, due to the presence of the feedback. Of course, in order for this suppressing to be satisfactory, the gains of the applied control law have to be chosen correctly.

Another deficiency is also the previously mentioned fact that the effective moment of inertia J_{ef} is not constant. Thus, it is not possible to choose the constant gains, that would provide for the desired constant value of the coefficient of the relative damping ζ. As the existence of the overshoot during the motion of the IR segments is undesirable, since it can lead to contact of the IR with some objects in its environment, the tendency is always for the response to be critically or over critically damped. Since the minimal value of the relative damping coefficient appears when $J_{ef} = J_{ef\,max}$, the values of gains are calculated in such way that the response is critically damped with respect to $J_{ef\,max}$. In this way, for the smaller values of the effective moment of inertia, it is ensured that the value of the relative damping factor is greater than unity, namely, the desired aperiodic response is ensured.

Another factor that should be taken into account is the value of the system's undamped natural frequency ω_n. It must be at least twice smaller than the joint's structural resonant frequency ω_r., which in IR can be extremely small (4 - 5 Hz).

The action of disturbance, that is the consequence of the mutual dynamic coupling of segments, can be divided into four components, different by their characters. From the dynamic equation of motion of the *i*-th segment (5), and in accordance with the previously stated treatment of individual terms of the equation, the disturbance torque, that acts upon the *i*-th segment, is of the form:

$$M_{Pi} = \sum_{\substack{j=1 \\ j\neq i}}^{n} H_{ij}\ddot{\theta}_j + \sum_{j=1}^{n} C_{ijj}\dot{\theta}_j^2 + \sum_{\substack{j=1 \\ j\neq k}}^{n}\sum_{k=1}^{n} C_{ijk}\dot{\theta}_j\dot{\theta}_k + G_i \tag{6}$$

The first component of the disturbance torque represents the inertial forces, and it is the function of inertial coefficients and segments' accelerations.

The second and the third components of the disturbance torque represent the action of the centrifugal and Coriolis' forces, respectively. They are functions of position and velocities of segments motions. It is important to notice that in the initial phase of the robot's motion, as well as in closing of the robot to its desired position, their influence is negligible, due to small velocities of motion.

When the robot reaches the desired position, namely, after the end of motion, the first three components of the disturbance moment are equal to zero, thus only remains the action of the gravitational force. The gravitational component is a function of the segments' positions, so it changes its value during the robot's motion, but it is important to notice that when reaching the desired position of a segment (in the stationary state) it has the constant value. Influence of the gravitational forces can be dominant, depending on the robot's configuration and instantaneous positions of its segments.

Now we shall concentrate on some conventional PID control structure of IR, where, for the sake of comparison of the PID control structures and the proposed PDF control structure, some simulation results will be presented for the case of concrete IR system, shown in Figure 3.

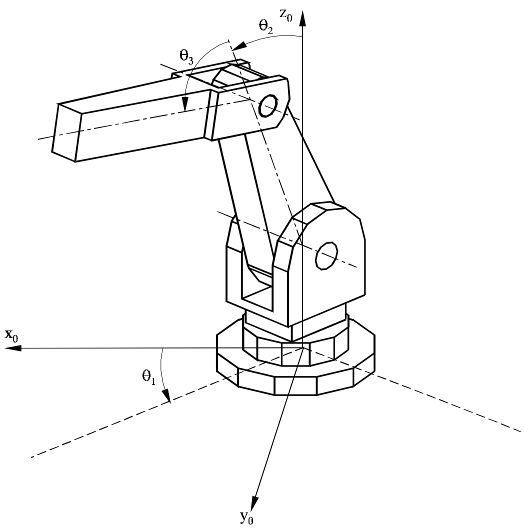

Fig. 3. The three segment industrial robot of the RzRyRy configuration.

Characteristic values of the shown robot are:

- lengths of segments	l_1= 0.75 m; l_2= 0.5 m; l_3= 0.5m
- positions of centers of masses	a_1= 0.4 m; a_2= 0.2 m; a_3= 0.2 m
- masses of segments	m_1= 2.27 kg; m_2= 15.91 kg; m_3= 6.82 kg
- moments of inertia, kgm^2 of the first segment	$J_{\xi 1}$ = 0.0194; $J_{\eta 1}$ = 0.0388; $J_{\zeta 1}$ = 0.0267
- of the second segment	$J_{\xi 2}$ = 0.01; $J_{\eta 2}$ = 3.7691; $J_{\zeta 2}$ = 3.6959
- of the third segment	$J_{\xi 3}$ = 0.0904; $J_{\eta 3}$ = 0.2245; $J_{\zeta 3}$ = 0.2842
- load mass	m_t= 2.5 kg
- reducer's transmission ratio for all the three segments	N = 40
- coefficient of the viscous friction in segments bearings	$B_{l1} = B_{l2} = B_{l3}$ = 0.2 Nms/rad

For the robot shown in Figure 5 the equations of motion are given as follows:
Dynamic equation of motion of the first segment:

$$\begin{gathered}
[J_{\zeta 1} + (m_2 a_2^2 + m_3 l_2^2 + m_t l_2^2 + J_{\xi 2}) \sin^2 \theta_2 + J_{\zeta 2} \cos^2 \theta_2 + \\
+ (J_{\xi 3} + m_3 a_3^2 + m_t l_3^2) \sin(\theta_2 + \theta_3) + J_{\zeta 3} \cos^2(\theta_2 + \theta_3) + \\
+ 2 l_2 (m_3 a_3 + m_t l_3) \sin \theta_2 \sin(\theta_2 + \theta_3)]\theta_1 + \\
[(J_{\xi 2} + m_2 a_2^2 - J_{\xi 2} + m_3 l_2^2 + m_t l_2^2 +) \sin 2\theta_2 + \\
+ 2 l_2 (m_3 a_3 + m_t l_3) \sin \theta_2 \sin(2\theta_2 + \theta_3) + \\
+ (J_{\xi 3} - J_{\varsigma 3} + m_3 a_3^2 + m_t l_3^2) \sin(2\theta_2 + 2\theta_3)]\theta_1 \theta_2 + \\
+ [(J_{\xi 3} - J_{\varsigma 3} + m_3 a_3^2 + m_t l_3^2) \sin(2\theta_2 + \theta_3) + \\
+ 2 l_2 (m_3 a_3 + m_t l_3) \sin \theta_2 \cos(2\theta_2 + \theta_3)]\theta_1 \theta_3 = M_1
\end{gathered} \tag{7}$$

Dynamic equation of motion of the second segment:

$$
\begin{aligned}
&[m_2 a_2^2 + J_{\eta 2} + m_3 a_3^2 + m_3 l_2^2 + m_t l_3^2 + J_{\eta 3} + 2 l_2 (m_3 a_3 + m_t l_3)\cos\theta_3]\ddot{\theta}_2 + \\
&+ [m_3 a_3^2 + m_t l_3^2 + J_{\eta 3} + l_2 (m_3 a_3 + m_t l_3)\cos\theta_3]\ddot{\theta}_3 - \\
&- 0.5[(J_{\xi 2} + m_2 a_2^2 - J_{\varsigma 2} + m_3 l_2^2 + m_t l_2^2]\sin 2\theta_2 + \\
&+ 2 l_2 (m_3 a_3 + m_t l_3)\sin\theta_2 \sin(2\theta_2 + \theta_3) + \\
&+ (J_{\xi 3} - J_{\varsigma 3} + m_3 a_3^2 + m_t l_3^2)\sin(2\theta_2 + 2\theta_3)]\ddot{\theta}_1^2 - \\
&- [2 l_2 (m_3 a_3 + m_t l_3)\sin\theta_3]\dot{\theta}_2\dot{\theta}_3 - [l_2 (m_3 a_3 + m_t l_3)\sin\theta_3]\dot{\theta}_3^2 - \\
&- (m_2 a_2 + m_3 l_2 + m_t l_2) g \sin\theta_2 - (m_3 a_3 + m_t l_3) g \sin(\theta_2 + \theta_3) = M_2
\end{aligned}
\tag{8}
$$

Dynamic equation of motion of the third segment:

$$
\begin{aligned}
&[m_3 a_3^2 + m_t l_3^2 + J_{\eta 3} + l_2 (m_3 a_3 + m_t l_3)\cos\theta_3]\ddot{\theta}_2 + [m_3 a_3^2 + m_t l_3^2 + J_{\eta 3}]\ddot{\theta}_3 - \\
&- 0.5[(J_{\xi 3} - J_{\varsigma 3} + m_3 a_3^2 + m_t l_3^2]\sin(2\theta_2 + \theta_3) + \\
&+ 2 l_2 (m_3 a_3 + m_t l_3)\sin\theta_2 \cos(\theta_2 + \theta_3)]\dot{\theta}_1^2 + \\
&+ [l_2 (m_3 a_3 + m_t l_3)\sin\theta_3]\dot{\theta}_2^2 - (m_3 a_3 + m_t l_3) g \sin(\theta_2 + \theta_3) = M_3
\end{aligned}
\tag{9}
$$

For the drive of the first and the third segment of IR the DC motor was chosen U9M4T, while for the drive of the second segment, which is the most exposed to influence of the gravitational forces, stronger motor was chosen of the U12M4T type. The characteristic values for the used motors are given in Table 1.

Table 1. Data for the used drive motors

Model	U9M4T	U12M4T
Moment of inertia of the rotor J_a , kgm^2	$56.484 \cdot 10^{-6}$	$233 \cdot 10^{-6}$
Coefficient of the viscous friction B_m , Nms/rad	$80.913 \cdot 10^{-6}$	$303.39 \cdot 10^{-6}$
Coefficient of torque K_i , Nm/A	0.043	0.10167
Back electro-motor force constant K_b , Vs/rad	0.04297	0.10123
Inductance of the rotor coil L, μH	100	100
Resistance of the rotor coil R, Ω	1.025	0.91
Maximum driving torque $M_{m\,max}$, Nm	1.4	2.8

According to the previously conducted analysis of equations of motion, we come up with the value for the moment of inertia J_{l1}:

$$
\begin{aligned}
J_{l_1} &= J_{\varsigma 1} + (m_2 a_2^2 + m_3 l_2^2 + m_t a_2^2 + J_{\xi 2})\sin^2\theta_2 + J_{\varsigma 1}\cos^2\theta_2 + \\
&+ (J_{\xi 3} + m_3 a_3^2 + m_t a_3^2)\sin^2(\theta_2 + \theta_3) + J_{\varsigma 3}\cos^2(\theta_2 + \theta_3) + \\
&+ 2 l_2 (m_3 a_3 + m_t l_3)\sin\theta_2 \sin(\theta_2 + \theta_3)
\end{aligned}
\tag{10}
$$

It was emphasized before that all other terms of the IR's dynamic equations of motion will be treated as the disturbing action. Considering that, for the concretely analyzed robot, expression for the disturbance torque that acts upon the first segment during its motion is of the following form:

$$\begin{aligned} M_{P1} &= [(J_{\xi 2} + m_2 a_2^2 - J_{\zeta 2} + m_3 l_2^2 + m_t l_2^2)\sin 2\theta_2 + \\ &+ 2l_2(m_3 a_3 + m_t l_3)\sin(2\theta_2 + \theta_3) + (J_{\xi 3} + m_3 a_3^2 + m_t l_3^2 - J_{\zeta 3})\sin(2\theta_2 + 2\theta_3)]\dot{\theta}_1\dot{\theta}_2 + \\ &+ [((J_{\xi 3} - J_{\zeta 3} + m_3 a_3^2 + m_t l_3^2)\sin(2\theta_2 + 2\theta_3) + \\ &+ 2l_2(m_3 a_3 + m_t l_3)\sin\theta_2 \cos(\theta_2 + \theta_3)]\dot{\theta}_1\dot{\theta}_3 ; \end{aligned} \tag{11}$$

(Expressions for the moments of inertia and the disturbance torques of the other two segments are analogous to (10) and (11), respectively, but are not given here for the sake of brevity).

3. CLASSICAL PROCEDURES AND CONTROLERS OF IRS

In literature that is dealing with the IR control problems as the most frequently applied are quoted conventional algorithms from the PID control algorithms. In [1], Luh, J.Y.S. proposes proportional control in the direct loop, modified by forming of the tachometer feedback. The deficiency of such a control structure is the fact that the system does not posses astatism with respect to the disturbing action, thus the satisfying accuracy of the system in the stationary state can be achieved only after introducing the feedforward compensation for the disturbance.

As it is known from the positive realization of control, the conventional design of controllers can not give the satisfactory control. Here it will be shown on the complete example of the controller with the PID action.

The positional servo-mechanism with the PID control is shown in Figure 4.

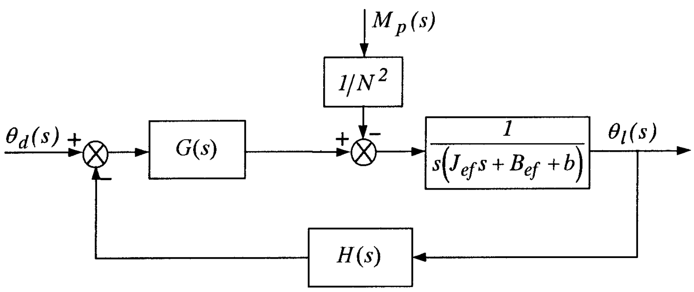

Fig. 4. Block diagram of the positional servomechanism.

The transfer function is:

$$G(s) = K_p \frac{K_I}{s} + K_D s \tag{12}$$

and the corresponding gains are:

$$K_p = \frac{K_{P1} K_i K_\theta}{NR}; \quad K_I = \frac{K_{P1} K_i K_\theta}{NR}; \quad K_D = \frac{K_{D1} K_i K_\theta}{NR}; \tag{13}$$

K_{PI}, $K_{II}$and K_{DI} denote the corresponding controllers' gains.
For the PID control is $H(s) = 1$.

The transfer function for the shown system is of the form:

$$\frac{\theta_l(s)}{\theta_d(s)} = \frac{K_D s^2 + K_P s + K_I}{J_{ef} s^3 + (B_{ef} + b + K_D) s^2 + K_P s + K_I} \tag{14}$$

and the function of the disturbance to the output is of the form:

$$\frac{\theta_l(s)}{M_P(s)} = \frac{-s^2 / N^2}{J_{ef} s^3 + (B_{ef} + b + K_D) s^2 + K_P s + K_I} \tag{15}$$

Each coefficient of control gain corresponds to one coefficient of characteristic equation, so the task of determination the values of gains is, from the aspect of satisfying the desired system performance, quite simple. However, the system transfer function (14) has the dynamics in the numerator what makes difficult the analysis of the considered system behavior. The choice of the values for gains will be done here by the transfer function poles placement method. The position of the transfer function zeros can not be influenced.

The characteristic equation of the considered transfer function reads:

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)(s + p_3) \tag{16}$$

By making equal the corresponding coefficients of the characteristic equation of the system transfer function (14) and equation (16) the following relations are obtained:

$$B_{ef} + b + K_D = J_{ef}(p_3 + 2\zeta\omega_n) \tag{17}$$

$$K_P = J_{ef}(2\zeta\omega_n p_3 + \omega_n^2) \tag{18}$$

$$K_I = J_{ef} p_3 \omega_n^2 \tag{19}$$

Let one of the requirements be that the relative damping factor for $J_{ef} = J_{efmax}$ is equal to unity. This ensures that, for the smaller values of the effective moment of inertia, the relative damping factor is greater than one, when all the three poles of the transfer function (14) are going to be real. Let us also require that the real pole p_3 distance from the imaginary axis in the s - plane be at least six times greater than those of the other two poles. Then its influence on the quality of response can be neglected, namely the pole p_3 shall not additionally damp the response. As a result three real poles are obtained $p_{1/2} = -\zeta\omega_{n1} \pm \sqrt{\zeta^2 - 1}$ i $p_3 = -6\zeta\omega_n$. In order for the position of the two dominant poles to be as distant as possible from the imaginary axis of the s - plane, i.e., in order for the response to be faster, the value of the undamped natural frequency ω_n will be increased up to the saturation limit of the executive organs.

By the computer simulation of the system behavior, the following values were obtained for undamped natural frequencies of the system, at which the executive organs are reaching the saturation limit: ω_{n1} = 0.835 rad/s; ω_{n2} = 0.79 rad/s and ω_{n3} = 1.833rad/s.

For thus set values of ζ, ω_n and position of pole p_3, based on relations (16) to (19) and (13), we come up with the needed values of the controllers gains for all the three control contours: $K_{P1} = 33.3$; $K_{P2} = 19.7$; $K_{P3} = 33.3$; $K_{I1} = 12.8$; $K_{I2} = 7.18$; K_{I3} 28.9; $K_{D1} = 22.6$; $K_{D2} = 11.1$; $K_{D3} = 9$.

The system responses and disturbing torques for application of the PID control are presented in Figure 5.

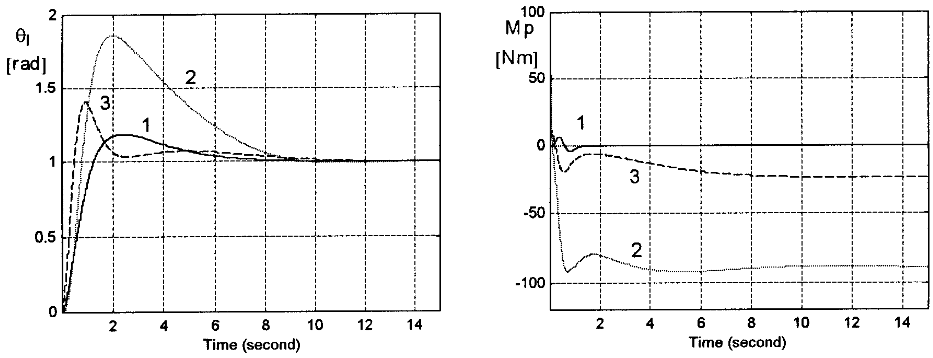

Fig. 5. System responses and variation of the disturbing torques for PID control.

Considering the presented responses one can notice that the error in stationary state is equal to zero (segments are reaching required positions), which is the consequence of the integral action which brings the astatism into the system, with respect to the disturbing action. By varying the values of controller gains in all the three control contours, the significantly better results can not be obtained. The more significant qualitative progress could be achieved only by application of the feedforward compensation.

The cause of such bad performances of the system's response is existence of dynamics in the numerator (existence of zeros) of the transfer function (14). The negative influence of dynamics in the numerator of the transfer function could not be taken into account by the applied method of transfer function poles placement (14). Thus as logical seems the idea to search for some other, modified control structure, that would keep the comfort of existence of the three gains for controllers, but where there would be no dynamics in the transfer function numerator.

4. APPLICATION OF THE PSEUDO-DERIVATIVE FEEDBACK IN IR AND DESIGN OF CONTROLERS IN THE S - DOMAIN

Result of search for the new, more convenient control structures, is the application of the fedback with pseudo-derivative action (PDF). There are several reasons that this way of control should be preferred with respect to conventional control algorithms.

One of the basic advantages lies in moving the D-action from the direct control branch into the feedback, thus keeping the advantages of the differential action, but without difficulties caused by differentiator placed in the direct branch. Namely, it is known that the D-action can be used for increasing the speed of the system's response. The reason for moving it form the direct branch is that it causes the sudden change in the error signal, where the physical limitations of the differentiating device cause the incorrect calculation of the derivative, thus, accordingly, the real performance of the system will be smaller than the ideal performance set by the model. Also, the presence of the D-action, and to the lesser extent also the P-action, in stepwise variation of the referent (input) signal, can cause the so called “differential peak” in the control variable, which can not be handled physically by the majority of the executive organs. In the PDF control the D-action is moved into the feedback by the output, and since the output value represents the result of several integrations, it will vary slower than the other signals in the system, and thus the differentiator’s response will be more realistic.

The application of the PDF control is also convenient from the reason of obeying the "one master principle" (according to terminology used in [7]), namely the principle of one action in the direct branch. The deficiency of the PID control laws family is also in that since the controller is required to simultaneously respond on signals that can be conflicting. If with $u(t)$ denotes the control signal, then the error signal $e(t)$ produces $u(t)$ according to equation:

$$\frac{du(t)}{dt} = K_I e(t) + K_P \frac{de(t)}{dt} + K_D \frac{d^2 e(t)}{dt^2} \tag{20}$$

If, for example, $e(t)$ is the sinusoidal signal, then the expressions for $de(t)/dt$ and $d^2e(t)/dt^2$ are moved for 90^0 and 180^0 with respect to $e(t)$, respectively, so the controller is forced to simultaneously process three different signals and generate $u(t)$.

Result of such an analysis, according to R. M. Phelan, is the conclusion that the most convenient is for the control algorithm not to contain more than one action in the direct branch. The application of this rule actually represents obeying "one master principle". Considering the previously presented analysis of the D-action, it is obvious that its application in the direct branch should be avoided. However, the proportional control can be used with the PDF in the case when the control object is of the first order with the negligible damping (the pure integrator), then when there is no disturbing action, and when the fast response is not required. Thus, for instance, in the case of existence of disturbance only of the P-action in the direct branch, the system would result which would not have astatism with respect to the disturbance signal, so it would always exist a certain error in the stationary state. Another deficiency of P-action application is also in that since it is not realistic to consider that the control element instantaneously responds to the stepwise response. Thus, as the most convenient solution will be the placement of the integral action in the direct branch.

4.1. Application of the PDF control of the d.c. motor for driving the IR segments

In order to present the PDF control concept better, one starts from the PID control, which was considered in Section 3. It was shown that the system can be reduced to the form shown in Figure 6. The transfer functions of the system, from the input to the output, and from the disturbing action to the output, are given by relations (14-15).

The transfer function by disturbance has one s in the numerator, thus the deviation of the stationary state due to stepwise disturbance will be equal to zero. However the presence of the D-action in the direct control branch, due to physical limitations of the differentiator, would not give the expected performances completely. Let us suppose that the differential and proportional actions are moved from the direct branch and placed into the feedback. Thus we come up with the hypothetical system that represents the introduction into the PDF. Such a system is shown in Figure 4 if $G(s) = \frac{K_I}{s}$ and the feedback has the transfer function:

$$H(s) = 1 + K_{D1} + K_{D2}s^2 \tag{21}$$

The transfer function by the output is of the form:

$$\frac{\theta_l(s)}{\theta_d(s)} = \frac{K_I}{J_{ef}s^3 + (B_{ef} + b + K_I K_{D2})s^2 + K_I K_{D1}s + K_I} \tag{22}$$

However, such a system requires the device not only for determination of the first but also of the second derivative of the output, which is not the most convenient solution. The considered structure has no physical sense, since the output signal is differentiated and then integrated. It is obvious that by removing one operator in expressions for K_{D1} and K_{D2} and closing the internal feedback loop behind the integrator (as shown in Figure 6), the effect is obtained that is equivalent to integration, except multiplication by K_I.

The structure presented in Figure 6 represents the realization of the PDF control applied to the actually considered system.

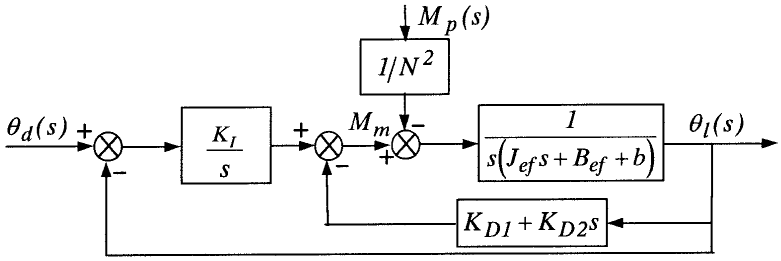

Figure 6. PDF object control

where: $K_I = \frac{K_I^{'} K_i K_\theta}{NR}$; $K_{D!} = \frac{K_{D1}^{'} K_i}{NR}$; $K_{D2} = \frac{K_{D2}^{'} K_i}{NR}$

and $K_I^{'}, K_{D1}^{'}, K_{D2}^{'}$ are the corresponding coefficients of the controllers gains.

The transfer function by the output is:

$$\frac{\theta_l(s)}{\theta_d(s)} = \frac{K_I}{J_{ef}s^3 + (B_{ef} + b + K_{D2})s^2 + K_{D1}s + K_I} \tag{23}$$

while the transfer function by the disturbance is:

$$\frac{\theta_l(s)}{M_P(s)} = \frac{-s / N^2}{J_{ef}s^3 + (B_{ef} + b + K_{D2})s^2 + K_{D1}s + K_I} \tag{24}$$

From the transfer functions given by (23) and (24) we can notice that only one gain makes each coefficient of the characteristic equation, which is the same as in the PID control law, but the dynamics in the numerator is eliminated. This is of a great importance for control of the motors for IR segments' drives, since in this way the phenomenon of overshoot in the stepwise response is avoided, what was not possible to achieve by application of the conventional PID control law.

4.2. Choice of the PDF gains

As requirements that the system should satisfy were previously defined, they could be reduced to one requirement for as fast as possible aperiodic response, with respecting all the restrictions that the system imposes. The values of gains can be determined analytically, however, since the considered system is of the third order, the expressions from which the values of gains K_I , K_{D1} and K_{D2} would be extremely complicated. Let us, thus consider the analyzed system in the form shown in Figure 7.

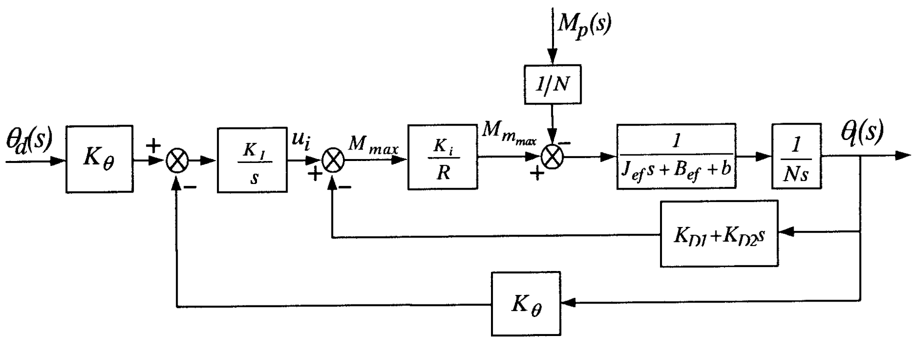

Figure 7. The proposed structure of the system with the PDF control.

For the case that there is no disturbance the interior feedback has the transfer function:

$$\frac{\theta_I(s)}{u_i(s)} = \frac{K_i}{RJ_{ef}N} \cdot \frac{1}{s^2 + \left(\frac{B_{ef}+b}{J_{ef}} + \frac{K_i K_{D2}}{RJ_{ef}N}\right)s + \frac{K_i K_{D1}}{RJ_{ef}N}} \tag{25}$$

The coefficient of the relative damping of the given transfer function (25) is of the form:

$$\zeta_i = \frac{(B_{ef}+b)RN + K_i K_{D2}}{2\sqrt{K_i K_{D1} RJ_{ef} N}} \tag{26}$$

It is certain that there the overshoot would not appear if $\zeta_i \geq 1$, but also the values $\zeta_i < 1$can be used since the input u_i , that represents the output from the integrator is never as serious as the step function. According to R. M. Phelan, results of numerous simulations have shown that for the wide range of values of gains, the smoothest and the fastest step response without overshoot appears for $\zeta_i \approx 0.707$. According to that, it is recommended that K_{D1} is determined with help of relation:

$$\zeta_i = \frac{(B_{ef}+b)RN + K_i K_{D2}}{2\sqrt{K_i K_{D1} RJ_{ef} N}} \tag{27}$$

where $M_{m\ max}$ is the maximum value of the motor's torque for which the saturation does not occur.

The design procedure unfolds in the following way: after the value for gain K_{D1} is determined from (27), by transformation of expression (26) the relation is obtained for determination of the needed value of gain K_{D2}:

$$K_{D2} = \frac{2\zeta\sqrt{K_i K_{D1} R J_{ef} N} - (B_{ef} + b)RN}{K_i} \tag{28}$$

With gains thus chosen, the response is analyzed to the input stepwise control for the much increased values of K_I until the saturation appears, or the overshoot of the response occurs. The higher values of K_I are desirable for compensation of the disturbance. Such a procedure in the majority of cases gives the good estimation of gains. Further simulations can be used for the more sophisticated adjustment of results.

The controller output is of the form:

$$M(t) = K_I \int e(t)dt - K_{D1}\theta_1(t) - K_{D2}\frac{d\theta_1(t)}{dt} \tag{29}$$

The only increase of the manipulative variable comes from the integral action, thus with increase of K_I also are increasing the response speed and possibility of overshoot. The value of $K_{D1}\theta_l$ is the most significant close to the stationary state when θ_l is large, so K_{D1} can be adjusted to eliminate the overshoot. In the initial phases of response θ_l and error integral have small values so the output from the controller to the greatest extent depends on term $K_{D2} \cdot \dot{\theta}_l$. With increase of K_{D2} , accordingly, the maximum value for $M(t)$ can be reduced, namely, the saturation can be prevented, considering that the highest values of the manipulative variable appear in the initial phases of response.

Further will be presented results of application of the PDF control obtained for the robot presented in section 2. Obtained values of gains are given in Table 2.

Table 2. Obtained values of gains

GAINS	K_I	K_{D1}	K_{D2}
Positional servomechanism of the first segment	288	170	33.4
Positional servomechanism of the second segment	125	127.6	20.4
Positional servomechanism of the third segment	395	170	13.8
Positional servomechanism of the fourth segment	2400	170	2.4

For the given values of gains the system responses and variation of the disturbance torque are given in Figure 8.

The disturbance compensation makes sense in real time for the part of the torque due to gravitation. With compensation of gravitation responses are improved for the second and the third joint, which is presented in Figure 10. For the case of addition of the fourth segment, due to additional masses and loads, for the first three segments, responses are somewhat slower, but for the fourth segment the exceptionally good tracking of the prescribed trajectory is achieved, what is shown in Figure 9.

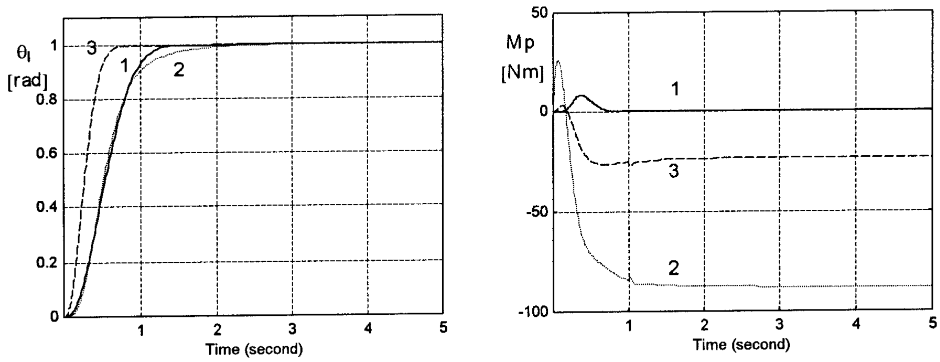

Figure 8. System responses and variation of the disturbance torque for the case of the PDF control.

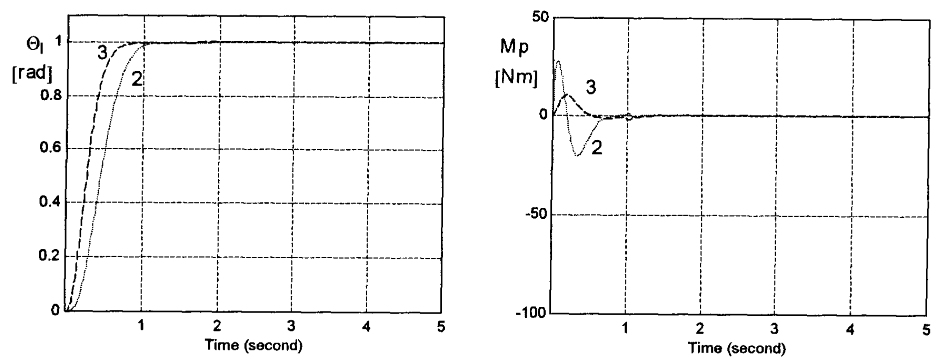

Figure 9. Response for PDF control with feedforward compensation of gravitation.

In order to compare the obtained results with results presented in [1], the robot shown in figure 3 will be considered, with the fourth segment added (the gripper segment), that performs the rotational motion about the longitudinal axis of segment 3. The characteristic values for the fourth segment are:

- length of segment $l_4 = 0.05$ m
- position of center of mass $a_4 = 0.025$ m
- mass of segment $m_4 = 1$ kg
- moments of inertia, kgm^2 $J_{\xi 4} = 0.0034$; $J_{\eta 4} = 0.0034$; $J_{\zeta 4} = 0.0005$
- coefficient of the viscous friction in segment bearings $B_{l4} = 0.2$ Nms/rad

For the fourth segment drive the same type of motor as for the drive of the first and the third segment. The transmission ratio of the reducer is $N = 40$, which is 2.5 less than the transmission ratio used in [1].

Since it is necessary to know the maximum values of the stepwise input for the calculation of the gains K_{Dl} according to (27), for all the four control contours, it shall be assumed that $\theta_{d\,max} = \pi/2$ [rad]. Let, for analogy with the previously analyzed conventional control laws, be required that all the first three segments be turned for *1* [rad] with respect to the initial position, and on the input of the positional servomechanism of the additional fourth segment is brought the sinusoidal referent signal with frequency of 2π [rad/s] and amplitude of $\pi/12$ [rad] (identical as in [1]).

Since all other needed data are already given in section 2, based on expression (27) the values of gains K_{D1} for all the three control contours can be determined. Based on expression (28), taking that $\zeta_i = 0.707$and $J_{ef} = J_{ef\,max}$, we come up with corresponding values of gains K_{D2}. For thus calculated gains, by computer simulations, for each control contour, values of integral action were increased all the way till the moment of appearance of overshoot in response, or entering of the executive organ into the saturation zone. The only changes are in values of integral gains, and they are: $K_{I1} = 288$, $K_{I2} = 125$, $K_{I3} = 395$. Calculated gains for the fourth segment are: $K_{I4} = 2400$, $K_{D1} = 170$, $K_{D2} = 2.4$.

For the case of addition of the fourth segment, due to additional masses and loads, for the first three segments, responses are somewhat slower, but for the fourth segment the exceptionally good tracking of the prescribed trajectory is achieved, which is shown in Figure 10.

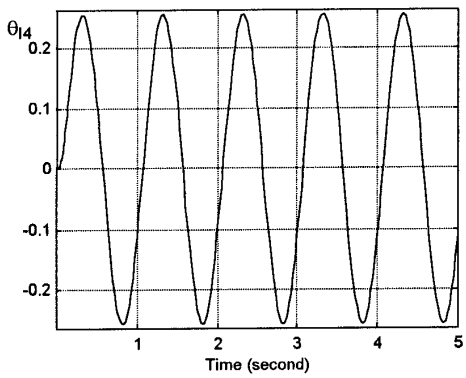

Figure 10. Tracking of the sinusoidal input for the fourth segment.

5. Concluding Remarks

Application of the PDF control in realization of the positional servomechanisms of industrial robot significantly increases the response characteristics, both in the transient process and in the stationary state. Responses of the first three segments are aperiodic, without appearance of overshoot, and despite that, significantly faster than responses obtained by application of some of the conventional control laws. Thus designed control systems can successfully overcome the disturbing action, so, as opposite to conventional control algorithms, where for the improvement of the system behavior is recommended to apply the feedforward compensation by disturbance, in this case this need for additional complicating of the system practically does not appear.

One deficiency of the PDF control structure should also be mentioned in realization of positional servomechanisms of industrial robots. It lies in the fact that the value of the integral gain is being determined experimentally or by the way of computer simulations, and not analytically. The consequence of that is that in the case of change of the referent input value, the values of integral gains would be inadequate for some other input referent value. If, as in the considered case, the value of integral action for referent input is θ_d = 1 [rad], then at smaller value of referent input (e.g. θ_d = 0.5 [rad]) the overshoot would appear in response, so it would be necessary to decrease the value of integral gain. On the other hand, if to input the referent value θ_d = 1.5 [rad] is brought, the response would be somewhat slower, so for improvement of response it would be necessary to increase the value of integral action. Thus, for each value of the referent input, there is the corresponding optimum value of integral gain, what, from the aspect of realization of controller, in continuous time domain, is very inconvenient. However, it should be said that nowadays the control of robots is based exclusively on application of digital control systems which, considering that the digital control algorithms are realized by software, enable very simple variation of the integral action coefficient values as a function of referent input values.

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PRIMENA PSEUDODERIVATNE POVRATNE SPREGE U KONTROLERIMA INDUSTRIJSKIH ROBOTA

Ilija Ž. Nikolić, Ivan Milivojević

Koncept upravljanja sa pseudoderivatnom povratnom spregom, koji je uveo R. M. Phelan, primenjen je za efikasno i praktično upravljanje industrijskim robotima. U radu je pokazano da je ovaj način upravljanja vrlo jednostavan i lako primenljiv, pošto motori jednosmerne struje industrijskih robota sadrže tahogeneratore. Dobijeni rezultati pokazuju da se i sa znatno manjim stepenom redukcije momenta opterećenja obezbeđuje brz aperiodični odziv sistema.