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CONTROLLABILITY OF DISTRIBUTED SYSTEMS WITH UNBOUNDED CONTROL ACTION

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Abstract. *The main result of this paper is to propose a new approach to study the exact controllability for linear distributed systems with unbounded input operators. Under appropriate assumption, we give the control which allows to reach a desired given state, the problem of minimum energy control is also examined.*

1. INTRODUCTION

The control processes for many dynamical systems are often severely limited. For example there may be delays in the control actuators. Also for systems described by partial differential equations it is not always possible to influence the state of the system at each point of the spacial domain. Instead control is restricted to a few points or part of boundaries. Modelling such limitations results in unbounded input operators. To study the exact controllability with unbounded input operators we use a new technique developped recently by J. L. Lions [4]; A. El Jai and A. Belfekih [3]. Finally we give an illustrative example.

2. PROBLEM STATEMENT

We consider the system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t)\,, \quad t \in [0,T]\,,\ T > 0 \\ x(0) = x_0 \end{cases} \tag{2.1}$$

Where A is the infinitesimal generator of strongly continuous semigroup $S(t)$ on a Hilbert space X, $u \in L^2(0, T;U)$, the operator B is unbounded and $B \in L(U,V)$ where U, V are

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Hilbert spaces such that $X \subset V$ with continuous dense injection, moreover $S(t)$ is a strongly continuous semigroup on V. We assume that there it exists a constant $b > 0$ such that

$$
\begin{aligned}
&- \int_0^t S(T-r)Bu(r)dr \in X\,, \quad \forall u \in L^2(0,T;U) \\
&- \| \int_0^T S(T-r)Bu(r)dr \|_X \le b \| u \|_{L^2(0,T;U)}
\end{aligned}
\tag{2.2}
$$

We interpret equation (2.1) in the mild form which means that

$$
x(t) = S(t)x_0 + \int_0^t S(t-r)Bu(r)dr\,, \quad 0 \le \mathrm{t} \le \mathrm{T} \tag{2.3}
$$

Remarks 2.1.

- In the following we identify the Hilbert X, U with their duals, then by duality we have $V^* \subset X$ with continuous dense embeding, moreover $S^*(t)$ is a strougly continuous semigroup on V^*

- The dual statement of (2.2.) is (see (5), (6))

$$
\| B^* S^*(T-.)x \|_{L^2(0,T;U)} \le b \| x \|_X\,, \quad \forall x \in V^* \tag{2.4}
$$

- If (2.2.) is satisfied, then for any $x \in X$ we will use the expression $B^*S^*(T-t)x$, $0 \le t \le T$; to denote the function in $L^2(0, T;U)$ which is obtained by continuous extention of the operator:

$$
\begin{aligned}
&X \to L^2(0,T;U) \\
&x \to B^* S^*(T-.)x
\end{aligned}
$$

We consider the following problem of exact controllability.

Is it possible to find a control $u^* \in L^2(0,T;U)$ steering the system (2.1) to a desired state x_d? ie such that

$$
x(T, x_0, u^*) = x_d \tag{2.5}
$$

$$
\| u^* \| = \min\{ \| v \| : x(T, x_0, v) = x_d \} \tag{2.6}
$$

where $x(T,x_0,v)$ is the solution of (2.3) corresponding to the control v.

Definition 2.1.

(i) the system (2.1) is said to be exactly controllable on $(0,T)$, if for every x_0, $x_d \in X$, $u \in L^2(0,T;U)$ such that $x(T, x_0, u) = x_d$

(ii) the system (2.1) is said to be approximately controllable on $(0,T)$, if for every x_0,$x_d \in X$ and $\varepsilon > 0$, there exists $u \in L^2(0,T;U)$ such that:

$$
\| x(T, x_0, u) - x_d \| < \varepsilon
$$

Let H be the operator defined by

$$H: \quad L^2(0,T;U) \to X$$
$$u \to \int_o^T S(T-r)Bu(r)dr$$

It follows from (2.2) that H is bounded and

$$\| Hu \|_X \le b \| u \|_{L^2(0,T;U)} .$$

Propostition 2.1.

The system (2.1) is approximately controllable on $[0,T]$ if and only if Ker $H^* = \{0\}$.

It is easy to establish that

$$H^* h = B^* S^* (T-.)h , \quad \forall h \in V^*$$

moreover by remark (2.1), H^* has and unique extention on X, we note it by

$$H^* x = B^* S^* (T-.)x , \quad \forall x \in X .$$

3. RESOLUTION OF THE PROBLEM

Consider the operator $\wedge$ defined by

$$\wedge: \quad X \to X$$
$$x \to \int_0^T S(T-r)BB^* S^* (T-r)xdr \tag{3.1}$$

it follows from (2.2) and the remark (2.1) that $\wedge$ is a bounded operator on X.

Lemma 3.1.

$\wedge$ is a self-adjoint and positive operator

Proof

For every $x, y \in V^*$ we have

$$< \wedge x, y >_{X,X} =< B^* S^* (T-.)x, \ B^* S^* (T-.)y >_{L^2(0,T;U)}$$

and by density

$$< \wedge x, y > = < x, \wedge y > , \quad \forall x, y \in X$$

moreover

$$< \wedge x, x > = \| H^* x \|^2 \ge 0 .$$

Lemma 3.2.

If the system (2.1) is apropcimately controllable on $[0,T]$ then $\wedge$ is injectif.

Proof

$\wedge x = 0$ implies that $<\wedge x, x> = 0$ then $\|H^* x\|^2 = 0$, and by the proposition (2.1) $x = 0$.

Let us define the inner product <<, >> on X by

$$<< x, y >>_{X,X} = < H^* x,\ H^* y >_{L^2(0,T;U)}$$

the associated semi-norm is

$$\| x \|_F = \| H^* x \|_{L^2(0,T;U)} .$$

Remarks 3.1.

-if the system (2.1) is approximately controllable then $\| \,.\, \|_F$ is a norm on X

- by remark (2.1) we have: $\|x\|_F \le b\ \|x\|_X$, $\forall x \in X$

If we denote F the completion of X for the norm $\| \,.\, \|_F$, we have the following result.

Lemma 3.3. [1]

If we identify X with its dual, such that $F^* \subset X \subset F$ then $\wedge$ is an isomorphism from F onto F^*, moreover $\| \wedge \|_{L(F,F^*)} = 1$ and $(\wedge x)(y) = << x, y >>$ for every $x, y \in F$.

Now we show the principle result of this paper.

Proposition 3.1.

If the system (1.1) is approximately controllable and $x_d - S(T)x_0 \in F^*$ then there exists a unique control $u^*(.)$. Solution of the problem (2.5)–(2.6). u^* is given by:

$$u^*(.) = H^* f \tag{3.2}$$

where f is the unique solution of the equation

$$\wedge f = x_d - S(t)x_0 \tag{3.3}$$

moreover $\| u^* \|_{L^2(0,T;U)} = \| f \|_F$.

Proof

$$x(T, x_0, u^*) = S(T)x_0 + Hu^* = S(T)x_0 + HH^* f = S(T)x_0 + \wedge f$$

and by (3.3), $x(T,x_0,u^*) = x_d$

on the other hand let C be the set:

$$C = \{u \in L^2(0,T;U) \,/\, x(T, x_0, u) = x_d\}$$

$C \ne \phi$ because $u^* \in C$. For every $u \in C$ we have $\int_0^T S(T - r)B(u - u^*)(r)dr = 0$, that implies

$$< f,\ \int_0^T S(T - r)B(u - u^*)(r)dr > = 0 .$$

Hence , $< H^* f, u - u^* >_{L^2(0,T;U)} = 0$ so $< u^*, u - u^* > = 0$

and finally $\| u^* \| \le \| u \|$, $\forall u \in C$

moreover $\| u^* \| \le \| H^* f \| = \| f \|_F$.

Proposition 3.2.

If we denote $E = \{x(T,x_0,u) / u \in L^2(0,T;U)\}$ the space of reachable states at time T, then

$E = S(T)x_0 + F^*$.

Proof

If $x \in S(T)x_0 + F^*$, then there exists $f \in F$ such that $\wedge f = x - S(T)x_0$

so $x = S(T)x_0 + \int_0^T S(T-r)Bu(r)dr = x(T, x_0 u) \in E$ where $u(.)$ is the control defined by $u(.) = B^* S^*(T-.)f$

Inversely, for $x(T,x_0,u) \in E$ we consider the linear form

$$L: \quad X \to |R$$
$$f \to \int_0^T < x(T, x_0, u) - S(T)x_0, f >$$

it is easy to see that $\| Lf \| \leq \| u \| \ \| f \|_F$.

Then, by density of X in F, (3.4) is true for all f in F, finally we deduce by Riesz theorem that $x(T, x_0, u) - S(T)x_0 \in F^*$.

Remark 3.2.

The resolution of the equation (3.3) is equivalent to minimize the functional.

$$J(x) = \frac{1}{2} < \wedge x, x > - < x_d - S(T)x_0, x >$$

and

$$J(f) = -\frac{1}{2} < \wedge f, f > -\frac{1}{2} \| H^* f \|^2 .$$

Example:

Let the controlled parabolic system be:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t), & t \in [0,T], \ T > 0 \\ x(0) = x_0 \end{cases} \tag{2.1}$$

where A is a self-adjoint operator on a real Hilbert space X, let λ_n, n $\in$ |N, be the simple eigenvalues of A with corresponding eigenvectors $\phi_n \in X$, $\|\phi_n\| = 1$ and assume $|\lambda_n| \to +\infty$, $\lambda_n < \lambda_{n-1} < \lambda_1$. We have

$$Ax = \sum_1^\infty \lambda_n < x, \ \phi_n > . \phi_n \tag{3.6}$$

with

$$\mathrm{Dom}(A) = \left\{ x \in X / \sum_1^\infty \lambda_n^2 < x, \ \phi_n >_X^2 < \infty \right\}$$

A generate the strongly continuos semi-group

$$S(t)x = \sum_1^\infty e^{\lambda_n t} < x, \ \phi_n > . \phi_n . \tag{3.7}$$

For any positive sequence $\alpha = (\alpha_n)_n$, $\alpha_n > 0$ we define the Hilbert space

$$X_\alpha = \left\{ x = (x_n)_n / \sum_1^\infty \alpha_n x_n^2 < \infty \right\} \tag{3.8}$$

with the inner product

$$< x, y >= \sum_1^\infty \alpha_n x_n y_n$$

Remarks 3.3.

- $X_\beta \subset X_\alpha$ with continuous dense injection if and only if the sequence $(\alpha_n/\beta_n)_n$ is bounded (thus $X \subset X_\alpha$ if $\alpha = (\alpha_n)_n$) is a bounded sequence)

$$X_\alpha^* = X_{\alpha^{-1}} \text{ where } \alpha^{-1} = (\alpha_n^{-1})_n$$

- we identify X with $X_1 = i^2$ via the isometric isomorphism

$$i: \quad X \to X$$
$$x \to \int_0^T (< x, \phi_n >)_n$$

The operator B is defined by

$$B: \quad U \to X$$
$$u \to \int_0^T (< b_n, u >_U)_n$$

where $b_n \in U$ and U a Hilbert space. It is easy to see that B is bounded if and only if $\sum \| b_n \|^2 < \infty$.

Let $N_0 = \max\{n \in | N / \lambda_n \geq 0\}$

Proposition 3.3. [5]

We assume the sequence $\beta = (\beta_n)$ is bounded, positive and $V = X_\beta$

if $\sum_1^\infty \beta_n \| b_n \|^2 < \infty$ then $B \in L(U,V)$

if $\sum_{N_0+1}^\infty \frac{\| b_n \|^2}{| \lambda_n |} < \infty$ then for every $T > 0$, there exists a constant $b > 0$ such that

$$\| \int_0^T S(T-r)Bu(r)dr \|_X \leq b \| u \|_{L^2(0,T;U)}, \forall u \in L^2(0,T;U)$$

Remarks 3.4.

- the operator $B^* \in L(X_\beta^*, U)$ is given by

$$B^*((x_n)_n) = \sum_1^\infty b_n x_n (B^* \phi_n = b_n)$$

- if $x = (x_n)_n \in \quad V^* \subset X$ then

$$H^* x = \sum_1^\infty b_n x_n e^{\lambda_n (T-.)}$$

In order to characterise the approximate controllability of the system (3.5), we must calculate $H^* x$ for $x \in X$. This is the object of the following results.

Lemma 3.4

If there exists $\varepsilon \in \,]0,2T[$ such that $\sum e^{\lambda_n (2T-\varepsilon)} \| b_n \|^2 < \infty$ and $\sum \beta_n \| b_n \|^2 < \infty$ then for every $x \in X$

$H^* x = \sum_1^\infty e^{\lambda_n (T-.)} x_n b_n$ on $[0, \frac{\varepsilon}{2}[$.

Proof

- if $x \in V^* \subset X$ it is obvious that

$H^* x = \sum_1^\infty e^{\lambda_n (T-.)} x_n b_n$ on $[0, T]$

- if $x \in X$, since V^* is dense in X there exists a sequence $(x^p)_p$ such that

$$\lim_p x^p = x \text{ , where } x^p = (x_n^p)_n \in V^*$$

for $s \in [0, \frac{\varepsilon}{2}[$ and $N > N_0$ we pose $K_N = \sum_1^N x_n e^{\lambda_n (T-s)} b_n$.

We have

$$\| K_N - K_M \| \leq \left(\sum_N^M \| b_n \|^2 \, e^{\lambda_n (2T-\varepsilon)} \right)^{1/2} \left(\sum_N^M x_n^2 \right)^{1/2}$$

as $\sum \| b_n \|^2 \, e^{\lambda_n (2T-\varepsilon)} < \infty$ and $\sum x_n^2 < \infty$, $(K_N)_N$ is a Cauchy's sequence, then

$$\lim_N K_N = \sum_1^\infty x_n e^{\lambda_n (T-s)} b_n \text{ .}$$

On the other hand it is easy to see that for fixed $p \in \mathbb{N}$ and $N > N_0$ we have

$$\| \sum_1^N x_n e^{\lambda_n (T-s)} b_n - \sum_1^N x_n^p e^{\lambda_n (T-s)} b_n \| \leq \left(\sum_1^N (x_n - x_n^p)^2 \right)^{1/2} \cdot \left[\left(\sum_1^{N_0} e^{\lambda_n T} \| b_n \|^2 \right)^{1/2} + \left(\sum_{N_0+1}^N \| b_n \|^2 \, e^{\lambda_n (2T-\varepsilon)} \right)^{1/2} \right] \tag{3.9}$$

Since the sequence $\sum_{N_0+1}^N \| b_n \|^2 \, e^{\lambda_n (2T-\varepsilon)}$ is bounded ($\sum_{N_0+1}^N \| b_n \|^2 \, e^{\lambda_n (2T-\varepsilon)} < \infty$) we obtain when $N \to +\infty$

$$\| \sum_1^N x_n e^{\lambda_n (T-s)} b_n - \sum_1^N x_n^p e^{\lambda_n (T-s)} b_n \| \leq c \left(\sum_1^N (x_n - x_n^p)^2 \right)^{1/2} \tag{3.10}$$

where c is a positive constant.

As $\lim_p x^p = x$, we deduce

$$\lim_p \sum_{n=1}^{\infty} e^{\lambda_n (T-s)} x_n^p b_n = \sum_{n=1}^{\infty} e^{\lambda_n (T-s)} x_n b_n \tag{3.11}$$

then, for every $s \in [0, \frac{\varepsilon}{2}]$

$$\lim_p (H^* x^p)(s) = \sum_1^{\infty} e^{\lambda_n (T-s)} x_n b_n$$

Since the function $\sum_{n=1}^{\infty} x_n^p e^{\lambda_n (T-.)} b_n = H^*(x^p) = B^* S^* (T-.) x^p$ is continuous on $[0, \frac{\varepsilon}{2}[$ for $A > 0$ and $s_0 \in [0, \frac{\varepsilon}{2}]$ there exists $\eta > 0$ such that $|s - s_0| < \eta$ implies that

$$\| \sum_1^{\infty} x_n^p e^{\lambda_n (T-s)} b_n - \sum_1^{\infty} x_n^p e^{\lambda_n (T-s_0)} b_n - \| < \frac{A}{3}$$

On the other hand $\exists\, p_0 \in |N$ such that $p > p_0 \to (\sum_1^{\infty} (x_n - x_n^p)^2) < \frac{A}{3c}$

then for fixed $p > p_0$ and $A > 0$ such that $|s - s_0| < \eta$ implies

$$\begin{aligned} &\| \sum_1^{\infty} e^{\lambda_n (T-s)} x_n b_n - \sum_1^{\infty} e^{\lambda_n (T-s_0)} x_n b_n \| \leq \| \sum_1^{\infty} e^{\lambda_n (T-s)} x_n b_n - \sum_1^{\infty} e^{\lambda_n (T-s)} x_n^p b_n \| + \\ &+ \| \sum_1^{\infty} e^{\lambda_n (T-s)} x_n^p b_n - \sum_1^{\infty} e^{\lambda_n (T-s_0)} x_n^p b_n \| + \| \sum_1^{\infty} e^{\lambda_n (T-s_0)} x_n^p b_n - \sum_1^{\infty} e^{\lambda_n (T-s_0)} x_n b_n \| \leq \\ &\leq \frac{A}{3} + \frac{A}{2} + \frac{A}{3} = 1 \end{aligned}$$

thus the function $\sum x_n e^{\lambda_n (T-.)} b_n$ is continuous on $[0, \frac{\varepsilon}{2}]$, using the moyenne formula we obtain

$$\| H^* x^p - \sum_1^{\infty} x_n e^{\lambda_n (T-.)} b_n \|^2_{L^2(0, \frac{\varepsilon}{2}, U)} = \frac{\varepsilon}{2} \| \sum_1^{\infty} e^{\lambda_n (T-\xi)} (x_n^p - x_n) b_n \|^2$$

where $\xi \in [0, \frac{\varepsilon}{2}]$, then by (3.11)

$$\lim_p H^* x^p = \sum_1^{\infty} e^{\lambda_n (T-.)} b_n \quad \text{in} \quad L^2(0, \frac{\varepsilon}{2}; U) \tag{3.12}$$

and finally by unicite of the limit, we deduce

$$H^* x = \sum_1^\infty x_n e^{\lambda_n (T-.)} b_n \quad \text{on} \quad [0, \frac{\varepsilon}{2}].$$

Lemma 3.5

If $\sum_{N_0+1}^\infty \frac{\| b_n \|^2}{|\lambda_n|} < \infty$, then $\sum_1^\infty e^{\lambda_n (2T-\varepsilon)} \| b_n \|^2 < \infty$, $\forall \varepsilon \in]0,2T[$.

Proof

As $\lim_n \frac{e^{\lambda_n (2T-\varepsilon)} \| b_n \|^2}{\frac{\| b_n \|^2}{|\lambda_n|}} = \lim_n |\lambda_n| e^{\lambda_n (2T-\varepsilon)} = 0$, we have

$\sum e^{\lambda_n (2T-\varepsilon)} \| b_n \|^2 \le (\text{cste}) \sum \frac{\| b_n \|^2}{|\lambda_n|} < \infty$.

Remark 3.5

It follows from the lemma 3.4 and the lemma 3.5 that under the assumptions $\sum_{N_0+1}^\infty \frac{\| b_n \|^2}{|\lambda_n|} < \infty$ $\sum \beta_n \| b_n \|^2 < \infty$ we have

$$H^* x = \sum_1^\infty e^{\lambda_n (T-.)} x_n b_n \quad \text{on} \quad [0, \frac{\varepsilon}{2}[\,, \ \forall x \in X, \ \ \forall \varepsilon \in [0,2T]$$

Proposition 3.4

If $\sum_{N_0+1}^\infty \frac{\| b_n \|^2}{|\lambda_n|} < \infty$, $\sum_1^\infty \beta_n \| b_n \|^2 < \infty$ and $b_n \neq 0$, $\forall n \in |N$ then the system (3.5) is approximately controllable on [0,T]

Proof

$x \in \text{Ker } H^*$ implies that

$$\sum e^{\lambda_n (T-s)} x_n b_n = 0\,, \quad \forall s \in [0, \frac{\varepsilon}{2}], \text{ where } \varepsilon \in [0,2T]\,. \tag{3.13}$$

Because of the analyticity we extend (3.13) for all $t \ge 0$, that implies

$x_n b_n = 0$, $\forall n \in |N$ (see [2]), then $x_n = 0$, $\forall n$.

Finally we deduce by the proposition (3.1) the following result.

Proposition 3.5

Under the assumptions $\sum_{N_0+1}^\infty \frac{\| b_n \|^2}{|\lambda_n|} < \infty$; $\sum_1^\infty \beta_n \| b_n \|^2 < \infty$; $b_n \neq 0$, $\forall n \in |N$ and $x_d - S(T)x_0 = z \in F^*$ the unique minimal norm control $u^*(.)$ which allows to reacht the desired state x_d is given by

$$u^*(.) = \lim u_n(.) \text{ in } L^2(0,T;U) \tag{3.14}$$

where

$$u_n(.) = \sum_{i=1}^{n} < \phi_i, r_n > e^{\lambda_i (T-.)} b_i$$

and r_n the unique solution of the linear finite system

$$A_n r_n = z_n \tag{3.15}$$

where $z_n = \sum_{i=1}^{n} < z, \phi_i >,\ \phi_i, A_n =< \wedge\phi_i, \phi_j >_{\leq i,j<n}$.

Proof

It follows from the proposition (3.1) that $u^* = H^* r$ where r is the unique solution of the equation

$$\text{(E)} \quad \wedge r = z$$

moreover

$$a(.,.): F \times F \to \mathbb{R}$$
$$(u,v) \to < \wedge u, v >_{F^*,F}$$

is a bilinear, continuous and coercive form ($a(u,v)$ = <<u,v>>) so we use the Galerkin method to resolve (E), ie $r = \lim r_n$ where r_n is the solution in the finite dimensional space

$\boldsymbol{V} = vect(\phi_1, \phi_2, ..., \phi_n)$ of the linear system: $A_n r_n = z_n$

as H^* is bounded, $\lim H^* r_n = H^* r$, hence $u^* = \lim u_n(.)$ in $L^2(0,T;U)$, where

$$u_n(.) = H_n^* r_n = \sum_{i=1}^{n} < \phi_i, r_n > e^{\lambda_i (T-.)} b_i \ .$$

Remark 3.6.

Under the assumptions of the proposition (3.5), if $x_d - S(T)x_0 \in \text{range}\,\overline{\wedge}$ where $\overline{\wedge} = \wedge / v^*$, the control u^* is exactly

$$u^* = \sum_{1}^{\infty} y_n e^{\lambda_n (T-.)} b_n \ .$$

and $(y_n)_n$ the unique solution of the infinit linear system

$$\overline{A} \begin{pmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ y_n \\ \cdot \\ \cdot \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ \cdot \\ \cdot \\ z_n \\ \cdot \\ \cdot \end{pmatrix}$$

where $\overline{A} = \left(\frac{1 - e^{(\lambda_i + \lambda_j)T}}{\lambda_i + \lambda_j} \cdot < b_i \cdot b_j > \right)_{1 \le i, j < \infty}$, $z_i = < z, \phi_i >$.

4. CONCLUSION

In this paper the exact controllability of distributed systems with unbounded control action has been considered, the minimal norm control which allows to reach a desired state and the space of reachable states are given. The result of this work can be applied to treat the hyperbolic, herioditary and other systems.

REFERENCES

1. Belfekih, A. 1990 *Nouvelle approche pour la contrôlabilité ef l'observabilité des systémes distribues*. These d'éetat, Kénitra-Maroc.
2. Cutain, R.F, and Pritichard, A.J., *Infinite dimensional linear systems theory*. Lecture notes in control and information sciences-vol. 8. Springer Verlag, Berlin.
3. El Jai, A, and Belfekih, A. 1990 *Exacte contrôlabilité et contrôle optimal des systèmes paraboliques*. Revue APII. Control/sistems Analysis.
4. Lions, J.L. 1988 *Contrôlabilitè exacte-Masson* R.M.A.
5. Pritchard, A.J. and Salamon, D. 1984 *The linear quadratic optimal control problem for infinite dimensional systems with unbounded input and output operators*, MRC, University of Wisconsin-Madison, TSR 2624.
6. Salamon. D. 1984. *On control and observation of neutral systems RNM91*, Pitman, London.

UPRAVLJIVOST RASPODELJENIH SISTEMA SA NEOGRANIČENOM UPRAVLJAČKOM AKCIJOM

Jamila Karrakchou, Mostafa Rachik

Glavni rezultat ovog rada je predlaganje novog pristupa izučavanju tačne upravljivosti linearnih raspodeljenih sistema sa neograničenim ulaznim operatorima. Pod odgovarajućim pretpostavkama, predložili smo upravljanje koje dozvoljava dostizanje datog željenog stanja. Takođe je ispitan i problem upravljanja minimalne energije.