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THE INVESTIGATION OF TEMPERATURE DISTRIBUTION AND STRESS WITHIN A TOOTH-RESTORATION CONSTRUCTION

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Abstract. *Article contain investigation results of temperature and stress field in composite cylindrical systems of bodies. Such system can be model of the tooth with filling. Use of real dental materials have been investigated. Dynamical model of the tooth have been constructed with use of the finite elements method and have been integrated on computer. Received results show that the most monolith constructions have greater stresses than microdentures with compensators.*

1. INTRODUCTION

A lot of the environmental objects can be modeled as composite elastic bodies separate parts of which have different adhesion forms and different degree and type of heat exchange. Sources of heat can be concentrated at the boundaries of bodies partition (e.g. friction of thise components) can be modeled; thermal insulation of various nature can be realized here too. Elastic interaction of components caused by difference in temperature patterns and thermoelastic characteristics within them may result in stress concentration and consequently necessitate investigation of the problem of construction rigidity and integrity. Similar problem arises in modern dentistry due to the sophistication of the inlays and fillings constructions as well as due to the widening the list of materials used. Investigation of physical properties of dental materials causes necessity of wide application of different physical and mathematical methods of investigation. While eating and even breathing the environment of "tooth-filling" system may have temperature substantially different from that of the human body. This temperature difference causes occurrence of nonhomogeneous temperature fields within a system resulting in thermo-elastic stress and the remedy is to use inlay with temperature compensator [5,6].

One of the places most subjected to destruction within a tooth-restoration

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construction system is the area of their contact. The most dangerous among all the variety of destructive factors are dinamatic loads from very hard feeding components and temperature fluctuations of the enveronment surrounding the system. If the first factor is of random character the second one is systematic nature though temperature fluctuations may depend on the different manifestations of subjective nature. That is why crack formation caused by thermoelastic deformations is the primary basic reason of dental destruction. A crack in the vertical zone of contact occurs as a result of inserting the inlay into the tooth cavity. After warming up or cooling the system as a result of temperature difference and difference of thermal expansion coefficients within the total deformed-stressed state occuring as a result of two deformations superposition normal stresses are turning into stretching stresses at the vertical contact boundary (i.e. there is no contact and the crack is being formed). Tooth and filling adhesion may be achieved due to the contact surfaces having ribs limiting relative displacements. Tangent stresses appearing on the boundaries may exceed the ultimate strength of the ribs and result in their destruction. In case the situation is repeated with sequential exposure to thermal action the processor rib destruction is being intensified and leads to crack formation. If in the two situations described above adhesion is being additionally supported with glue destruction or crack formation takes place when the corresponding stresses exceed the overall strength of the factors providingg adhesion. In case adhesion is achieved structurally only with the help of the glue depending on the relations between ultimate tensile strength and spear strength and its stresses at the boundaries cracks may appear too. Hence the main and the most difficult task is to find out stresses along the contact boundaries. As soon as occuring thermoelastic stresses drastically depend on relation between coefficients of thermal expansion of the separate parts of the tooth-filling system, a composite filling consisting of two constituent parts has been offered. The purpose of such structure of the filling is to minimize the difference between coefficients of linear expansion of the tooth and the filling preserving sufficient strength of the last one which can be broken as a result of mechanical action upon it. Taking into consideration mechanical properties it follows that maximum stresses appear when the system is characterized by increased adhesion rigidity limiting relative displacements of its arts. The given situation may take place in case when the cavity within the tooth has no outlet through the side tooth surface but it can be considered to have central position for stressed deformed conditions due to the small eccentricity in cavity layout. It is possible to receive quite exact description of stresses distribution within the "tooth-filling" system if to limit oneself to the given case. A tooth is considered to be a circular cylinder with cylindrical blind hole alined with tooth axis. This hole is filled with composite filling consisting of an insert and an inlay. An insert presents a circuular cylinder with cylindrical blind hole aligned with an insert axis and filled with an inlay. A filling is being put into the tooth cavity in such a way that the insert occurs at the bottom and touches the tooth surface. Materials of which a tooth, an insert and an inlay consist of are considered to be homogeneous, isotropic and their properties don't depend on temperature. Cylindrical system of coordinates (r,θ,x) is connected with the system of tooth and filling bodies. The plane (r,θ) corresponds to the upper section of the tooth and the filling. The axis x is directed inside the system and coincide with the axis of symmetry of the tooth, filling and inlay. There exists and axially symmetric system of cylindrical bodies consisting of a tooth, a filling and an inlay. A cylindrical tooth has radius R, height h_z and a cylindrical hole with radius R_p and depth h_p. The tooth is inserted into the gum for the depth of $h_z - h_t$. A filling having radius R_p, height h_p and cylindrical hole filled with an inlay with depth h_p - h_v and radius R_v is inserted into the tooth ($h_z > h_t > h_p > h_v$, $R > R_p > R_v$).

2. Formulation of the problem

The problem is being considered to find out the temperature fields and stressed-deformed state of this system caused by an instantaneous change of the temperature at the external surface of the system. The part of the tooth immersed into the gum has constant temperature of the body. The gum influence on the tooth deformation is so small that the system may be considered to be free. Taking into consideration material properties and the form of the system the problem is axially symmetrical. All the values depend on r and x and don't depend on θ. For displacement $U_\theta = 0$; for deformations $\varepsilon_{r\theta} = \varepsilon_{x\theta} = 0$ [1]. Neither of the forces influence on the surface of the system $\sigma_n = 0$. Functional expressing the total potential energy of the elastic system has the form [1]

$$\Pi = \int_S [\frac{1}{2}\varepsilon_{ij}\sigma_{ij} - \frac{1}{2}\alpha\delta_{ij}\sigma_{ij}]ds - \int_S u_i F_i ds - \int_{\partial S}(u_i\sigma_{ni})d\tau ,$$

where F_i - volume forces (in our case $F_i = 0$),
σ_{ni} - forces applied to the surface, also equal to 0,
α- coefficient of volumetric expansion,
T - temperature charge,
σ_{ij} - stresses,
ε_{ij} - deformations,
u_i - displacements,
S - area filled with the body,
δS - surface limiting the body.

Functional corresponding to our problem statement has the folloving form

$$\Pi = \int_S [\frac{1}{2}\varepsilon_{ij}\sigma_{ij} - \frac{1}{2}\alpha\delta_{ij}\sigma_{ij}]ds$$

To solve this problem it is necessary to find temperature distribution. To find the temperature we have the problem [2]: to minimize the functional

$$\chi = \int_S \frac{r}{2}\left[K_{rr}\left(\frac{\partial T}{\partial r}\right)^2 + \frac{1}{2}K_{xx}\left(\frac{\partial T}{\partial x}\right)^2 - 2\rho c\left(\frac{\partial T}{\partial t}\right)T ds\right] + \int_{\Gamma_1}\frac{\gamma}{2}(T - T_c)^2 d\tau$$

under conditions $T\big|_{G_2} = T_t$ and $T\big|_{t=0} = T_0(r,x)$where T_0 - the initial distribution of the temperature giving minimum to the functional [2]

$$\chi_0 = \int_S \frac{r}{2}\left[K_{rr}\left(\frac{\partial T}{\partial r}\right)^2 + \frac{1}{2}K_{xx}\left(\frac{\partial T}{\partial x}\right)^2\right] + \int_{\Gamma_1}\frac{\gamma}{2}(T - T_c)^2 d\tau$$

under conditions $T\big|_{\Gamma_2} = T_t$, where

$$S = \{(r,x) : x \in (0,h_z), r \in (0,R)\}$$
$$\Gamma_1 = \{(r,x) : x = 0, r \in (0,R]; r = R, x \in (0,h_t]\}$$
$$\Gamma_2 = \{(r,x) : x = h_z, r \in (0,R]; r = R, x \in (h_t,h_z]\}$$

and $T_{c0} \neq T_c$.

3. Description of the method of solution

Having this setting we will solve all three problems using the finite elements method. In the finite elements method S area is being devided into elements. In the studied case the area within the coordinates plane r and x is a union rectangles that is why we devide it into rectangles. Numbering of the node points - vertexes of the element is of great importance. Let's devide an interval of the charge r into $m_r - 1$ portions and on the x coordinate into $m_x - 1$. We will get the total number of rectangular elements $(m_r - 1)(m_x - 1)$ having $m_r m_x$ node points. Let a node point with coordinates (0,0) be denoted first. Then let's take a node with the same x coordinate and r coordinate larger by the width of the element and continue up to the node m_r. Then a node with zero r coordinate and with x coordinate larger by the height of the element denote $m_r + 1$. Let's continue in this way up to $m_r m_x$ node point. Let the number of the upper left node point (node point having the least values of r and x) be the number od the element. Let's assume that for each element value distribution has the form $\varphi = Ar + Bx + Crx + d$. φ is unambiguously defined here by the values in the node points - vertexes of the element [3]

$$\varphi = \frac{\varphi_1(x_2 - x)(r_2 - r) + \varphi_2(x_2 - x)(r - r_1) + \varphi_3(x - x_1)(r - r_1) + \varphi_4(x - x_1)(r_2 - r)}{(x_2 - x_1)(r_2 - r_1)}$$

where φ_i - quantity value in the node points of the element,
(x_i, r_j) - coordinates of the corresponding nodes.

Functional reffering to the value to be determined may be repsented as a sum [3,4] (formula), where $\chi^{(e)}$ functional is written for n - element within which integration is done by the area of an element. $\chi^{(e)}$ value can be found for each element which is to represent a quadratic function of the node points values φ_i. Let's find minimum of functional χ using a set of node values. Let's differentiate $\varphi_i \frac{\partial \chi}{\partial \varphi_i} = \sum \frac{\partial \chi^{(i)}}{\partial \varphi_i}$ - we will get as set of linear equations for φ_i

$$K\Phi = F$$

where $\Phi = \begin{pmatrix} \varphi_1 \\ \vdots \\ \varphi_n \end{pmatrix}$ - node point values,

K - a global matrix of rigidity, dimension of n at n,
$n = m_r m_x$,
F - a vector column of the right parts.

Besides this, $K = \Sigma K^{(e)}$ and $K = \Sigma F^{(e)}$ - where $K^{(e)}$ and $F^{(e)}$ - are corresponding matrixes for each element. Each $K^{(e)}$ has the same dimension as K, though there are only 16 non-zero coefficients within $K^{(e)}$, the rest are zeroes as $\frac{\partial \chi_{ij}}{\partial \varphi_{mn}} = 0$ if $m \neq i$ or $i - 1$ and $n \neq j$ or $j - 1$. It is advisable to construct matrix K using direct rigidity method. In our case this can be done easier as soon as each node point may belong to not more than 4 elements. Then

$$\frac{\partial \chi}{\partial \varphi_{ij}} = \frac{\partial \chi_{ij}}{\partial \varphi_{ij}} + \frac{\partial \chi_{i-1j}}{\partial \varphi_{ij}} + \frac{\partial \chi_{ij-1}}{\partial \varphi_{ij}} + \frac{\partial \chi_{i-1j-1}}{\partial \varphi_{ij}}.$$

K_{nm} depends on relative location of n and m nodes. If n and m nodes don't belong to the same element K_{nm} will equal to zero. If n and m nodes are located in the opposite angles of the same element, it means that K_{nm} will equal to the corresponding coefficient for for this element only. If n and m nodes are located on the same side of the of the element then K_{nm} will be equal to sum of two coefficients for the elements containing these nodes. And if $m = n$, then K_{nm} will be equal to the sum of 4 corresponding coefficients for each 4 elements having this node. It is possible to receive ecpression of these coefficients as (r_1, x_1) - coordinates of the upper left vartex of the element and r_e, h – width and height of the element with the help of two programmes using the language of analitic computation.

In this case matric K will be of tape type with the width of the strip $m_r + 2$. Besides this it will be symmetrical and positively defined which decreases the quantity of calculations necessary to get solution of the system of equations. Probability of serious errors due to the rounding is also decreasing. But before starting solution of the system it is necessary to assume some node values of φ_i. There is a procedure giving an opportunity to do this without worsening the properties of the matrix of rigidity. For example, if we assume φ_j, then j-th equation is substituted for and identity, i.e. φ_j is transposed into the right part $F_j = \varphi_j$, while $K_{jj} = 1$ and $K_{ij} = 0$ at $i = \overline{j - m_r - 2, j + m_r + 2}$. All the other equations are transformed by substraction of $K_{ij}\varphi_j$ from F_i and substitution of $K_{ij} = 0$. As a result of such transformation matrix K remains to be of tape, symmetrical and positively defined type. One of the most effective method of solving the system of equations received while using the finite elements method is the Gauss elimination method. In this way it is possible to receive the initial temperature distribution in the form of the table representing the node points values Φ_0. Using the same discretization of the field as in the standard task on heat conductivity, solution of non-stationary task on heat conductivity will come to the solution of the system of linear differential equations [3,4].

$$[C]\frac{\partial\{\Phi\}}{\partial t} + [K]\{\Phi\} = \{F\}$$

where K - global matrix of rigidity,
F - vector column of the right sides,
C - damped matrix.

Calculation of matrix C is similar to calculation of matrix K.

To receive values of $\{\Phi\}$ in every point of the time interval it is necessary to solve the linear differential equation. Let's do it numerically. Let's substitute $\frac{\partial \varphi}{\partial t} = \frac{(\varphi_1 - \varphi_2)}{\Delta t}$

where Δt – an interval, φ_1, φ_0 - the same node points values at different time intervals. In matrix form this will be written as [4] $\frac{\partial\{\Phi\}}{\partial t} = \frac{1}{\Delta t}(\{\Phi_1\} - \{\Phi_2\})$

Substituting this expression in the system of differential equations we'll get:

$$\left([K] + [C]\frac{1}{\Delta t}\right)\{\Phi_1\} = \frac{1}{\Delta t}[C]\{\Phi_0\} + \{F\}\,.$$

The final form of the system will be [4] $[A]\{\Phi\}_{n+1}=[P]\{\Phi\}_n+[F]$.

As $\{\Phi\}_0$ let's take the solution of the problem about the initial temperature distribution.

To solve the problem of the elasticity theory it is also possible to use division of the area into rectangle elements. But displacements will be the unknown quantity in this case. As compared to temperature displacement is a vector quantity that is why 2 unknown quantities will correspond to the each node point. Let $U_{2n-1} = U_{rn}$, and $U_{2n} = U_{xn}$. In this case dimension of the global matrix of rigidity will be $2m_r m_x$, and a strip width will be $2m_r + 3$. To solve the system of linear equations connected with the elasticity theory problem it is possible to use the same subprogrammes which are used to solve the problem of heat conductivity but taking into consideration that a displacement is a vector quantity it is necessary to change the function of filling the global matrix of rigidity. Expressions for these coefficients in terms of (r_1, x_1) - coordinates of the upper left vertex of the element and r_e, h - a width and a height of an element are received using the programm written on the lanuage of analitic computation. It is implied in this programme that functional for a separate element has the form:

$$\Pi^{(e)} = \int_{S^{(e)}} \{\sigma\}^T \{\varepsilon\} ds - \int_{S^{(e)}} \{\sigma\}^T \{\varepsilon_0\} ds$$

where $\{\sigma\}^{\mathrm{T}} = \{\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{xx}, \sigma_{rx}\}$ - stress,

$\{\varepsilon\}^{\mathrm{T}} = \{\varepsilon_{rr}, \varepsilon_{\theta\theta}, \varepsilon_{xx}, \varepsilon_{rx}\}$ - deformation,

$\{\varepsilon_0\} = \{\alpha\mathrm{T}, \alpha\mathrm{T}, \alpha\mathrm{T}, 0\}$ deformation resulted from temperature change.

Displacements within an element can be defined from the equality:

$$\begin{Bmatrix} U_r \\ U_x \end{Bmatrix} = \begin{bmatrix} n_1 & 0 & n_2 & 0 & n_3 & 0 & n_4 & 0 \\ 0 & n_1 & 0 & n_2 & 0 & n_3 & 0 & n_4 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix}$$

n_i – functions of the form, u_i – node values of displacements [4].

To find $\{\varepsilon\}$ let's use Cauchy relationships

$$\begin{Bmatrix} e_{rr} \\ e_{\theta\theta} \\ e_{xx} \\ e_{rx} \end{Bmatrix} = [B] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \end{Bmatrix}$$

Matrix B is received by differentiation of the function of the form [4]. As it goes from Hooke's law: $\{\sigma\} = [D]\{\varepsilon\}$. Having substituted the found quantities in the functional and differentiated with respect to u_i we'll receive:

$$[K^{(e)}] = \int_{S^{(e)}} [B]^T [D][B] ds \, .$$

By analogy expressions for the right side of each linear system are received

$$\{\Pi^{(e)}\} = \int_{S^{(e)}} [B]^T [D]\{\varepsilon_0\} ds \, .$$

The algorythm of finding the stress-deformed state and temperature field is realized in the form of the programme package based on Pascal. Several designs of fillings with compensator and without it made of different materials and under different temperature conditions have been calculated. Having analysed the constructed temperature and stress fields of the tooth-microdenture system it is possible to state that the greatest stresses are located at the boundary of the system and on the tooth-microdenture contact surface.

4. Analysis of Received Results

Numerical model allowed to receive data on stabilization period of the temperature conditions within a tooth-microdenture system. Analyses of the data received gives a large variation of this index in the range from 0,5 up to 120 seconds while heating and from 1.5 up to 120 seconds while cooling. Minimum heating and cooling period has a negative effect upon the connection strength between the microdenture and dentin. This is connected with the fact that the problem is turning into dynamic one in case of large heating and cooling period redistribution of stresses and decrease of its concentration is possible.The least indexes were found in metal constructions especially in cases of one-piece filling made of silver-palladium alloy. This is due to high heat conductivity of metal alloys and that of silver-palladium alloy in particular. Large stabilization period has been found in the systems with plastic and composite constructions. The largest period of stabilization has been noticed in case of composite insert with cement compensator. There is no difference in the temperature stabilizing periods of the system depending on construction in case of inserts made of gold, porcelain and stainless steel. It has been noticed that the heating and cooling periods are larger in case of using plastic and composite inserts and inserts made of silver-palladium alloy. It has been found that the heating period is greater by 3 folds in case of using composite inserts with metal and cement compensator and 2 folds in case of plastic insert with cement compensator. Data of theoretical investigations received show that the offered constructions with compensator allow to increase greatly temperature stabilizing period witin the tooth-microdenture system in case of using plastic and composite inserts ensuring the construction reliability due to the opportunity of stress redistribution and decrease of its concentration.

Besides data on depth of localization of maximum normal and tangent stresses at the boundary of microdenture-dentine while heating and cooling the tooth have been received. Appearence of maximum stresses at the contact boundary of the microdenture-tooth closer to the tooth face influences negatively upon connection between them. It is due to the fact that if crack is taking place there secondary (biological) factors of the system destruction to fact leading to the occurrence of the secondary caries. In case maximum stress is localized in the depth of the connection microdenture and dentine worseing biomechanical fixation of microdenture within the tooth. Such localization within the tooth-microdenture system is preferable from the view-point of construction reability. Data received on maximum normal and tangent stresses at the tooth-microdenture boundary as a result of such temperature influences as heating and cooling have shown these stresses occur on all the surfaces. So, while cooling maximum normal stresses at the external boundary of the junction of the tooth-microdenture system localize only near the surface in case of porcelain inserts with metal compensator, plastic insert with cement compensator and an insert made of silver-palladium alloy with cement compensator. While heating - in case of porcelain insert, porcelain insert with cement compensator, plastic insert with metal compensator, stainless stell inserts, inserts of stainless steel with cement compensator, silver-palladium alloy insert. Maximum tangent stresses at the external boundary of the joint of the tooth-microdenture system near the surface while cooling have been found in case of porcelain, composite and insert made of stainless steel as well as of composite insert, insert with metal and cement compensator. While cooling maximum tangent stress near the surface has been found in case with plastic inserts only. Analyses of localization of the stresses occured along the juncture into the depth are different for normal and tangent stresses. So location of maximum tangent stress into the depth at the end of the junction while-heating the system has been found only in case of composite insert with metal compensator.

Analyses of data on the depth of localization of maximum normal and tangent stresses at the microdenture dentin boundary while heating and cooling didn't allow to find any preferences depending on design of restoration construction microdenture-insert and insert with compensator. That is why estimation of microdenture constructions has been done based on their analyses taking into account absolute values of maximum stresses at the tooth-microdenture junction. Stress index occuring within a system without compensator referring to which index of other constructions has been counted was recognized as a 1. Comparative data on maximum stresses at the tooth-microdenture boundary show that the most monolith constructions have greater stresses than microdentures with compensators. The greatest difference was noticed in case of gold microdentures. Only in case with porcelain insert with metal compensator, silver-palladium insert with cement compensator greater stresses have been noticed as compared with monolith constructions. The greatest difference have been found in case of composite insert with metal compensator.

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ISTRAŽIVANJE RASPODELA TEMPERATURA I NAPONA U KONSTRUKCIJI ZUBNE PROTEZE

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U radu su prikazani rezultati istraživanja u temperaturskom i naponskom polju kompozitnih cilindričnih sistema. Ovo može biti model za zub sa plombom. Dinamički model zuba napravljen je korišćenjem metoda konačnih elemenata i rešen je na računaru. Dobijeni rezultati pokazuju da monolitne konstrukcije imaju veće napone nego mikrodenture sa kompenzatorima.