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Address: Univerzitetski trg 2, 18000 Niš, YU, Tel: (018) 547-095, Fax: (018)-547-950
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ON THE PROBLEM OF MICROPOLAR FLUID FLOW*

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Predrag Cvetković

Faculty of Transport and Trafic Engineering
Vojvode Stepe 305, Belgrade, Yugoslavia

Abstract. *In the present paper the theory of the micopolar fluid has been applied for analysis of stationary flow between two parallel plates. Possibility for application of new dynamic boundary condition which is presumed that the couple stress on the boundary surfaces has a certain value is being considered. The obtained results for the velocity and the microrotation velocity has been compared with known results and conclusion is that they are special case of results from this paper.*

1. INTRODUCTION

The theory of micro-fluids, introduced by A. C. Eringen [1], deals with a class of fluids which exhibit certain microscopic effects arising from the local structure and the micromotions of the fluid elements. A subclass of these is the micropolar fluids which have the microrotational effects and microrotational inertia [2]. This class of fluids possesses a certain simplicity and elegance in their mathematical formulation. The micropolar fluids can support the couple stress, the body couples, and the nonsymmetric stress tensor, and posses a rotation field, which is independent of the velocity fluid. The theory, thus, has two independent kinematic variables: the velocity vector $\vec{\upsilon}$, and the spin or microrotation velocity vector $\vec{\nu}$.

The linear constitutive equations for a nonsymmetric stress tensor contain an additional viscosity coefficient k, which describes the coupling between the velocity and the field. The linear constitutive equation for the couple stress also contains three additional viscosity coefficients α, β and γ.

In the present paper we are considering the possibility of applying a new boundary condition for the velocity of microrotation. Namely, we attempt to answer the question whether it is physically justifiable to take the couple stress on the boundary to be zero [3].

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The expression obtained for the velocity is graphically represented and compared with the corresponding expression for the case of the classical theory.

2. Equations of motion of a micropolar fluid

2.1 Constitutive Equations

The constitutive equations for the stress tensor t_{kl} and the couple stress tensor m_{kl} are given as [1]

$$t_{kl} = (-\pi + \lambda \upsilon_{r,r})\delta_{kl} + \mu(\upsilon_{k,l} + \upsilon_{l,k}) + k(\upsilon_{l,k} - \varepsilon_{klr}\nu_r) \tag{2.1}$$

$$m_{kl} = \alpha \nu_{r,r}\delta_{kl} + \beta \nu_{k,l} + \gamma \nu_{l,k}\,, \tag{2.2}$$

where the comma denotes the partial differentiation, and δ_{kl} and ε_{klr} are the Kronecker delta and the alternating tensor respectively.

2.2 Field Equations

The field equations for micropolar fluids in the vectorial form are given by:

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \vec{\upsilon}) = 0\,, \tag{2.3}$$

$$(\lambda + 2\mu + k)\nabla\nabla\vec{\upsilon} - (\mu + k)\nabla\times\nabla\times\vec{\upsilon} - k\nabla\times\vec{\nu} - \nabla\pi + \rho\vec{f} = \rho\dot{\vec{\upsilon}}\,, \tag{2.4}$$

$$(\alpha + \beta + \gamma)\nabla\nabla\vec{\nu} - \gamma\nabla\times\nabla\times\vec{\nu} + k\nabla\times\vec{\upsilon} - 2k\vec{\nu} + g\vec{l} = gj\dot{\vec{\nu}}\,, \tag{2.5}$$

where ρ, j, $\vec{f}$, $\vec{l}$ and π are the mass density, microinertia, body force per unit mass, body couple per unit mass and the thermodynamic pressure respectively, λ and μ are the viscosity coefficients of the classical fluid mechanics, and k, α, β and γ are the new viscosity coefficients for micropolar fluids. The dot denotes the material time derivative. From the local Clausius-Duhem inequality, the material coefficients must be subjected to the restrictions:

$$\begin{aligned} &k \geq 0\,,\ \ \mu \geq 0\,,\ \ \gamma \geq 0\,, \\ &3\lambda + 2\mu \geq 0\,,\ \ 3\alpha + \beta + \gamma \geq 0\,,\ \ -\gamma \leq \beta \leq \gamma\,. \end{aligned} \tag{2.6}$$

By the incompressibility assumption, the thermodynamic pressure π is an undetermined pressure p, which must be determined by the given boundary conditions, and the equation (2.3) is replaced by

$$\nabla\vec{\upsilon} = 0\,, \tag{2.7}$$

which is automatically satisfied.

2.3. Boundary conditions

By their own character, the boundary conditions can be twofold: kinematic and dynamic. The kinematic boundary conditions lie in the fact that the kinematic values, velocity and the angular velocity have definite quantities at the boundary. In the dynamic boundary conditions, however, on the boundary surfaces the values of the stress and the couple stress are fixed.

Let us suppose, as we do in the classical fluid mechanics, that the stress velocities of the fluid particles on the boundary surface are equal to that of the boundary surface itself, i.e.,

$$\upsilon^i = V^i , \tag{2.8}$$

where υ^i represents the velocity of the fluid particles, and V^i the boundary surface velocity.

In order to realize the existence of the couple stresses on the boundary surface, it must be assumed that there exists a rotational friction between the fluid particles and the boundary surface. If we assume that the rotational friction in the whole area is the greatest just on the boundary surface, then, for the dynamic boundary condition referring to the microrotation velocity, it will be assumed to have a certain fixed values, i.e.

$$M_k = m_{kl} n_l = const. , \tag{2.9}$$

where M_k is the couple stress, m_{kl} is the couple stress tensor, and n_k is the unit normal vector to the boundary.

3. THE FLOW BETWEEN TWO PARALLEL PLATES

The flow between two stationary parallel plates at the distance of $2h$ is defined by the pressure gradient (*Fig. 1*). The *Ox* axis overlaps the main line, the *Oy* axis is perpendicular to the flow, whereas the *Oz* axis is perpendicular to the plane of the flow. In this case the velocity components and the microrotation velocities are

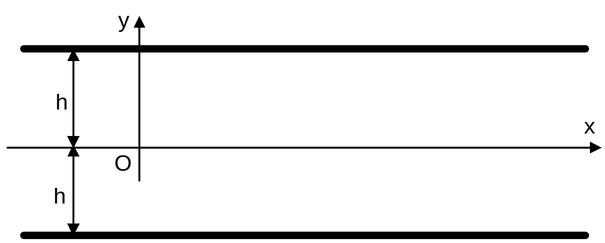

Fig. 1.

$$\upsilon_y = \upsilon_z = 0 , \quad \upsilon_x = \upsilon(y) , \tag{3.1}$$

$$\nu_x = \nu_y = 0 , \quad \nu_z = \nu(y) . \tag{3.2}$$

The conservation law is identically satisfied with $\rho = const$. By neglecting the body forces and body couples the equations of motion (2.4) and (2.5) are reduced to the form

$$(\mu + k)\frac{d^2\upsilon}{dy^2} + k\frac{d\nu}{dy} - \frac{dp}{dx} = 0 , \tag{3.3}$$

$$\gamma \frac{d^2 \nu}{dy^2} + k\frac{d\upsilon}{dy} - 2k\nu = 0\,. \tag{3.4}$$

The general solutions of the equation (3.3) and (3.4), for the velocity and microrotation velocity are

$$\begin{gathered}\upsilon = \frac{\gamma}{k}k(A\sin hky + B\cos hky) - \frac{2A}{\mathrm{k}}\sin hky - \frac{2B}{\mathrm{k}}\cos hky + \\ + \frac{2C}{2\mu + k}y + 2\frac{dp}{dx}\frac{1}{2\mu + k}y\,,\end{gathered} \tag{3.5}$$

$$\nu = A\cos hky + B\sin hky - \frac{C}{2\mu + k}y - \frac{dp}{dx}\frac{1}{2\mu + k}y\,, \tag{3.6}$$

where $k \equiv \left(\frac{2\mu + k}{\mu + k}\frac{k}{\gamma}\right)^{\frac{1}{2}}$, and A, B, C and D are the arbitrary integral constants. For the given case of the flow, the boundary conditions, according to (2.1), (2.2) and (2.7), are

$$y = h: \quad \upsilon(h) = 0\,, \quad \gamma\frac{d\nu}{dy}\bigg|_{y=h} = M\,, \tag{3.7}$$

$$y = -h: \quad \upsilon(\text{-}h) = 0\,, \quad \gamma\frac{d\nu}{dy}\bigg|_{y=-h} = M\,. \tag{3.8}$$

By using the boundary conditions (3.7) and (3.8), we can obtain the values for the integration constants

$$\begin{gathered}A = 0, \quad B = \left(\frac{M}{\gamma} + \frac{1}{2\mu + k}\frac{dp}{dx}\right)\left(\frac{\lambda}{h}\cos \mathrm{h}\lambda\right)^{-1}, \\ C = 0, \quad \mathrm{D} = \left(\frac{M}{\gamma} + \frac{1}{2\mu + k}\frac{dp}{dx}\right)\left(-\frac{\gamma}{k} + \frac{2h^2}{\lambda^2}\right) - \frac{1}{2\mu + k}h^2\frac{dp}{dx},\end{gathered} \tag{3.9}$$

where $\lambda \equiv kh$.

By substituting the integration constant values in (3.7) and (3.8), we obtain the solutions for the velocity and microrotation velocity for the case of the micropolar fluid between two parallel plates

$$\upsilon = \left(\frac{M}{\gamma} + \frac{1}{2\mu + k}\frac{dp}{dx}\right)\left(\frac{\cos\lambda y}{\cos\lambda h} - 1\right)\left(\frac{\gamma}{k} - \frac{2h^2}{\lambda^2}\right) + \frac{1}{2\mu + k}\frac{dp}{dx}(y^2 - h^2)\,, \tag{3.10}$$

$$\nu = \frac{1}{2\mu + k}\frac{dp}{dx}\left(\frac{\sin\lambda y}{\sin\lambda h} - y\right) + \frac{M}{\gamma}\frac{h\sin\lambda y}{\lambda\sin\lambda h}\,. \tag{3.11}$$

Unlike the results known in the literature, in the solutions for velocity and the microrotation velocity, there appears a term, as a consequence of the assumption of the existence of the couple stresses both in the fluid and on the boundary surfaces.

If in the solutions quoted above we take that $M = 0$, we obtain for the microrotation velocity the expressions identical to those given in the paper [4], i.e.,

$$\upsilon = \frac{1}{2\mu + k}\frac{dp}{dx}\left[(y^2 - h^2) + \left(\frac{\cos\lambda y}{\cos\lambda h} - 1\right)\left(\frac{\gamma}{k} - \frac{2h^2}{\lambda^2}\right)\right], \tag{3.12}$$

$$\nu = \frac{1}{2\mu + k}\frac{dp}{dx}\left(\frac{\sin\lambda y}{\sin\lambda h} - y\right). \tag{3.13}$$

If in the solutions (3.12) and (3.13) we take $k = 0$, they are then reduced to the case of the classical fluid flow,

$$\upsilon = \upsilon_{kl} = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - h^2), \quad \nu = 0. \tag{3.14}$$

In the addition to that, if in the expression for velocity (3.10), we take

$$M = -\frac{\gamma}{2\mu + k}\frac{dp}{dx}, \tag{3.15}$$

we obtain the solution for the velocity of the classical fluid flow.

The expression (3.15) calls for a discussion. Namely, let us suppose that the couple stress varies within the limits

$$-\frac{\gamma}{2\mu + k}\frac{dp}{dx} \le M \le 0. \tag{3.16}$$

For each M higher than $-\frac{\gamma}{2\mu + k}\frac{dp}{dx}$, the velocity values are lower than classical velocity v_{kl}, whereby the lowest velocities are obtained for $M = 0$, as illustrated in (*Fig. 2*). The classical velocity is obtained for $k/\mu = 0$, whereas in the case when $M = 0$, the lowest velocity is obtained for $k/\mu = \infty$.

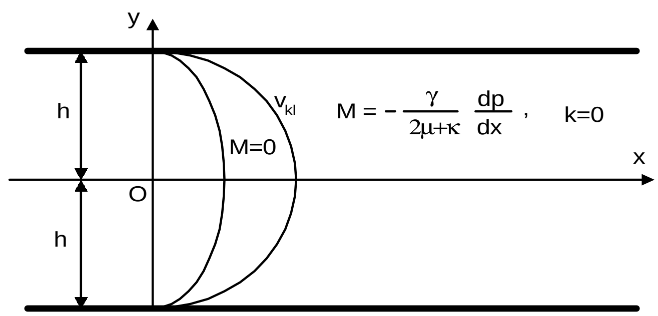

Fig. 2.

From (3.10) we obtain volumetric flow rate

$$Q = Q_0\left[1 - \frac{3A}{2h^4}\left(\frac{\gamma}{k} - \frac{2}{\mathrm{k}^2}\right)\mathrm{tg}\, h\lambda - \frac{3}{2}\frac{2\mu + k}{h^4}\frac{M\lambda}{\gamma}\,\mathrm{tg}\, h\lambda\left(\frac{\gamma}{k} - \frac{2}{\mathrm{k}^2}\right)\right], \tag{3.17}$$

where $Q_0 = -\frac{4}{3}\frac{h^3}{2\mu + k}\frac{dp}{dx}$. Note that with $M = 0$ we obtain for volumetric flow expression identical to those given in the paper [4], and for $k = 0$ flow is described by classic Poiseuille formula.

4. CONCLUSION

In the present paper the theory of the micropolar fluid has been applied to an actual flow. The assumption that the couple stress on the boundary surfaces has a certain value, which has been taken as a dynamic boundary condition, has caused some differences from the results for velocity and the microrotation velocity of the flow under consideration known up to the present. In the boundary conditions formulated in this way, there has been raised the question of how high the couple stresses on the boundary surfaces are, but such assumption has, nevertheless, made it possible to incite a discussion on the character of the effects of the surface couple stresses on the given flow of a micropolar fluid.

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O PROBLEMU STRUJANJA MIKROPOLARNOG FLUIDA

Predrag Cvetković

U radu je primenjena teorija mikropolarnih fluida za analizu stacionarnog strujanja između dve paralelne ploče. Razmatra se mogućnost primene novog dinamičkog graničnog uslova kojim se pretpostavlja da na graničnim površinama naponski spreg ima neku vrednost. Dobivena rešenja za brzinu i brzinu mikrorotacije upoređuju se sa do sada poznatim rešenjima i zaključuje se da su ona specijalni slučaj rešenja iz rada.