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HOPF BIFURCATIONS AND FLUTTER INSTABILITY OF AUTONOMOUS POTENTIAL DISSIPATIVE SYSTEMS

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Abstract. *This paper deals with perfect bifurcational discrete dissipative systems with trivial precritical states under mainly conservative compressive loading. Attention is focused on the conditions under which these symmetric autonomous, weakly damped systems can exhibit a limit cycle response due to a Hopf bifurcation or to a double zero eigenvalue or to flutter instability.*

1. INTRODUCTION

In various recent studies of the author and his associates [1,2,3,4] criteria for the occurrence of *limit cycles* in *non-potential* (nonconservative), weakly damped, systems in regions of divergence were presented.

In this work a thorough local analysis for seeking the conditions of existence of Hopf bifurcations, in *symmetric*, weakly damped, systems as well as flutter (existence of a pair of complex conjugate eigenvalues with positive real part) or *coupled flutter-divergence instability* (associated with a double zero eigenvalue) are comprehensively discussed. To this end attention is focused on the nature of the damping matrix, whose effect is of paramount importance for the occurrence of the above instability phenomena.

2. MATHEMATICAL ANALYSIS

Consider a general N-DOF, N-Mass, nonlinear discrete dissipative system with trivial precritical state under partial follower compressive *load* λ with *nonconservativeness* loading parameter η *Lagrange* equations of motion of this autonomous nonpotential dissipative system in terms of generalized displacements q_i and generalized velocities $\dot{q}_i$, $(i = 1,...,n)$ are given by

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$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}_i}\right)-\frac{\partial K}{\partial q_i}-\frac{\partial F}{\partial \dot{q}_i}+\frac{\partial U}{\partial q_i}-Q_i=0\,,\quad i=1,...n \tag{1}$$

where the dots denote differentiation with respect to time t; $K=(1/2)\alpha_{ij}\dot{q}_i\dot{q}_j$ is the positive definite function of the total kinetic energy (in tensor Einstein's notation) with diagonal elements, being functions of masses m_i (i.e. $\alpha_{ii}=\alpha_{ii}(m_i)$) and non-diagonal elements α_{ij}, being functions of m_i and q_i (i.e. $\alpha_{ii}=\alpha_{ii}(m_i,q_i)$ for $i\neq j$); $F=(1/2)c_{ij}\dot{q}_i\dot{q}_j$, is a positive definite, or positive semi-definite or indefinite *dissipation* function with elements c_{ij}, being functions only of damping coefficients c_i (i.e. $c_{ij}=c_{ij}(c_i)$); $U=U(q_i;k_i)$ is the positive definite function of the strain energy, being a nonlinear function with respect to q_i and *linear* with respect to stillness coefficients k_i; $Q_i=\lambda\overline{Q}_i(q_i;\eta)$ are generalized *non-potential* external forces, being nonlinear functions of q_i and η. For a certain value $\eta=\eta_c$, the external forces become potential (conservative), while for $\eta\neq\eta_c$ the system is non-potential (asymmetric). Clearly, the damping matrix $[c_{ij}]$ may be positive definite, positive semi-definite or indefinite.

The static *equilibrium* equations and *buckling* equations are given by

$$\left.\begin{aligned} &V_i=U_i-\lambda\overline{Q}_i=0,\quad i=1,..,n\\ &\det[V_{ij}]=\det([U_{ij}]-\lambda[\overline{Q}_{ij}])=0\end{aligned}\right\} \tag{2}$$

where $U_i=\partial U/\partial q_i$, and $[U_{ij}]$ is *symmetric* matrix, while $[\overline{Q}_{ij}]=[\partial\overline{Q}_i/\partial q_i]$ is a *square* non-singular **asymmetric** matrix for $\eta\neq\eta_c$ (non-self-adjoint system), while for $\eta=\eta_c$ the matrix $[\overline{Q}_{ij}]$ becomes equal to the *identity* matrix; $[\overline{Q}_{ij}(\eta_c)]\equiv I$. Thus, the final stiffness matrix $[V_{ij}]$, if λ is a compressive partial follower force, due to the 2^{nd} of eqs (2) has elements equal to the elements of the **stiffness matrix** $[U_{ij}]$ minus the corresponding elements of the matrix $\lambda[\overline{Q}_{ij}]$. Hence, $[V_{ij}(\lambda)]$ is an *asymmetric* matrix, becoming symmetric for $\eta=\eta_c$. Clearly, the determinant of the **generalized stiffness** matrix $[V_{ij}(\lambda)]$ *decreases* as the compressive load λ *increases*. The second of eqs(2), being the *buckling equation*, is a n^{th} degree algebraic polynomial with respect to λ. From this equation we get $\lambda^C_{(i)}$ ($i=1,...,n$), i.e. the successive buckling loads. Obviously, $\det[V_{ij}(\lambda)]>0$ for $\lambda<\lambda^C_{(1)}$, $\det[V_{ij}(\lambda)]<0$ for $\lambda>\lambda^C_{(1)}$ [excluding the case of a double point $0(\eta_o,\lambda^C_o)$ mentioned below], while $\det[V_{ij}(\lambda)]=0$ for $\lambda=\lambda^C_{(1)}$. Recall that for a 3-DOF [2] and a 2-DOF [3] cantilever model we have respectively

$$[V_{ij}]=\begin{bmatrix}\overline{k}_1+\overline{k}_2-\lambda & -\overline{k}_2 & \lambda(1-\eta)\\ -\overline{k}_2 & \overline{k}_2+1-\lambda & -1-\lambda(\eta-1)\\ 0 & -1 & 1-\lambda\eta\end{bmatrix},\quad [V_{ij}]=\begin{bmatrix}\overline{k}_1+1-\lambda & -1-\lambda(\eta-1)\\ -1 & 1-\lambda\eta\end{bmatrix} \tag{3}$$

where $\overline{k}_1$ and $\overline{k}_2$ are stiffness ratios.

The *boundary* between existence and *non-existence* of adjacent equilibria (η_o,λ^C_o) is established by employing the theorem for *implicit* functions [5] via the solution of the system of equations

$$\det[V_{ij}]^C = \frac{d}{d\lambda}\left(\det[V_{ij}]^C\right) = 0 \tag{4}$$

evaluated at the critical **divergence** state C. A typical plot η vs λ^C showing this boundary (η_o, λ_o^C) denoted by point 0 is shown in Fig. 1. Usually η_o is a minimum in the curve η vs λ^C. To the left of point 0 we have always *dynamic* (flutter) instability, while to the right of this point according to *classical* analyses we have *divergence*. However, as was shown by Kounadis [1], this is *not always true*, since in the shaded area of Fig. 1 both types (i.e. static and dynamic) of instability may occur. As aconsequence of this various phenomena may occur, as for instance:

- Possible *failure* of Ziegler criterion for establishing the actual critical load in the *shaded* area.
- The degree of (nonconservativeness) *asymmetry* of the stiffness matrix *may not* be related to the *static* (divergence) type of instability, contrary to Huseyin & Leipholz analysis [6], since as was shown in previous studies a non-potential system with given degree of asymmetry may exhibit both types of dynamic instability.
- *Asymmetric* (non-potential) systems with *symmetrizable* damping and stiffness matrices may exhibit (contrary to Inman study [7]) *dynamic* instability.

Subsequently, the *nature* of the *damping* matrix $[c_{ij}]$ in connection with the existence of a *Hopf* bifurcation will be discussed. The pertinent study will be based on a stiffness matrix $[V_{ij}]$ of general nature and not on the above one of eq. (3) associated with a concrete cantilever model.

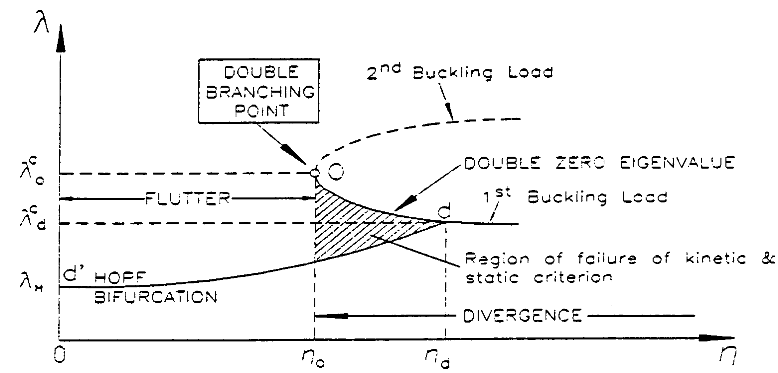

Fig. 1. Typical curve $\eta = \eta(\lambda_C)$ showing point 0 (boundary between flutter and divergence instability) and point d (defining the region $\eta_o \le \eta \le \eta_d$ for a double zero eigenvalue or a Hopf bifurcation).

Hopf bifurcation and matrix $[c_{ij}]$

Lagrange equation of motion, eq. (1), after *linearization*, leads to the following equation of motion

$$[\alpha_{ij}]\ddot{\mathbf{q}} + [c_{ij}]\dot{\mathbf{q}} + [V_{ij}]\mathbf{q} = 0 \tag{5}$$

One can seek solutions of eq.(5) in the form

$$\mathbf{q} = \mathbf{r}e^{\rho t} \tag{6}$$

where ρ is, in general, a *complex* number and $\mathbf{r}$ the corresponding *complex* vector, independent of t.

Substituting this expression of $\mathbf{q}$ into eq.(5) leads to

$$L(\rho)\mathbf{r} = ([\alpha_{ij}]\rho^2 + [c_{ij}]\rho + [V_{ij}])\mathbf{r} = 0 \tag{7}$$

where $L(\rho) = [\alpha_{ij}]\rho^2 + [c_{ij}]\rho + [V_{ij}]$ is a matrix-valued function. Solutions of eq. (5) are intimately related to the algebraic properties of $L(\rho)$, and more specifically to the nature of *Jacobian* eigenvalues ρ_i (i=1,...,2n) obtained by solving the characteristic (secular) equation

$$\det L(\rho)\mathbf{r} = |[\alpha_{ij}]\rho^2 + [c_{ij}]\rho + [V_{ij}]| = 0 \tag{8}$$

which guarantees the existence of *nontrivial* solutions of eq. (5) or (7). Expansion of eq. (8) leads to the following equation [1]

$$\rho^{2n} + \alpha_1\rho^{2n-1} + \alpha_2\rho^{2n-2} + ... + \alpha_{2n-1}\rho + \alpha_{2n} = 0 \tag{9}$$

where α_i (i=1,...,2n) are determined through Bocher recurrence formulae.

As stated above eigenvalues (roots) of eq. (8) occur in complex conjugate pairs $\rho_i = \nu_i + \mu_i j$ (where $j = \sqrt{-1}$, i = 1,...,n and μ, ν real numbers) with corresponding *complex* eigenvectors. Hence, solutions of eq. (5) associated with eq. (6) are of the form

$$Ae^{\nu_i t}\cos\mu_i t \quad \text{and} \quad Be^{\nu_i t}\sin\mu_i t \tag{10}$$

which are *bounded* tending to zero αs $t \to \infty$, if all eigenvalues of eq. (9) have negative real parts [8]. According to Routh-Hurvitz *stability criteria* a *necessary* condition in order that all eigenvalues have *negative* real parts is $\alpha_i > 0$ for all i, while a *sufficient* condition assuring this is all Routh-Hurwitz determinants Δ_i of even (or odd) order to be *positive* [8]. Moreover, a necessary and sufficient condition for all eigenvalues to lie in the left-hand side of the complex plane is $\Delta_i > 0$ for all i. Then, eq. (9) has complex conjugate eigenvalues of the above form, i.e. $\rho_i = \nu_i \pm \mu_i j$, where $\nu_i < 0$ and $\mu_i > 0$ (i = 1,...,n). Subsequently, we focuse attention on *symmetric* (potential) systems occurring for $\eta = \eta_C$ (which implies $V_{ij} = V_{ji}$).

3. Symmetric systems

Premultiplying eq. (7) by $\mathbf{r}^T$, the transpose vector of $\mathbf{r}$, one can obtain

$$\mathbf{r}^T([\alpha_{ij}]\rho^2 + [c_{ij}]\rho + [V_{ij}])\mathbf{r} = 0 \tag{11}$$

where clearly all *quadratic* forms are real *scalar* quantities. Thus, eq. (11) is a 2nd degree polynomial with respect to ρ from which we get

$$\rho = \frac{1}{2\mathbf{r}^T[\alpha_{ij}]\mathbf{r}}\left[-\mathbf{r}^T[c_{ij}]\mathbf{r} \pm \sqrt{(\mathbf{r}^T[c_{ij}]\mathbf{r})^2 - 4(\mathbf{r}^T[\alpha_{ij}]\mathbf{r})(\mathbf{r}^T[V_{ij}]\mathbf{r})}\right] = 0 \tag{12}$$

Since both $[\alpha_{ij}]$ and $[c_{ij}]$ are given *symmetric* matrices with constant real elements, only the stiffness matrix $[V_{ij}] = [V_{ij}(\lambda)]$ depends on λ. As the loading increases from zero, the variation of the madix $[V_{ij}(\lambda)]$ influences the complex conjugate eigenvalues ρ and the corresponding complex eigenvectors $\mathbf{r}$.

Assuming, due to physical considerations weak damping, from eq. (12) one can observe the following:

Case (a)

If matrix $[c_{ij}]$ is *positive definite*, the quadratic form $\mathbf{r}^T[c_{ij}]\mathbf{r}$ is always a *positive* quantity for $\mathbf{r} \neq 0$, while the quantity under the radical of eq. (12) is *negative*. Then, eq. (12) yields *complex* conjugate eigenvalues with *negative* real parts as long as $\lambda < \lambda^C_{(1)}$, due to which $\det[V_{ij}] > 0$. At the static (divergence) *critical state* C, occurring $\lambda = \lambda^C_{(1)}$ we have, as stated above, $\det[V_{ij}(\lambda^C_{(1)})] = \det[V_{ij}]^C = 0$. Then, eq. (12) has one *zero* eigenvalue and one *negative* eigenvalue. Hence, if the nondissipative system is *stable*, the dissipative system is asymptotically stable provided that the damping matrix $[c_{ij}]$ is *positive definite*.

Case (b)

We consider now the case for which matrix $[c_{ij}]$ is *positive semi-definite* (i.e. when $\det[c_{ij}]=0$). If $\lambda < \lambda^C_{(1)}$, matrix $[V_{ij}]$ is positive definite and then the eigenvalues (depending on λ) of eq. (9) and the corresponding eigenvectors are associated with $\mathbf{r}^T[c_{ij}]\mathbf{r} > 0$. Thus, from eq. (12) it follows that all eigenvalues have *negative* real parts. At a *certain* value of λ, the corresponding quadratic quantity becomes equal to zero, i.e.

$$\mathbf{r}^T[c_{ij}]\mathbf{r} = 0 \tag{13}$$

Since matrix $[c_{ij}]$ is positive semi-definite, eq. (13) implies

$$[c_{ij}]\mathbf{r} = 0 \tag{14}$$

Introducing eq. (14) into eq. (11) it is deduced that $\mathbf{r}$ is also an eigenvector of the conservative system [9]

$$([\alpha_{ij}]\rho^2 + [V_{ij}])\mathbf{r} = 0 \tag{15}$$

Clearly, if $\mathbf{r}$ is an eigenvector of the nondissipative system (15) satisfying eq. (14), then $\mathbf{r}$ is also an eigenvector of the dissipative system of eq. (7) with corresponding eigenvalue (resulting from eq. (12)) which is *imaginary*. Setting $\rho = \pm j\mu$ into eq. (15) one can determine the eigenvector which is *real*. Clearly, since $[c_{ij}]$ is a positive *semi-definite* matrix one of its eigenvalues is zero, and hence

$$([c_{ij}]\text{-}0I_n)\mathbf{r} = 0 \tag{16}$$

From this equation [or eq.(14)] one can establish the real eigenvector $\mathbf{r}$ for a given damping matrix $[c_{ij}]$. Then, eq. (12) leads to

$$-\rho^2 = \mu^2 = \frac{\mathbf{r}^T[V_{ij}(\lambda)]\mathbf{r}}{\mathbf{r}^T[\alpha_{ij}]\mathbf{r}} \tag{17}$$

from which we can establish μ as a function of the load λ for a given matrix $[\alpha_{ij}]$.

Introducing $\mathbf{r}$ [obtained from eq. (16)] and μ^2 [obtained from eq. (17)] into eq. (15) one can determine the *critical flutter* load λ_F which is accepted if

$$0 < \lambda_F < \lambda_{(1)}^C = 0.381966011 \tag{18}$$

For instance, for a 2-DOF system related to a cantilever damped model [1,2] we have

$$[\alpha_{ij}] = \begin{bmatrix} 1+m & 1 \\ 1 & 1 \end{bmatrix}, \ (m>0), \ [c_{ij}] = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \text{ and } [V_{ij}] = \begin{bmatrix} 2-\lambda & -1 \\ -1 & 1-\lambda \end{bmatrix} \tag{19}$$

We choose the following positive semi-definite matrix: $c_{11} = 0.2$, $c_{22} = 0.05$, $c_{12} = c_{21} = 0.1$.

Clearly, the mass ratio m is a free ranging parameter which will be adjusted so that inequality (18) be satisfied.

Using eq. (16) we find the eigenvector

$$r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = -\begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{or} \quad r_1/r_2 = -0.5 \tag{20}$$

On the other hand, from eqs (15) and (19) we also obtain

$$r_1/r_2 = -\frac{V_{12} - \mu^2\alpha_{12}}{V_{11} - \mu^2\alpha_{11}} = \frac{\mu^2 + 1}{-\mu^2(m+1) + 2 - \lambda} \tag{21}$$

Combining eqs (20) and (21) we find

$$\lambda = 4 - (m-1)\mu^2 \tag{22}$$

From eq. (17) we can determine μ^2 as follows

$$\mu^2 = \frac{10 - 5\lambda}{m+1} \tag{23}$$

Introducing this expression of μ^2 into eq. (22) we find

$$\lambda = \frac{7-3m}{3-2m}, \quad (\text{for } m \neq 1.5) \tag{24}$$

Due to eq. (18), eq. (24) yields

$$m > 7/3 \tag{25}$$

Clearly, this value of m depends on the structure of the damping matrix $[c_{ij}]$. For $m = 2.5$ we get from eq. (24) the *flutter* load

$$\lambda = \lambda_F = 0.25 < \lambda_{(1)}^C = 0.5(3 - \sqrt{5}) = 0.381966011 \tag{26}$$

The last result can also be obtained as follows.
With the aid of the characteristic equation

$$\rho^4 + \alpha_1\rho^3 + \alpha_2\rho^2 + \alpha_3\rho + \alpha_4 = 0 \tag{27}$$

one can also establish the above value of the flutter load λ_F taking into account that

$$\left.\begin{aligned} \alpha_1 &= \frac{1}{m}[0.5(1+m)] = \frac{m+1}{20m} \\ \alpha_2 &= \frac{1}{m}[m+5-\lambda(m+2)] \\ \alpha_3 &= \frac{1}{m}(0.5-0.25\lambda. \\ \alpha_4 &= \frac{1}{m}(\lambda^2-3\lambda+1) \end{aligned}\right\} \tag{28}$$

Introducing $\rho = \pm\mu i$ into eq. (27) we find

$$\left.\begin{aligned} \mu^2 &= \alpha_3/\alpha_1 \\ \text{and}\quad \mu^4 + \alpha_2\mu^2 + \alpha_4 &= 0 \end{aligned}\right\} \tag{29}$$

from which we obtain the *necessary condition* for a Hopf bifurcation [1]

$$(\alpha_1\alpha_2 - \alpha_3)\alpha_3 - \alpha_1^2\alpha_4 = 0 \tag{30}$$

Eq. (30) due to relations (28) leads to the following 2nd degree algebraic equation

$$(2m^2 - 6m + 4.5)\lambda^2 - (6m^2 - 23m + 21)\lambda + 4.5m^2 - 21m + 24.5 = 0 \tag{31}$$

which has a *d o u b l e* root, and hence

$$\lambda_{(1)} = \lambda_{(2)} = \lambda_F = \frac{6m^2 - 23m + 21}{4m^2 - 12m + 9} = \frac{7-3m}{3-2m} \tag{32}$$

Thus, we have rederive formula (24).

From the above analysis, one can infer the following important *conclusions*:
If the damping matrix $[c_{ij}]$ is positive semi-definite, then under certain conditions associated with the magnitude of the mass ratio m, the *s y m m e t r i c* (potential) systems, contrary to widely accepted (classical) findings, may exhibit a Hopf bifurcation (i.e. limit cycles).

Moreover, it is worth observing that although such a local bifurcation corresponds to a pair of *purily imaginary* eigenvalues, the corresponding eigenvector is r e a l. Furthermore, in case of a 2-DOF model the necessary condition(*) for a Hopf bifurcation leads to a 2nd degree algebraic polynomial in λ which has a d o u b l e root, being the *critical flutter* load λ i.e. $\lambda = \lambda_F$. Note also that if m is kept *constant* and λ is slightly

(*) This condition is also satisfied in case of existence of equal and opposite sign roots [8].

higher than λ_F (i.e. $\lambda^C_{(1)} > \lambda > \lambda_F$) the system yields a *point attractor* response. This is so, since excluding the case $\lambda = \lambda_F$ (i.e. for $\lambda \neq \lambda_F$), the quadratic form $\bar{\mathbf{r}}^T[c_{ij}]\mathbf{r}$ in eq. (12) is positive and thus its *real* part is *negative*. Hence, as the loading λ increases gradually from zero, at a certain value of λ, the symmetric system exhibits an *isolated* Hopf bifurcation with a double root of eq. (31).

In closing with case (b), one can observe that if the generalized stiffness matrix $[V_{ij}]$ is positive semi-definite (occurring for $\lambda = \lambda^C_{(1)}$), eq. (12) yields a *zero* eigenvalue corresponding to *divergence* (static) instability.

Case (c)

We now consider the case of an *indefinite* (symmetric) damping matrix $[c_{ij}]$. From a brief discussion one can observe the following:

For λ **sufficiently** small, the quadratic complex form $\mathbf{r}^T[c_{ij}]\mathbf{r}$ takes positive values and thus from eq. (12) it is clear that all eigenvalues ρ have *negative* real parts. Then, the system exhibits a *point attractor* response. At a certain value of λ, this form *vanishes* yielding a pair of *purily imaginary* eigenvalues (case of a Hopf bifurcation, i.e. $\lambda = \lambda_F$). For λ slightly higher than λ_F, the quadratic form becomes *negative* and thus a pair of eigenvalues has *positive* real part. The trivial state is *locally unstable* but *globally stable*. This situation of *bounded* amplitude oscillation is called *flutter*. Flutter occurs also when there exists one pair of complex conjugate eigenvalues with positive real part.

Case (d)

Another important case is associated with a positive *semi-definite* stiffness matrix $[V_{ij}]$ (i.e. $|V_{ij}(\lambda^C_1)| = 0$) which will be discussed in connection with a *positive semi-definite* damping matrix $[c_{ij}]$. The question that now arises is which must be the structure of the matrix $[c_{ij}]$ so that

$$\lambda_F = \lambda^C_{(1)} \tag{33}$$

From eq. (12) it follows that in this case we have a double zero eigenvalue (clearly, $\nu = \mu = 0$). On the other hand the necessary condition for a *Hopf bifurcation* in terms of *Routh-Hurwitz* determinants is given by

$$\Delta_{2n-1} = \alpha_{2n-1} M_{2n-1} - \alpha_{2n} M_{2n} = 0 \tag{34}$$

where M_{2n-1}, and M_{2n} are the *minors* (determinants) of the elements α_{2n-1}, and α_{2n} of the matrix Δ_{2n-1}, whose expression is [2]

$$\Delta_{2n-1} = \begin{vmatrix} \alpha_1 & 1 & 0 & 0 & 0 & \cdots \\ \alpha_3 & \alpha_2 & \alpha_1 & 1 & 0 & \cdots \\ \alpha_5 & \alpha_4 & \alpha_3 & \alpha_2 & 1 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & \cdots & \cdots & 0 & \alpha_{2n} & \alpha_{2n-1} \end{vmatrix} \tag{35}$$

Clearly, if $\alpha_{2n-1} = \alpha_{2n} = 0$, eq. (9) has a *double zero* eigenvalue [2,10] but at the same time the equation for a Hopf bifurcation is satisfied. Then, the corresponding solution is associated with a limit cycle response [1]. The critical (trivial) state is *unstable* but the global response is stable. Such a situation is called *coupled divergence-flutter instability*. Namely, the last case corresponds to a *special* type of Hopf bifurcation.

4. Numerical results

In this section numerical results under graphical form in the form of phase-plane portraits illustrate and confirm the above theoretical findings. *Symmetric* (potential) systems of 2-DOF, weakly damped, cantilever (Ziegler's) models experiencing stable limit cycles related to a *Hopf* bifurcation (Fig. 2), to *flutter* (Fig. 3) or to a *double zero* eigenvalue (Fig. 4) are presented below.

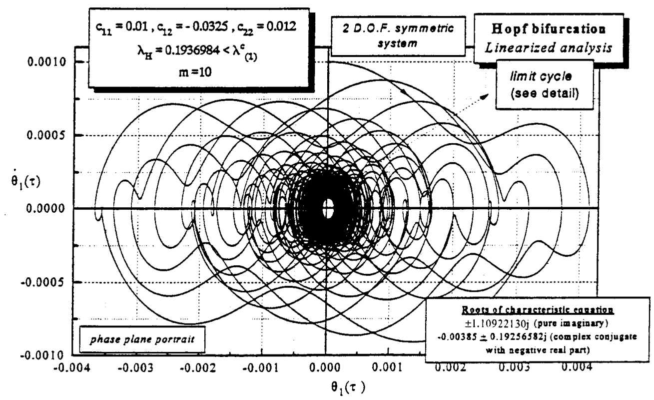

Fig. 2. Hopf bifurcation for a 2-DOF symmetric damped system.

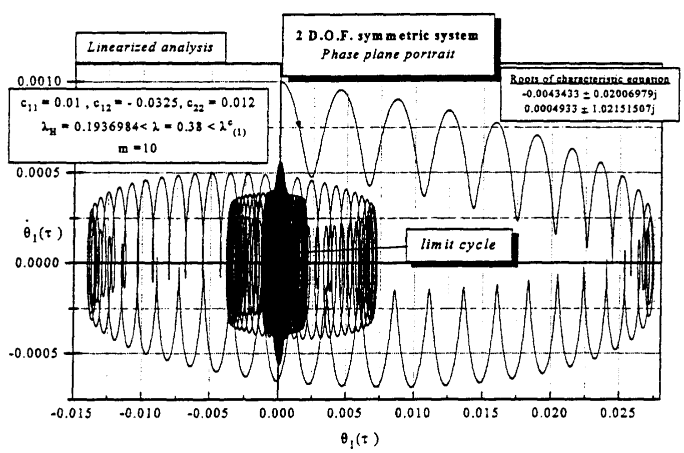

Fig. 3. Flutter instability (with a pair of complex conjugate eigenvalues with positive real part) associated with stable limit cycles.

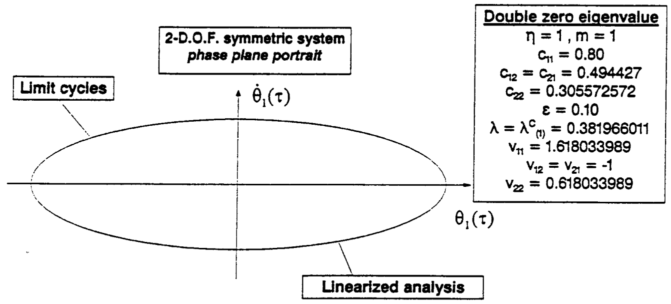

Fig. 4. Double zero eigenvalue bifurcation for a 2-DOF symmetric damped system associated with stable limit cycles.

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HOPF-OVE BIFURKACIJE I NESTABILNOSTI FLATERA AUTONOMNIH POTENCIJALNIH DISIPATIVNIH SISTEMA

Anthony N. Kounadis

Ovaj rad se bavi idealnim bifurkacionim diskretnim disipativnim sistemima sa trivijalnim prekritičnim stanjima pod dejstvom glavnog konzervativnog pritisnog opterećenja. Pažnja je usmerena na uslove pod kojima ti simetrični autonomni slabo prigušeni sistemi mogu ispoljiti odgovor tipa graničnog ciklusa usled Hopf-ove bifurkacije ili usled dvostruke nule sopstvene urednosti ili nestabilnosti tipa flatera.