HOPF BIFURCATIONS AND FLUTTER INSTABILITY OF AUTONOMOUS POTENTIAL DISSIPATIVE SYSTEMS

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Abstract. This paper deals with perfect bifurcational discrete dissipative systems with trivial precritical states under mainly conservative compressive loading. Attention is focused on the conditions under which these symmetric autonomous, weakly damped systems can exhibit a limit cycle response due to a Hopf bifurcation or to a double zero eigenvalue or to flutter instability.

1. INTRODUCTION

In various recent studies of the author and his associates [1,2,3,4] criteria for the occurrence of limit cycles in non-potential (nonconservative), weakly damped, systems in regions of divergence were presented. In this work a thorough local analysis for seeking the conditions of existence of Hopf bifurcations, in symmetric, weakly damped, systems as well as flutter (existence of a pair of complex conjugate eigenvalues with positive real part) or coupled flutter-divergence instability (associated with a double zero eigenvalue) are comprehensively discussed. To this end attention is focused on the nature of the damping matrix, whose effect is of paramount importance for the occurrence of the above instability phenomena.

2. MATHEMATICAL ANALYSIS

Consider a general N-DOF, N-Mass, nonlinear discrete dissipative system with trivial precritical state under partial follower compressive load \( \lambda \) with nonconservativeness loading parameter \( \eta \) Lagrange equations of motion of this autonomous nonpotential dissipative system in terms of generalized displacements \( q_i \) and generalized velocities \( \dot{q}_i \), \( (i = 1, \ldots, n) \) are given by

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\[
\frac{d}{dt} \left( \frac{\partial K}{\partial \dot{q}_i} \right) - \frac{\partial K}{\partial q_i} + \frac{\partial U}{\partial q_i} - Q_i = 0, \quad i = 1,...,n
\]

where the dots denote differentiation with respect to time \(t\); \(K = (1/2)\alpha_{ij}q_iq_j\) is the positive definite function of the total kinetic energy (in tensor Einstein's notation) with diagonal elements, being functions of masses \(m_i\) (i.e. \(\alpha_{ii} = \alpha_i(m_i)\)) and non-diagonal elements \(\alpha_{ij}\), being functions of \(m_i\) and \(q_i\) (i.e. \(\alpha_{ij} = \alpha_{ij}(m_i,q_i)\) for \(i \neq j\)); \(F = (1/2)c_{ij}q_iq_j\) is a positive definite, or positive semi-definite or indefinite dissipation function with elements \(c_{ij}\), being functions only of damping coefficients \(c_i\) (i.e. \(c_{ij} = c_{ij}(c_i)\)); \(U = U(q_i;k_i)\) is the positive definite function of the strain energy, being a nonlinear function with respect to \(q_i\) and linear with respect to stillness coefficients \(k_i\); \(\eta_i(q_i;\lambda)\) are generalized non-potential external forces, being nonlinear functions of \(q_i\) and \(\eta\). For a certain value \(\eta = \eta_c\), the external forces become potential (conservative), while for \(\eta \neq \eta_c\) the system is non-potential (asymmetric). Clearly, the damping matrix \([c_{ij}]\) may be positive definite, positive semi-definite or indefinite.

The static equilibrium equations and buckling equations are given by

\[
\begin{align*}
V_i = U_i - \lambda \bar{Q}_i &= 0, \quad i = 1,...,n \\
\text{det}[V_{ij}] = \text{det}([U_{ij}] - \lambda [\bar{Q}_{ij}]) &= 0
\end{align*}
\]

where \(U_i = \partial U/\partial q_i\), and \([U_{ij}]\) is symmetric matrix, while \([\bar{Q}_{ij}] = [\partial \bar{Q}_i/\partial q_j]\) is a square non-singular asymmetric matrix for \(\eta \neq \eta_c\) (non-self-adjoint system), while for \(\eta = \eta_c\) the matrix \([\bar{Q}_{ij}]\) becomes equal to the identity matrix; \([\bar{Q}_{ij}(\eta_c)] = I\). Thus, the final stiffness matrix \([V_{ij}]\), if \(\lambda\) is a compressive partial follower force, due to the 2\(^{nd}\) of eqs (2) has elements equal to the elements of the stiffness matrix \([U_{ij}]\) minus the corresponding elements of the matrix \(\lambda [\bar{Q}_{ij}]\). Hence, \([V_{ij}(\lambda)]\) is an asymmetric matrix, becoming symmetric for \(\eta = \eta_c\). Clearly, the determinant of the generalized stiffness matrix \([V_{ij}(\lambda)]\) decreases as the compressive load \(\lambda\) increases. The second of eqs(2), being the buckling equation, is a \(n\)\(^{th}\) degree algebraic polynomial with respect to \(\lambda\). From this equation we get \(\lambda^{(i)}(\eta_c) (i = 1,...,n)\), i.e. the successive buckling loads. Obviously, \(\text{det}[V_{ij}(\lambda)] > 0\) for \(\lambda < \lambda^{(1)}\), \(\text{det}[V_{ij}(\lambda)] < 0\) for \(\lambda > \lambda^{(1)}\) [excluding the case of a double point \(0(\eta_o,\lambda_o)\) mentioned below], while \(\text{det}[V_{ij}(\lambda)] = 0\) for \(\lambda = \lambda^{(1)}\). Recall that for a 3-DOF [2] and a 2-DOF [3] cantilever model we have respectively

\[
[V_{ij}] = \begin{bmatrix}
\bar{k}_1 + \bar{k}_2 - \lambda & -\bar{k}_2 & \lambda(1-\eta) \\
-\bar{k}_2 & \bar{k}_2 + 1 - \lambda & -1 - \lambda(\eta - 1) \\
0 & -1 & 1 - \lambda \eta
\end{bmatrix}, \quad [V_{ij}] = \begin{bmatrix}
\bar{k}_1 + 1 - \lambda & -1 - \lambda(\eta - 1) \\
-1 & 1 - \lambda \eta
\end{bmatrix}
\]

where \(\bar{k}_1\) and \(\bar{k}_2\) are stiffness ratios.

The boundary between existence and non-existence of adjacent equilibria \((\eta_o,\lambda_o)\) is
established by employing the theorem for implicit functions [5] via the solution of the system of equations

\[ \det[V_j^C] = \frac{d}{d\lambda} \left( \det[V_j^C] \right) = 0 \]  

(4)
evaluated at the critical divergence state C. A typical plot \( \eta \) vs \( \lambda_C \) showing this boundary \((\eta_0, \lambda_C^0)\) denoted by point 0 is shown in Fig. 1. Usually \( \eta_0 \) is a minimum in the curve \( \eta \) vs \( \lambda_C \). To the left of point 0 we have always dynamic (flutter) instability, while to the right of this point according to classical analyses we have divergence. However, as was shown by Kounadis [1], this is not always true, since in the shaded area of Fig. 1 both types (i.e. static and dynamic) of instability may occur. As a consequence of this various phenomena may occur, as for instance:

- Possible failure of Ziegler criterion for establishing the actual critical load in the shaded area.
- The degree of (nonconservativeness) asymmetry of the stiffness matrix may not be related to the static (divergence) type of instability, contrary to Huseyin & Lipholz analysis [6], since as was shown in previous studies a non-potential system with given degree of asymmetry may exhibit both types of dynamic instability.
- Asymmetric (non-potential) systems with symmetrizable damping and stiffness matrices may exhibit (contrary to Inman study [7]) dynamic instability.

Subsequently, the nature of the damping matrix \([c_{ij}]\) in connection with the existence of a Hopf bifurcation will be discussed. The pertinent study will be based on a stiffness matrix \([V_j]\) of general nature and not on the above one of eq. (3) associated with a concrete cantilever model.

Fig. 1. Typical curve \( \eta = \eta(\lambda_C) \) showing point 0 (boundary between flutter and divergence instability) and point d (defining the region \( \eta_0 \leq \eta \leq \eta_d \) for a double zero eigenvalue or a Hopf bifurcation).

**Hopf bifurcation and matrix \([c_{ij}]\)**

Lagrange equation of motion, eq. (1), after linearization, leads to the following equation of motion

\[ [\alpha_j] \ddot{q} + [c_{ij}] \dot{q} + [V_{ij}] q = 0 \]  

(5)
One can seek solutions of eq.(5) in the form
\[ q = re^{\omega t} \]  
(6)
where \( \rho \) is, in general, a complex number and \( r \) the corresponding complex vector, independent of \( t \).

Substituting this expression of \( q \) into eq.(5) leads to
\[ L(\rho)r = ([a_{ij}]\rho^2 + [c_{ij}]\rho + [V_{ij}]\rho + \alpha) = 0 \]  
(7)
where \( L(\rho) = [a_{ij}]\rho^2 + [c_{ij}]\rho + [V_{ij}] \) is a matrix-valued function. Solutions of eq. (5) are intimately related to the algebraic properties of \( L(\rho) \), and more specifically to the nature of Jacobian eigenvalues \( \rho_i \) (i=1,...,2n) obtained by solving the characteristic (secular) equation
\[ \det L(\rho)r = ([a_{ij}]\rho^2 + [c_{ij}]\rho + [V_{ij}]) = 0 \]  
(8)
which guarantees the existence of nontrivial solutions of eq. (5) or (7). Expansion of eq. (8) leads to the following equation [1]
\[ \rho^{2n} + \alpha_1\rho^{2n-1} + \alpha_2\rho^{2n-2} + ... + \alpha_{2n-1}\rho + \alpha_{2n} = 0 \]  
(9)
where \( \alpha_i \) (i=1,...,2n) are determined through Bocher recurrence formulae.

As stated above eigenvalues (roots) of eq. (8) occur in complex conjugate pairs \( \rho_i = \nu_i + \mu_i j \) (where \( j = \sqrt{-1} \), i = 1,...,n and \( \mu, \nu \) real numbers) with corresponding complex eigenvectors. Hence, solutions of eq. (5) associated with eq. (6) are of the form
\[ Ae^{\nu_i t}\cos\mu_i t \quad \text{and} \quad Be^{\nu_i t}\sin\mu_i t \]  
(10)
which are bounded tending to zero as \( \omega t \to \infty \), if all eigenvalues of eq. (9) have negative real parts [8]. According to Routh-Hurwitz stability criteria a necessary condition in order that all eigenvalues have negative real parts is \( \alpha_i > 0 \) for all \( i \), while a sufficient condition assuring this is all Routh-Hurwitz determinants \( \Delta_i \) of even (or odd) order to be positive [8]. Moreover, a necessary and sufficient condition for all eigenvalues to lie in the left-hand side of the complex plane is \( \Delta_i > 0 \) for all \( i \). Then, eq. (9) has complex conjugate eigenvalues of the above form, i.e. \( \rho_i = \nu_i \pm \mu_i j \), where \( \nu_i < 0 \) and \( \mu_i > 0 \) (i = 1,...,n).

Subsequently, we focus attention on symmetric (potential) systems occurring for \( \eta = \eta_c \) (which implies \( V_{ij} = V_{ji} \)).

### 3. Symmetric Systems

Premultiplying eq. (7) by \( r^T \), the transpose vector of \( r \), one can obtain
\[ r^T ([a_{ij}]\rho^2 + [c_{ij}]\rho + [V_{ij}])r = 0 \]  
(11)
where clearly all quadratic forms are real scalar quantities. Thus, eq. (11) is a 2nd degree polynomial with respect to \( \rho \) from which we get
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\[
\rho = \frac{1}{2r^T [\alpha_{ij}] r} \left[ r^T [c_{ij}] r \pm \sqrt{(r^T [c_{ij}] r)^2 - 4(r^T [\alpha_{ij}] r)(r^T [V_{ij}] r)} \right] = 0 \quad (12)
\]

Since both \([\alpha_{ij}]\) and \([c_{ij}]\) are given symmetric matrices with constant real elements, only the stiffness matrix \([V_{ij}] = [V_{ij}(\lambda)]\) depends on \(\lambda\). As the loading increases from zero, the variation of the matrix \([V_{ij}]\) influences the complex conjugate eigenvalues \(\rho\) and the corresponding complex eigenvectors \(r\).

Assuming, due to physical considerations weak damping, from eq. (12) one can observe the following:

**Case (a)**

If matrix \([c_{ij}]\) is positive definite, the quadratic form \(r^T [c_{ij}] r\) is always a positive quantity for \(r \neq 0\), while the quantity under the radical of eq. (12) is negative. Then, eq. (12) yields complex conjugate eigenvalues with negative real parts as long as \(\lambda < \lambda_{(1)}^C\), due to which \(\text{det}[V_{ij}] > 0\). At the static (divergence) critical state \(C\), occurring \(\lambda = \lambda_{(1)}^C\) we have, as stated above, \(\text{det}[V_{ij}(\lambda_{(1)}^C)] = \text{det}[V_{ij}]^C = 0\). Then, eq. (12) has one zero eigenvalue and one negative eigenvalue. Hence, if the nondissipative system is stable, the dissipative system is asymptotically stable provided that the damping matrix \([c_{ij}]\) is positive definite.

**Case (b)**

We consider now the case for which matrix \([c_{ij}]\) is positive semi-definite (i.e. when \(\text{det}[c_{ij}] = 0\)). If \(\lambda < \lambda_{(1)}^C\), matrix \([V_{ij}]\) is positive definite and then the eigenvalues (depending on \(\lambda\)) of eq. (9) and the corresponding eigenvectors are associated with \(r^T [c_{ij}] r > 0\). Thus, from eq. (12) it follows that all eigenvalues have negative real parts. At a certain value of \(\lambda\), the corresponding quadratic quantity becomes equal to zero, i.e.

\[
r^T [c_{ij}] r = 0 \quad (13)
\]

Since matrix \([c_{ij}]\) is positive semi-definite, eq. (13) implies

\[
[c_{ij}] r = 0 \quad (14)
\]

Introducing eq. (14) into eq. (11) it is deduced that \(r\) is also an eigenvector of the conservative system (9)

\[
([\alpha_{ij}] p^2 + [V_{ij}]) r = 0 \quad (15)
\]

Clearly, if \(r\) is an eigenvector of the nondissipative system (15) satisfying eq. (14), then \(r\) is also an eigenvector of the dissipative system of eq. (7) with corresponding eigenvalue (resulting from eq. (12)) which is imaginary. Setting \(\rho = \pm j\mu\) into eq. (15) one can determine the eigenvector which is real. Clearly, since \([c_{ij}]\) is a positive semi-definite matrix one of its eigenvalues is zero, and hence

\[
([c_{ij}] - 0I_r) r = 0 \quad (16)
\]

From this equation [or eq.(14)] one can establish the real eigenvector \(r\) for a given damping matrix \([c_{ij}]\). Then, eq. (12) leads to
\[
-\rho^2 = \mu^2 = \frac{r^T[V_0(\lambda)]r}{r^T[a_{ij}]r}
\]  

(17)

from which we can establish \( \mu \) as a function of the load \( \lambda \) for a given matrix \([a_{ij}]\).

Introducing \( r \) [obtained from eq. (16)] and \( \mu^2 \) [obtained from eq. (17)] into eq. (15) one can determine the critical flutter load \( \lambda_F \) which is accepted if

\[ 0 < \lambda_F < \lambda_{C(1)} = 0.381966011 \]  

(18)

For instance, for a 2-DOF system related to a cantilever damped model [1,2] we have

\[
[a_{ij}] = \begin{bmatrix} 1 + m & 1 \\ 1 & 1 \end{bmatrix}, \quad \text{(m > 0)}, \quad \left[c_{ij}\right] = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \quad \text{and} \quad \left[V_{ij}\right] = \begin{bmatrix} 2 - \lambda & -1 \\ -1 & 1 - \lambda \end{bmatrix}
\]  

(19)

We choose the following positive semi-definite matrix: \( c_{11} = 0.2, \ c_{22} = 0.05, \ c_{12} = c_{21} = 0.1 \).

Clearly, the mass ratio \( m \) is a free ranging parameter which will be adjusted so that inequality (18) be satisfied.

Using eq. (16) we find the eigenvector

\[
r = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \text{or} \quad r_1/r_2 = -0.5
\]  

(20)

On the other hand, from eqs (15) and (19) we also obtain

\[
r_1/r_2 = -\frac{V_{12} - \mu^2 a_{12}}{V_{11} - \mu^2 a_{11}} = \frac{\mu^2 + 1}{\mu^2 (m + 1) + 2 - \lambda}
\]  

(21)

Combining eqs (20) and (21) we find

\[
\lambda = 4 - (m - 1)\mu^2
\]  

(22)

From eq. (17) we can determine \( \mu^2 \) as follows

\[
\mu^2 = \frac{10 - 5\lambda}{m + 1}
\]  

(23)

Introducing this expression of \( \mu^2 \) into eq. (22) we find

\[
\lambda = \frac{7 - 3m}{3 - 2m}, \quad \text{(for } m \neq 1.5) \]  

(24)

Due to eq. (18), eq. (24) yields

\[
m > 7/3
\]  

(25)

Clearly, this value of \( m \) depends on the structure of the damping matrix \([c_{ij}]\). For \( m = 2.5 \) we get from eq. (24) the flutter load

\[
\lambda = \lambda_F = 0.25 < \lambda_{C(1)} = 0.5(3 - \sqrt{5}) = 0.381966011
\]  

(26)
The last result can also be obtained as follows. With the aid of the characteristic equation

$$\rho^4 + \alpha_1 \rho^3 + \alpha_2 \rho^2 + \alpha_3 \rho + \alpha_4 = 0 \quad (27)$$

one can also establish the above value of the flutter load $\lambda_F$ taking into account that

$$\begin{align*}
\alpha_1 &= \frac{1}{m} \left[ 0.5(1 + m) \right] = \frac{m + 1}{20m} \\
\alpha_2 &= \frac{1}{m} [m + 5 - \lambda(m + 2)] \\
\alpha_3 &= \frac{1}{m} (0.5 - 0.25\lambda) \\
\alpha_4 &= \frac{1}{m} (\lambda^2 - 3\lambda + 1)
\end{align*} \quad (28)$$

Introducing $\rho = \pm \mu i$ into eq. (27) we find

$$\begin{align*}
\mu^2 &= \frac{\alpha_1}{\alpha_2} \\
and \quad \mu^4 + \alpha_3 \mu^2 + \alpha_4 &= 0
\end{align*} \quad (29)$$

from which we obtain the necessary condition for a Hopf bifurcation [1]

$$\left( \alpha_1 \alpha_3 - \alpha_2 \alpha_4 \right) = 0 \quad (30)$$

Eq. (30) due to relations (28) leads to the following 2nd degree algebraic equation

$$(2m^2 - 6m + 4.5)\lambda^2 - (6m^2 - 23m + 21)\lambda + 4.5m^2 - 21m + 24.5 = 0 \quad (31)$$

which has a double root, and hence

$$\lambda_{(1)} = \lambda_{(2)} = \lambda_F = \frac{6m^2 - 23m + 21}{4m^2 - 12m + 9} = \frac{7 - 3m}{3 - 2m} \quad (32)$$

Thus, we have rederive formula (24).

From the above analysis, one can infer the following important conclusions:

If the damping matrix $[c_{ij}]$ is positive semi-definite, then under certain conditions associated with the magnitude of the mass ratio $m$, the symmetric (potential) systems, contrary to widely accepted (classical) findings, may exhibit a Hopf bifurcation (i.e. limit cycles).

Moreover, it is worth observing that although such a local bifurcation corresponds to a pair of purely imaginary eigenvalues, the corresponding eigenvector is real. Furthermore, in case of a 2-DOF model the necessary condition\(^(*)\) for a Hopf bifurcation leads to a 2nd degree algebraic polynomial in $\lambda$ which has a double root, being the critical flutter load $\lambda$, i.e. $\lambda = \lambda_F$. Note also that if $m$ is kept constant and $\lambda$ is slightly

\(^(*)\) This condition is also satisfied in case of existence of equal and opposite sign roots [8].
higher than \( \lambda_F \) (i.e. \( \lambda^C \geq \lambda > \lambda_F \)) the system yields a point attractor response. This is so, since excluding the case \( \lambda = \lambda_F \) (i.e. for \( \lambda \neq \lambda_F \)), the quadratic form \( \mathbf{F}^T [c_{ij}] \mathbf{r} \) in eq. (12) is positive and thus its real part is negative. Hence, as the loading \( \lambda \) increases gradually from zero, at a certain value of \( \lambda \), the symmetric system exhibits an isolated Hopf bifurcation with a double root of eq. (31).

In closing with case (b), one can observe that if the generalized stiffness matrix \([V_{ij}]\) is positive semi-definite (occurring for \( \lambda = \lambda^C \)), eq. (12) yields a zero eigenvalue corresponding to divergence (static) instability.

**Case (c)**

We now consider the case of an indefinite (symmetric) damping matrix \([c_{ij}]\). From a brief discussion one can observe the following:

For \( \lambda \) sufficiently small, the quadratic complex form \( \mathbf{r}^T [c_{ij}] \mathbf{r} \) takes positive values and thus from eq. (12) it is clear that all eigenvalues \( \rho \) have negative real parts. Then, the system exhibits a point attractor response. At a certain value of \( \lambda \), this form vanishes yielding a pair of purely imaginary eigenvalues (case of a Hopf bifurcation, i.e. \( \lambda = \lambda_F \)). For \( \lambda \) slightly higher than \( \lambda_F \), the quadratic form becomes negative and thus a pair of eigenvalues has positive real part. The trivial state is locally unstable but globally stable. This situation of bounded amplitude oscillation is called flutter. Flutter occurs also when there exists one pair of complex conjugate eigenvalues with positive real part.

**Case (d)**

Another important case is associated with a positive semi-definite stiffness matrix \([V_{ij}]\) (i.e. \( |V_{ij}(\lambda^C)| = 0 \)) which will be discussed in connection with a positive semi-definite damping matrix \([c_{ij}]\). The question that now arises is which must be the structure of the matrix \([c_{ij}]\) so that

\[ \lambda_F = \lambda^C \]  

(33)

From eq. (12) it follows that in this case we have a double zero eigenvalue (clearly, \( \nu = \mu = 0 \)). On the other hand the necessary condition for a Hopf bifurcation in terms of Routh-Hurwitz determinants is given by

\[ \Delta_{2n-1} = \alpha_{2n-1} M_{2n-1} - \alpha_{2n} M_{2n} = 0 \]  

(34)

where \( M_{2n-1} \) and \( M_{2n} \) are the minors (determinants) of the elements \( \alpha_{2n-1} \) and \( \alpha_{2n} \) of the matrix \( \Delta_{2n-1} \), whose expression is [2]

\[ \Delta_{2n-1} = \begin{vmatrix} \alpha_1 & 1 & 0 & 0 & 0 & \ldots \\ \alpha_3 & \alpha_2 & \alpha_1 & 1 & 0 & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ \alpha_{2n} & \alpha_{2n-2} & \alpha_{2n-3} & \alpha_{2n-4} & \alpha_{2n-5} & \alpha_{2n-1} \end{vmatrix} \]  

(35)
Clearly, if \( \alpha_{2n-1} = \alpha_{2n} = 0 \), eq. (9) has a \textit{double zero} eigenvalue [2,10] but at the same time the equation for a Hopf bifurcation is satisfied. Then, the corresponding solution is associated with a limit cycle response [1]. The critical (trivial) state is \textit{unstable} but the global response is stable. Such a situation is called \textit{coupled divergence-flutter instability}. Namely, the last case corresponds to a \textit{special} type of Hopf bifurcation.

4. NUMERICAL RESULTS

In this section numerical results under graphical form in the form of phase-plane portraits illustrate and confirm the above theoretical findings. \textit{Symmetric} (potential) systems of 2-DOF, weakly damped, cantilever (Ziegler's) models experiencing stable limit cycles related to a \textit{Hopf} bifurcation (Fig. 2), to \textit{flutter} (Fig. 3) or to a \textit{double zero} eigenvalue (Fig. 4) are presented below.

Fig. 2. Hopf bifurcation for a 2-DOF symmetric damped system.

Fig. 3. Flutter instability (with a pair of complex conjugate eigenvalues with positive real part) associated with stable limit cycles.
Fig. 4. Double zero eigenvalue bifurcation for a 2-DOF symmetric damped system associated with stable limit cycles.

REFERENCES


HOPF-OVE BIFURKACIJE I NESTABILNOSTI FLATERA AUTONOMNIH POTENCIJALNIH DISIPATIVNIH SISTEMA

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Ovaj rad se bavi idealnim bifurkacionim diskretnim disipativnim sistemima sa trivijalnim prekritičnim stanjima pod dejstvom glavnog konzervativnog pritisnog opterećenja. Pažnja je usmerena na uslove pod kojima ti simetrični autonomni slabo prigušeni sistemi mogu ispoljiti odgovor tipa graničnog ciklusa usled Hopf-ove bifurkacije ili usled dvostruke nule sopstvene urednosti ili nestabilnosti tipa flatera.