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SOME ALGORITHMS FOR OPTIMAL STATE ESTIMATION AND OPTIMAL CONTROL CHOICE FOR STOCHASTIC SYSTEMS

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Abstract. *In this paper some approaches to the control of stochastic systems have been considered. Algorithms for optimal state and parameters estimation for stochastic systems, with constant and changeable structure, have been presented. For systems with changeable structure it is assumed that change of structure is random and represents Markov's process. On the basis of algorithms for optimal state variables and parameters estimation, corresponding algorithms for choice of the optimal control have been formed. Challenge for investigations in this field is to find adaptive algorithms for improving the accuracy of estimation, and thus of control, in cases when priors describing stochastic disturbances are not exact enough. In such cases alternative techniques based on application of fuzzy logic in control systems can be used. Performance and robustness of the fuzzy PI controller and stochastic LQG optimal controller have been compared for a pneumatic servo system, in case when random disturbances act upon the system.*

Key words: *control, stochastic systems, state estimation, optimal control, fuzzy control, filtration, estimation, LQG control*

1. INTRODUCTION

Systems with random disturbances are step towards the accurate mathematical description of the object and observation process, and are very common in all technical fields today. Therefore, for the last twenty years this area of automatic control has been deserving special attention, what yielded significant results. It is most likely that research in this area will be important and also combined with alternative techniques from the domain of soft computing, especially with fuzzy logic control structures.

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In this paper authors have reviewed some results which they have obtained while researching this area. Results in this paper prove that results obtained in the stochastic control theory generally provide very good quality of the control for the systems exposed to random disturbances [21] [28]. However, attention are gaining alternative control techniques, based on the elements of soft computing [22] [23] [24]. Way to practical applications have found fuzzy controllers above all [25] [26] [27], so further expansion of their presence in control systems can be expected. Some results that indicate possibilities for application of the fuzzy control structures as alternative to conventional stochastic systems are also presented in the paper.

2. OPTIMAL STATE AND PARAMETERS ESTIMATION AND CHOICE OF THE OPTIMAL CONTROL FOR STOCHASTIC SYSTEMS

Algorithms for optimal state and parameters estimation of linear and non-linear dynamic systems [7] [8] [9] [10] with one structure, which are based on Kalman algorithm [12] are considered. Attention are deserving adaptations of this algorithms, what represents dominate task in further research of this area. On the basis of well known results from deterministic [1] [4] [5] and stochastic [6] [12] control theory, two simple but highly applicable algorithms have been considered.

Random structure of stochastic processes in control systems [20] raises a range of new tasks that need to be solved in the modern stochastic theory of dynamic systems. Some results concerning problems of estimation-optimization and design of the corresponding control laws [13] are presented. State estimation for the non-linear systems with random structure, as well as design of the optimal control for such systems, by using algorithms of the direct and inverse dynamic programming, have been considered.

3. CONTINUOUS AND DISCRETE KALMAN STATE ESTIMATOR (FILTER) AND THEIR ADAPTATIONS

By application of classic variance calculus [4], Kalman and Busy have formed non-linear Riccati differential equation for determination of error covariance matrix of the optimal continuos filter [12]. Although discrete version of the filter is dominant in practical applications because of its suitability for implementation on digital computers, theoretical significance of the continuos Kalman-Bucy filter is large. Basic information is object model, defined with state and output equations:

$$\dot{\underline{x}}(t) = \underline{A}\underline{x}(t) + \underline{B}\underline{u}(t) + \underline{G}\underline{w}(t), \quad \underline{y}(t) = \underline{C}\underline{x}(t) + \underline{v}(t). \tag{1}$$

Statistical characteristics of the disturbance and measurement noise (priors) are:

$$M\{\underline{w}(t)\} = \underline{0}; M\{\underline{v}(t)\} = \underline{0}; \ \text{cov}[\underline{w}(t), \underline{w(\tau)}] = \underline{Q}\delta(t-\tau), \ \text{cov}[\underline{v}(t), \underline{v}(\tau)] = \underline{R}\delta(t-\tau)$$

$$\text{cov}[\underline{w}(t), \underline{v}(\tau)] = \text{cov}[\underline{x}(t), \underline{w}(\tau)] = \text{cov}[\underline{x}(t), \underline{v}(\tau)] = \underline{0}, \tag{2}$$

where $\underline{Q}(t)$ and $\underline{R}(t)$ are continuos positively defined covariance matrices, and $\delta(t\text{-}\tau)$ is Dirac's function. Optimal state estimator is defined with differential equation:

$$\dot{\underline{\hat{x}}}(t) = \underline{A}\underline{\hat{x}}(t) + \underline{K}(t)[\underline{y}(t) - \underline{C}\underline{\hat{x}}(t)] + \underline{B}\underline{u}(t) . \quad (3)$$

Matrix of the Kalman gains $\underline{K}(t)$ is defined with:

$$\underline{K}(t) = \underline{\Sigma}(t)\underline{C}^T \underline{R}(t)^{-1}, t \geq t_0 . \quad (4)$$

Estimation error covariance matrix $\underline{\Sigma}(t) = M\{\underline{\tilde{x}}(t)\underline{\tilde{x}}^T(t)\}$ satisfies Riccati's equation:

$$\dot{\underline{\Sigma}}(t) = \underline{A}\Sigma(t) + \underline{\Sigma}(t)\underline{A}^T + \underline{G}\underline{Q}(t)\underline{G}^T - \underline{\Sigma}(t)\underline{C}^T \underline{R}(t)^{-1}\underline{C}\underline{\Sigma}(t) . \quad (5)$$

Also

$$\underline{\hat{x}}(t_0) = M\{\underline{x}(t_0)\}, \underline{\Sigma}(t_0) = \underline{\Sigma}_0 = \text{cov}[\underline{x}_0, \underline{x}_0] . \quad (6)$$

Steady state Kalman-Bucy filter is defined with:

$$\underline{A}\underline{\Sigma} + \underline{\Sigma}\underline{A}^T + \underline{G}\underline{Q}\underline{G}^T - \underline{\Sigma}\underline{C}^T \underline{R}^{-1}\underline{C}\underline{\Sigma} = \underline{0} , \ \underline{K} = \underline{\Sigma}\underline{C}^T \underline{R}^{-1} , \ \dot{\underline{\hat{x}}}(t) = \underline{A}\underline{\hat{x}}(t) + \underline{K}[\underline{y}(t) - \underline{C}\underline{\hat{x}}(t)] + \underline{B}\underline{u}(t) . \quad (7)$$

Among the proposed recursive algorithms based on different approaches to the estimation problem, algorithm based on the minimal estimation variance is commonly used. System behavior and observation process are defined with:

$$\underline{x}_{k+1} = \underline{A}\underline{x}_k + \underline{B}\underline{u}_k + \underline{G}\underline{w}_k , \quad \underline{y}_k = \underline{C}\underline{x}_k + \underline{D}\underline{u}_k + \underline{v}_k . \quad (8)$$

Statistical characteristics of the disturbances (priors) are the same as previously defined.

Recursive algorithm for estimation of the state vector $\underline{x}_k$ of dynamical Markov's stochastic process given with the equations (8) is defined with:

$$\underline{\bar{x}}_{k+1} = \underline{A}\underline{\hat{x}}_k + \underline{B}\underline{u}_k , \ \underline{\bar{\Sigma}}_{k+1} = \underline{A}\underline{\Sigma}_k \underline{A}^T + \underline{G}\underline{Q}_k \underline{G}^T , \ \underline{K}_k = \underline{\bar{\Sigma}}_k \underline{C}^T (\underline{C}\underline{\bar{\Sigma}}_k \underline{C}^T + \underline{R}_k)^{-1} ,$$

$$\underline{\hat{x}}_k = \underline{\bar{x}}_k + \underline{K}_k (\underline{y}_k - \underline{C}\underline{\bar{x}}_k - \underline{D}\underline{u}_k) , \ \underline{\Sigma}_k = (\underline{I} - \underline{K}_k \underline{C})\underline{\bar{\Sigma}}_k , \ \underline{\hat{y}}_k = \underline{C}\underline{\hat{x}}_k + \underline{D}\underline{u}_k . \quad (9)$$

If controlled object is stationary, and disturbances and measurement noises are also stationary stochastic processes, non-stationary Kalman filter reduces to steady-state filter.

For discrete model of water treatment plant described with equations (8), where:

$$\underline{A} = \begin{bmatrix} 0.18 & 0 & 0 & 0 \\ -0.25 & 0.27 & 0 & 0 \\ 0.55 & 0 & 0.18 & 0 \\ 0 & 0.55 & -0.25 & 0.27 \end{bmatrix}; \ \underline{B} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}; \ \underline{G} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \ \underline{C} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \ \underline{D} = \underline{0};$$

$$\underline{Q} = \begin{bmatrix} 0.005 & 0 & 0 & 0 \\ 0 & 0.005 & 0 & 0 \\ 0 & 0 & 0.005 & 0 \\ 0 & 0 & 0 & 0.005 \end{bmatrix}; \quad \underline{R}_k = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, k = \overline{1,50}; \ \underline{R}_k = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \ k = \overline{50,100};$$

in Figures 1 and 2 real system output, sensor signals, estimated output and change of estimation error are shown, while change of the elements of the output estimation error covariance matrix is shown in Figure 3.

Besides the algorithm of the so-called *b*-modification which authors developed for the non-linear systems [11], and that can also be used for linear systems, two similar algorithms are presented here. First adaptive algorithm starts from the system description:

$$\underline{x}_{k+1} = \underline{A}_k \underline{x}_k + \underline{G}_k \underline{w}_k , \quad \underline{y}_k = \underline{C}_k \underline{x}_k + \gamma_k \underline{\alpha}_k + (1-\gamma_k)\underline{\beta}_k , \tag{10}$$

Array γ_k is formed with independent random variables that can have values 0 and 1 with probabilities q_k and p_k:

$$P[\gamma_k = 1] = p_k = 1 - q_k . \tag{11}$$

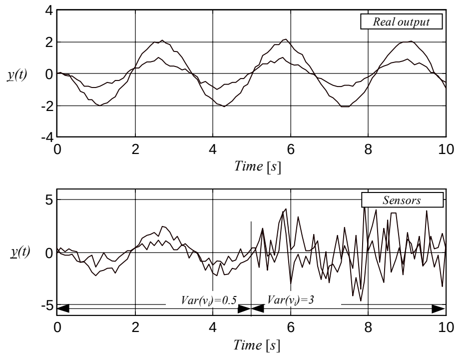

Fig. 1. Real system outputs and signal $\underline{y}(t)$ generated by sensors

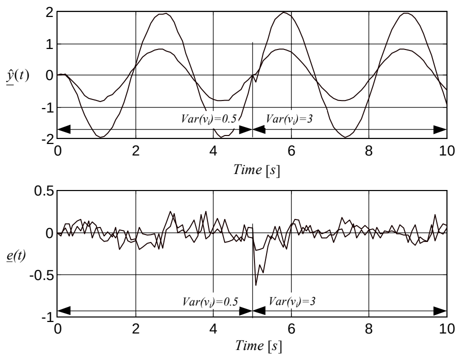

Fig. 2. Outputs estimated by Kalman filter and estimation error

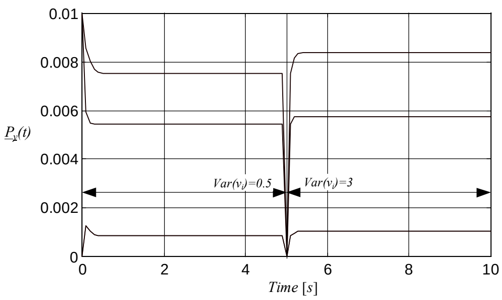

Fig. 3. Change of elements of the output estimation error covariance matrix

Random *m*-dimensional vectors $\underline{\alpha}_k$ and $\underline{\beta}_k$ are independent Gaussian random values with covariance matrices $\underline{R}_{\alpha k}$ and $\underline{R}_{\beta k}$ respectively. It is assumed that $p_k << q_k$, what corresponds to case when in measurement channel common noises $\underline{\beta}_k$ are present, with rare occurrence of noises of vast intensity $\underline{\alpha}_k$, which correspond to the anomalies in observation model.

Algorithm of the adaptive estimation is based on the idea of classification of observation results $\underline{y}_k$ in two groups, that correspond to disturbances $\underline{\alpha}_k$ and $\underline{\beta}_k$, with modification of the estimation algorithm by considering real disturbance model. Methodology of Bayesian solutions is used [10].

Adaptive algorithm for obtaining stable solution $\hat{\underline{x}}_k$ is described with:

$$\overline{\underline{x}}_{k+1} = \underline{A}_k \hat{\underline{x}}_k , \quad \overline{\underline{\Sigma}}_{k+1} = \underline{A}_k \underline{\Sigma}_k \underline{A}_k^T + \underline{G}_k \underline{Q}_k \underline{G}_k , \quad \hat{\underline{x}}_k = \overline{\underline{x}}_k + \underline{K}_k\left(\hat{\underline{\Gamma}}_k^k\right)\underline{C}_k \overline{\underline{\Sigma}}_k ,$$

$$\underline{K}_k\left(\hat{\underline{\Gamma}}_k^k\right) = \overline{\underline{\Sigma}}_k \underline{C}_k [\underline{C}_k \overline{\underline{\Sigma}}_k \underline{C}_k^T + \hat{\gamma}_k \underline{R}_{\underline{\alpha}_k} + (1-\hat{\gamma}_k)\underline{R}_{\underline{\beta}_k}]^{-1} , \tag{12}$$

where

$$\underline{\hat{\Gamma}}^{k} = \{\hat{\gamma}(0),\ldots,\hat{\gamma}_k\}\,, \tag{13}$$

while estimate $\hat{\gamma}_k$ of the γ_k is determined by using Bayesian solutions [10].

If discrete linear system is described with difference equations

$$\underline{x}_{k+1} = \underline{A}_k \underline{x}_k + \underline{G}_k \underline{w}_k\,,\ \ \underline{y}_k = \underline{C}_k \underline{x}_k + \underline{v}_k\,, \tag{14}$$

Kalman algorithm for state estimation significantly relies on exact knowledge of covariance matrices $\underline{Q}$ and $\underline{R}$. However, in many practical cases those matrices are not known, or can be known only approximately. In such cases adaptive modification of the classical Kalman filter can be formed [16], that can be effectively applied when information on matrices $\underline{Q}$ and $\underline{R}$ are not known. Algorithm of the adaptive estimation starts from the relation:

$$\Delta\underline{y}_k = \underline{y}_k - \underline{C}\underline{x}_k\,, \tag{15}$$

Value $\Delta\underline{y}_k$ involves new statistical information, which is gained from the observation $\underline{y}_k$, and estimates of the covariance matrices $\underline{Q}$ and $\underline{R}$ can be obtained from it.

In steady state matrix of the filter gains $\underline{K}$ and also matrix $\underline{\Sigma}$ remain constant:

$$\underline{K} = \hat{\underline{\Sigma}}\underline{C}^T(\underline{C}\Sigma\underline{C}^T + \underline{R})^{-1}\,,\ \ \hat{\underline{\Sigma}} = \underline{A}(\underline{I} - \underline{K}\underline{C})\underline{\Sigma}(\underline{I} - \underline{K}\underline{C})^T\underline{A}^T + \underline{A}\underline{K}\underline{R}\underline{K}^T\underline{A}^T + \underline{G}\underline{Q}\underline{G}^T\,. \tag{16}$$

From the relation

$$\underline{C}_k = M\left[\Delta\underline{y}_i\Delta\underline{y}_{(i-k)}^T\right], \tag{17}$$

it is obtained

$$\underline{C}_0 = \underline{C}\hat{\underline{\Sigma}}\underline{C}^T + \underline{R}\,,\ \ \underline{C}_0 = \underline{C}[\underline{A}(\underline{I} - \underline{K}\underline{C})]^{k-1}\underline{A}[\underline{\Sigma}\underline{C}^T - \underline{K}\underline{C}_0], k \geq 1\,. \tag{18}$$

Essence of this algorithm of adaptive filtration is that by using priors about $\underline{Q}_0$ and $\underline{R}_0$, unknown covariance matrices, to obtain their more accurate estimates and accordingly calculate corresponding state estimates.

Estimates of the covariance matrices arme realized by using the following algorithm:

$$\hat{\underline{C}}_k = \frac{1}{N}\sum_{i=k}^{N}\Delta\underline{y}_i\Delta\underline{y}_{(i-k)}^T\,,\ \ \underline{C}_1 = \underline{C}\underline{A}^n\underline{\Sigma}\underline{C}^T - \underline{C}\underline{A}\underline{K}\underline{C}_0\,,\ \ldots$$

$$\ldots\ ,\ \underline{C}_n = \underline{C}\underline{A}^n\underline{\Sigma}\underline{C}^T - \underline{C}\underline{A}\underline{K}\underline{C}_{n-1} - \ldots - \underline{C}\underline{A}\underline{K}^n\underline{C}_0 \tag{19}$$

$$\hat{\underline{\Sigma}\underline{C}}^T = \underline{T}^+\begin{bmatrix}\hat{\underline{C}}_1 + \underline{C}\underline{A}\underline{K}\hat{\underline{C}}_0 \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \hat{\underline{C}}_n + \underline{C}\underline{A}\underline{K}\hat{\underline{C}}_{n-1} + \ldots + \underline{C}\underline{A}^n\underline{K}\hat{\underline{C}}_0\end{bmatrix}, \tag{20}$$

where $\underline{T}^+$ is pseudoinversion of matrix

$$\underline{T} = \left[\underline{C} \mid \underline{C}\underline{A} \mid \cdots \mid \underline{C}\underline{A}^{n-1}\right]^T\underline{A}\,. \tag{21}$$

Estimate $\hat{\underline{R}}$ of covariance matrix $\underline{R}$ is obtained from matrix equation:

$$\hat{\underline{R}} = \hat{\underline{C}}_0 - \underline{C}\left(\hat{\underline{\Sigma C}}^T\right). \quad (22)$$

Estimate $\hat{\underline{Q}}$ of covariance matrix $\underline{Q}$ is obtained from the system of matrix equations:

$$\sum_{j=0}^{k-1} \underline{C}\underline{A}^j \underline{G}\hat{\underline{Q}}\underline{G}^T (\underline{A}^{j-k})\underline{C}^T = \left(\hat{\underline{\Sigma C}}^T\right)^T (\underline{A}^{-k})^T \underline{C}^T - \underline{C}\underline{A}^k \hat{\underline{\Sigma C}}^T - \sum_{j=0}^{k-1} \underline{C}\underline{A}^j \hat{\underline{S}}(\underline{A}^{j-k})\underline{C}^T, k = 1,2,\ldots n, \quad (23)$$

$$\hat{\underline{S}} = \underline{A}\left[-\underline{K}\left(\hat{\underline{\Sigma C}}^T\right)^T - \Sigma\hat{\underline{C}}^T \underline{K}^T + \underline{K}\hat{\underline{C}}_0 \underline{K}^T\right]\underline{A}^T. \quad (24)$$

4. Optimal control for stochastic systems

There are various possibilities for state estimation with linear and non-linear control systems to be used for design of the optimal control laws [1] [6] [17].

Filter and controller can be combined so that resulting control law is proportional-integral, similar as with observer. Therefore, if system described with equations (1) is considered, where $\dot{\underline{w}} = \underline{\xi}(t)$, linear optimal controller for maintaining of the desired steady state $\underline{x}_d = 0$ and $\underline{u}_d = 0$ is determined with:

$$\underline{u} = -\underline{K}_1 \hat{\underline{x}} - \underline{K}_2 \hat{\underline{w}}. \quad (25)$$

Estimation of the state vector and disturbance is realized by the algorithm:

$$\dot{\hat{\underline{x}}} = \underline{A}\hat{\underline{x}} + \underline{B}\underline{u} + \underline{G}\hat{\underline{w}} + \underline{K}_{e1}[\underline{y} - \underline{C}\hat{\underline{x}}], \quad \dot{\hat{\underline{w}}} = \underline{K}_{e2}(\underline{y} - \underline{C}\hat{\underline{x}}), \quad (26)$$

where matrix coefficients $\underline{K}_{e1}$ and $\underline{K}_{e2}$ reach steady states in time, and control law becomes:

$$\underline{u} = -\underline{K}_1 \hat{\underline{x}} + \underline{K}_2 \underline{K}_{e2} \underline{C}\int_0^t \hat{\underline{x}} dt - \underline{K}_2 \underline{K}_{e2} \int_0^t \underline{y} dt. \quad (27)$$

For stochastic systems described with equations (1) optimality criterion [6]

$$J = M\left\{\frac{1}{2}\underline{x}^T(t_f)\underline{S}_f \underline{x}(t_f) + \frac{1}{2}\int_0^{t_f}[\underline{x}^T \underline{F}\underline{x} + \underline{u}^T \underline{E}\underline{u}]dt\right\}, \quad (28)$$

optimal control is determined with relation:

$$\underline{u}(t) = -\underline{K}(t)\hat{\underline{x}}(t) = -\underline{E}^{-1}\underline{B}^T \underline{S}(t), \quad \dot{\underline{S}} = -\underline{S}\underline{A} - \underline{A}^T \underline{S} + \underline{S}\underline{B}\underline{E}^{-1}\underline{B}^T \underline{S} - \underline{F}; \underline{S}(t_f) = \underline{S}_f. \quad (29)$$

For asynchronous motor [2][3] with characteristics $P_n = 100\ kW$, $\omega_s = 78{,}53\ rad/s$, $M_N = 1300\ Nm$, $J_N = 6\ kgm^2$, $f_N = 50\ Hz$, $J = 2\ J_N$, change of the motor speed for uncontrolled and optimally controlled motor is shown in Figures 4 and 5.

Optimal control has been determined by the expression (29), for linearized motor model, while the state estimates are determined with non-linear filter [10]. It can be concluded that optimally controlled system is insensitive to the variations of random moment acting upon the output motor shaft.

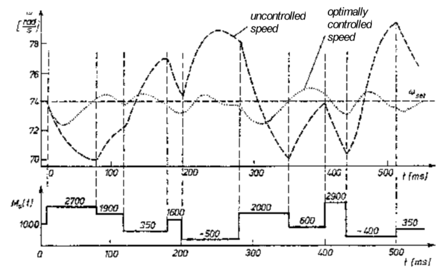

Fig. 4. Variation of the motor speed upon the act of stochastic load moment

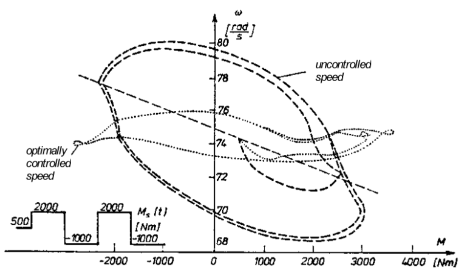

Fig. 5. Motor speed change as function of the stochastic load moment

5. OPTIMAL CONTROL FOR STOCHASTIC SYSTEMS WITH RANDOM STRUCTURE

State estimation for the systems with random structure represent separate problem. Case of the Poisson type structure change with functions $\nu_{\ell,r}(t)$, $\ell,r\in[1,s]$ of the new structure occurrence is considered here. Functions of the disappearance and occurrence in Gaussian approximation are of the form:

$$\nu_{\ell r}(\underline{x},t)=\nu_{\ell r}(t)\hat{P}_{10}^{(\ell)}(\underline{x},t)\,,\ \nu_{r\ell}(\underline{x},t)=\nu_{r\ell}(t)\hat{P}_{10}^{(r)}(\underline{x},t)\,, \tag{30}$$

where $P_{10}^{(\ell)}(\underline{x},t)$ is Gaussian probability distribution. Corresponding algorithm of the optimal estimation [20] for non-linear system defined with equations:

$$\dot{\underline{x}}=\underline{f}^{(\ell)}[\underline{x},t]+\underline{g}^{(\ell)}[\underline{x},t]\underline{w}(t)\,,\qquad \underline{x}=\underline{h}[\underline{x},t]+\underline{v}(t)\,, \tag{31}$$

where $\underline{w}(t)$ and $\underline{v}(t)$ are Gaussian processes, white noises with zero mathematical expectations, is of the form:

$$\dot{\hat{\underline{x}}}^{(\ell)}=\underline{f}_0^{(\ell)}[\hat{\underline{x}}^{(\ell)},\underline{\Sigma}^{(\ell)},t]+\underline{\Sigma}^{(\ell)}\underline{C}^{(\ell)T}\underline{R}^{-1}[\underline{z}\underline{C}^{(\ell)}\hat{\underline{x}}^{(\ell)}]+\Sigma\frac{\hat{P}_r(t)}{\hat{P}(t)}\nu_{r\ell}(t)[\hat{\underline{x}}^{(r)}-\underline{x}^{(\ell)}]\,, \tag{32}$$

$$\dot{\underline{\Sigma}}^{(\ell)}=\underline{G}^{(\ell)}(t)\underline{Q}(t)\underline{G}^{(\ell)T}(t)+\frac{\partial\underline{f}_0^{(\ell)}}{\partial\hat{\underline{x}}^{(\ell)}}+\underline{\Sigma}^{(\ell)}\cdot\left[\frac{\partial\underline{f}_0^{(\ell)}}{\partial\hat{\underline{x}}^{(\ell)}}\right]-\underline{\Sigma}^{(\ell)}\underline{C}^{(\ell)T}\underline{R}^{-1}\underline{C}^{(\ell)}\underline{\Sigma}^{(\ell)}+\sum_{r=1}^{s}\frac{\hat{P}_r(t)}{\hat{P}_l(t)}\nu_{r\ell}(t)\cdot \tag{33}$$

$$\cdot[\underline{\Sigma}^{(r)}-\underline{\Sigma}^{(\ell)}+(\hat{\underline{x}}^{(r)}-\hat{\underline{x}}^{(\ell)})(\hat{\underline{x}}^{(r)}-\hat{\underline{x}}^{(\ell)})^T],$$

$$\underline{h}_x=\underline{C}\,,\qquad \hat{P}_\ell=-\sum_{r=1}^{s}[\hat{P}_\ell\nu_{\ell r}(t)-\hat{P}_r\nu_{r\ell}(t)]+\frac{1}{2}\hat{P}_\ell\sum_{r=1}^{s}[\hat{P}_r f^{(r)}(\underline{z},t)],\ \ell\in[1,s]\,, \tag{34}$$

where

$$g^{(r)}(\underline{z},t)=\sum_{\rho,\nu=1}^{m}\frac{\overline{R}_{\rho\nu}}{|\underline{R}|}\left\{\left(z_\rho-\sum_{i=1}^{n}C_{\rho i}^{(r)}\hat{x}_i^{(r)}\right)\left(z_\nu-\sum_{i=1}^{n}C_{\nu i}^{(r)}\hat{x}_i^{(r)}\right)+\sum_{q,j=1}^{n}C_{\rho q}^{(r)}C_{\nu j}^{(r)}\Sigma_{qj}^{(r)}\right\}. \tag{35}$$

Equations (32) describe filtration block (Figure 6) and for various values l are connected with function N_ℓ of the form:

$$\underline{N}_\ell = \sum_{r=1}^{s} \frac{\hat{P}_r(t)}{\hat{P}_\ell(t)} \nu_{r\ell}(t)[\underline{\hat{x}}^{(r)} - \underline{\hat{x}}^{(l)}]. \tag{36}$$

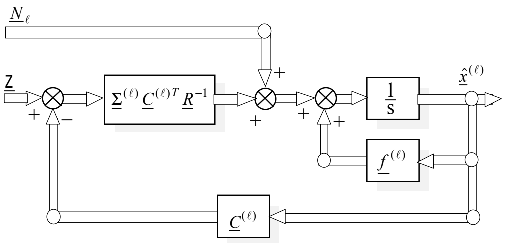

Fig. 6. Block diagram of state estimation

Output signals from filter represent the best estimation of state vector of each structure. Equations (33) determine fltration accuracy (correlation matrices) while relations (34) describe block of structure estimation (identificator) and realize control by switching on the filter output.

In [18] [19] [20] algorithms of continuos-discrete dynamic programming for optimal control choice for non-linear systems with one structure have been proposed. In the similar manner relations can be obtained that form the algorithm for optimal control choice for systems with random structure when each structure is considered independently. In that case, just as it is when algorithms for systems with one structure are considered, optimal estimations of the state variables or discrete distributions of the probability densities can be used alternatively. To the contrary to the system of equations (31) that has been used as basis for most of the optimization methods described in the literature [4][15], methodology for optimal control choice described here starts from the stochastic non-linear model of the most general type:

$$\underline{\dot{x}}^{(\ell)} = \underline{f}^{(\ell)}[\underline{x}^{(\ell)}, \underline{u}, \underline{w}]. \tag{37}$$

Optimality criterion has one of the following four forms:

$$J(t) = M_w\{\int_t^T f^{0(\ell)}[\underline{x}^{(\ell)}(\tau), \underline{u}^{(\ell)}(\tau), \underline{w}(\tau)]d\tau\} \to \min, \quad t \in [0,T],$$

$$J(t) = \int_t^T f^{0(\ell)}[\underline{\hat{x}}^{(\ell)}(\tau), \underline{u}^{(\ell)}(\tau)]d\tau \to \min,$$

$$J = \sum_{k=0}^{N-1} M_w[f^{0(\ell)}(\underline{x}_k^{(\ell)}, \underline{u}_k^{(\ell)}, \underline{w}_k)] \to \min, \quad J = \sum_{k=0}^{N-1} f^{0(\ell)}(\underline{\hat{x}}_k^{(\ell)}, \underline{u}_k^{(\ell)}) \to \min, \tag{38}$$

where $f^{\,0}$ is a scalar function, M_w mathematical expectancy and N number of the discretization steps of constant length.

Direct algorithm of continuos-discrete dynamic programming which uses distribution of probability densities is described with equations:

$$J_{(\mu+1)d}[\underline{x}_\mu^{(\ell)}(d)] = \inf_{\overline{u}_\mu^{(\ell)} \in \overline{D}^{(\ell)}[\mu d,(\mu+1)d]} \{\int_0^d \sum_{j=1}^{K} P_{\mu j} \cdot f^{0(\ell)}[\underline{x}_\mu^{(\ell)}, \overline{u}_\mu^{(\ell)}, \underline{w}_{\mu j}(t_\mu)]dt_\mu + J_{\mu d}[\underline{x}_\mu^{(\ell)}(0)]\},$$

$$\underline{x}_\mu^{(\ell)}(t_\mu) = \underline{x}_\mu^{(\ell)}(0) + \int_0^{t_\mu} \underline{f}^{(\ell)}[\underline{x}_\mu^{(\ell)}(\tau_\mu), \overline{u}_\mu^{(\ell)}, \underline{w}_{\mu j}(\tau_\mu)]d\tau_\mu,$$

$$\underline{x}_{\mu}^{(\ell)}(d) = \underline{x}_{\mu}^{(\ell)}(0) + \int_{0}^{d} \sum_{j=1}^{K} P_{\mu j} \underline{f}^{(\ell)}[\underline{x}_{\mu}^{(\ell)}(t_{\mu}), \overline{\underline{u}}_{\mu}^{(\ell)}, \underline{w}_{\mu j}(t_{\mu})] dt_{\mu} ,$$

$$J_{1d}[\underline{x}_{0}^{(\ell)}(d)] = \inf_{\overline{\underline{u}}_{0}^{(\ell)} \in \overline{D}^{(\ell)}[0,d]} \int_{0}^{d} \sum_{j=1}^{K} P_{0j} f^{0(\ell)}[\underline{x}_{0}^{(\ell)}(t_{0}), \overline{\underline{u}}_{0}^{(\ell)}, \underline{w}_{0j}(t_{0})] dt_{0} ,$$

$$t_{\mu} \in [0,d], \mu = 0,1,2,...,N-1; \quad j = 1,2,...,K; \ell \in [1,s]. \tag{39}$$

Direct algorithm of continuos-discrete dynamic programming with optimal estimations is of the form:

$$J_{(\mu+1)d}[\hat{\underline{x}}_{\mu}^{(\ell)}(d)] = \inf_{\overline{\underline{u}}_{\mu}^{(\ell)} \in \overline{D}^{(\ell)}[\mu d,(\mu+1)d]} \{ \int_{0}^{d} f^{0(\ell)}[\hat{\underline{x}}_{\mu}^{(\ell)}(t_{\mu}), \overline{\underline{u}}_{\mu}^{(\ell)}] dt_{\mu} + J_{\mu d}[\hat{\underline{x}}_{\mu}^{(\ell)}(0)] \} ,$$

$$J_{\mu d}[\hat{\underline{x}}_{\mu}^{(\ell)}(0)] = J_{\mu d}[\hat{\underline{x}}_{\mu-1}^{(\ell)}(d)] = \inf_{\overline{\underline{u}}_{i}^{(\ell)} \in \overline{D}^{(\ell)}[id,(i+1)d]} \{ \sum_{i=0}^{\mu-1} \int_{0}^{d} f^{0(\ell)}[\hat{\underline{x}}_{i}^{(\ell)}(t_{i}), \overline{\underline{u}}_{i}^{(\ell)}] dt_{i} \} ,$$

$$\hat{\underline{x}}_{\mu}^{(\ell)}(t_{\mu}) = \hat{\underline{x}}_{\mu}^{(\ell)}(0) + \hat{\underline{x}}_{\mu}^{(\ell)} \Big|_{0}^{t_{\mu}}, \quad \hat{\underline{x}}_{\mu}^{(\ell)}(d) = \hat{\underline{x}}_{\mu}^{(\ell)}(0) + \hat{\underline{x}}_{\mu}^{(\ell)} \Big|_{0}^{d} . \tag{40}$$

For μ=0:

$$J_{1d}[\hat{\underline{x}}_{0}^{(\ell)}(d)] = \inf_{\overline{\underline{u}}_{0}^{(\ell)} \in \overline{D}^{(\ell)}[0,d]} \{ \int_{0}^{d} f^{0(\ell)}[\hat{\underline{x}}_{0}^{(\ell)}(t_{0}), \overline{\underline{u}}_{0}^{(\ell)}] dt0 + J_{0d}[\hat{\underline{x}}_{0}^{(\ell)}(0)] \} ,$$

$$J_{0d}[\hat{\underline{x}}_{0}^{(\ell)}(0)] = 0 , \quad t_{\mu} \in [0,d], \mu = 0,1,2,...,N-1; \quad j = 1,2,...,K; \ell \in [1,s]. \tag{41}$$

Estimates of the state variables are obtained with described algorithm for optimal state estimation for systems with random structure.

Authors have also published corresponding variants of the inverse algorithms of the continuos-discrete dynamic programming for optimal control choice for systems with random structure [14]. Algorithms of inverse and direct continuos-discrete dynamic programming can be used in cases when choice of optimal control is not performed for each structure independently. In that case discrete form of the used optimality criterion is:

$$J = \sum_{\ell=1}^{s} \sum_{k=0}^{N-1} M_{w}[f^{0(\ell)}(\underline{x}_{k}^{(\ell)}, \underline{u}_{k}^{(\ell)}, \underline{w}_{k})] \quad \text{or} \quad J = \sum_{\ell=1}^{s} \sum_{k=0}^{N-1} f^{0(\ell)}(\hat{\underline{x}}_{k}^{(\ell)}, \underline{u}_{k}^{(\ell)}) , \tag{42}$$

depending of whether the probability density distributions or optimal state estimations are used.

Corresponding algorithms for optimal control choice are obtained in the similar manner as when each structure is considered independently, except that equations that take structure change into account are also added, that is equations from which state probabilities are determined.

Described techniques can be demonstrated on the example of mechanical system shown in Figure 6, which has two structures described with equations:

$$f^{(1)}\dot{x}^{(1)} + c^{(1)}x^{(1)} = u^{(1)}(t) + w(t),$$
$$f^{(2)}\dot{x}^{(2)} + c^{(2)}x^{(2)} = u^{(2)}(t) + w(t).$$

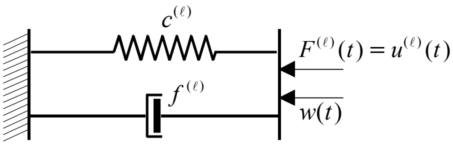

Fig 6. Mechanical system with 2 structures

Task is to determine optimal control for the first structure (procedure for the 2nd is the same) that minimizes function:

$$J = M_w\{\int_0^T \{[x^{(1)}]^2 + [u^{(1)}(t)]^2\}dt\} \quad \text{or} \quad J = \int_0^T \{[\hat{x}^{(1)}]^2 + [u^{(1)}(t)]\}dt\,. \tag{43}$$

Following parameters are known:

$$f^{(1)} = 1;\; c^{(1)} = 1;\; u^{(1)}(t) \in [-1,1];\; t \in [0,T];\; T = 0{,}4;\; w(t) = \begin{cases} +1 & \text{with probability } p_1 = 0{,}7 \\ -1 & \text{with probability } p_2 = 0{,}3 \end{cases}$$

$$u_\mu^{(1)}(t_\mu) = \bar{u}_\mu^{(1)} = \begin{cases} -1 \\ 0 \\ +1 \end{cases};\quad w_\mu(t_\mu) = \begin{cases} w_\mu(t_\mu) = +1, p_{\mu 1} = 0{,}7 \\ w_\mu(t_\mu) = -1, p_{\mu 2} = 0{,}3 \end{cases},\ \mu = \overline{0{,}7};\quad t_\mu \in [0,d],\, d = 0{,}2;\quad N = 8\,.$$

Task of the optimal control choice is solved by application of the inverse algorithm of the continuos-discrete dynamic programming [14], while as the optimality criterion in the first case expression (43a) has been used, and in the second case optimality criterion expressed with the optimal estimation of the state variable $\hat{x}^{(1)}$, defined with (43b).

Under described conditions it follows:

$$J_{\mu d}[x_\mu^{(1)}(0)] = \inf_{|\bar{u}_\mu| \le 1}\{\int_0^d \{\sum_{j=1}^2 p_{\mu j}\{[x_{\mu j}^{(1)}(t_\mu)]^2 + [\bar{u}_\mu^{(1)}]^2\}\}dt_\mu + J_{(\mu+1)d}[x_\mu^{(1)}(d)]\};\ \mu = 0,\dots,7;$$

$$x_{\mu j}^{(1)}(d) = x_{\mu j}^{(1)}(0) + \int_0^{t_\mu}[-x_{\mu j}^{(1)}(\tau_\mu) + \bar{u}_\mu^{(1)} + w_{\mu j}(\tau_\mu)]d\tau_\mu\,;\quad t_\mu \in [0,d]; \mu = 0,\dots,7; j = 1,2;$$

$$x_\mu^{(1)}(d) = x_{\mu j}^{(1)}(0) + \int_0^d \{\sum_{j=1}^2 p_{\mu j}[-x_{\mu j}^{(1)}(t_\mu) + \bar{u}_\mu^{(1)} + w_{\mu j}(t_\mu)]\}dt_\mu; \mu = 0,\dots,7\,;\quad J_{8d}[x_7^{(1)}(d)] = 0\,.$$

Table 1. Values of the optimal control

$\hat{x}_\mu^{*(1)}(0)$	$x_\mu^{*(1)}(0)$	$\bar{u}_\mu^{*(1)}(0)$
1,0282	1,0000	-1
1,2281	1,2040	-1
1,3376	1,3174	-1
1,4213	1,4380	-1
1,5485	1,5620	-1
1,6708	1,6835	-0,90
1,7739	1,7852	-0,90
1,9167	1,9272	0

Realization of this algorithm yields values of the optimal control $\bar{u}_\mu^{*(1)}$ and optimal trajectories for μ=0,1,2,3,4,5,6, 7. The same values, shown in Table 1, are obtained no matter whether the algorithm with the probability density distribution or with optimal estimation of state variables is used.

In the similar manner task can be solved by application of the presented direct algorithms of the continuos-discrete dynamic programming.

6. CONTROL OF PNEUMATIC POSITION SERVOSYSTEM: STOCHASTIC VS. FUZZY

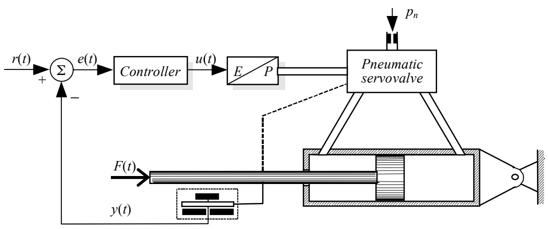

Fig. 7. Pneumatic position servosystem

In Figure 7 pneumatic position servosystem is shown, which is based on pneumatic servo cylinder. Approaches to the control of the above system are considered in [32] [31] [33], while in [29] an analytical-experimental linearized model of the pneumatic servo cylinder of the maximal stroke of 150 *mm* has been presented. When system is exposed to random disturbances, that is external force $F(t)$ has random character and measurement of the cylinder position is not accurate but disturbed with noise, control has to provide for the minimization of the act of random disturbances upon the system.

For the considered system optimal stochastic LQG controller can be proposed, and its design can conveniently be carried out by the so-called polynomial approach [30], developed by Grimble [28]. As the natural opponent to the optimal LQG controller, for the considered pneumatic servo system fuzzy PI controller can be proposed, which represents standard solution from the field of fuzzy control.

Polynomial approach to the LQG optimal controller design is based on general representation of the discrete scalar time invariant system in polynomial form. Problem of the synthesis of the optimal stochastic LQG controller is reduced to determination of the controller transfer function $C_0(z)$ that minimizes given quadratic optimality criterion:

$$J = E\{Q_c e^2(t) + R_c u^2(t)\}, \quad \text{or} \quad J = E\{Q_c y^2(t) + R_c u^2(t)\} \ \text{(for zero reference)}, \quad (44)$$

where $E\{.\}$ denotes unconditional expectation, and Q_c, R_c are positive weighting factors. Optimum has to be reached while minimizing effects of the disturbance and measurement noise. Transfer function of the LQG optimal controller, on the basis of polynomial approach and for the considered system description and criterion (44) is obtained by finding the solution of the corresponding Diophantine equation. Discrete model of the considered pneumatic servo in the form of transfer functions suitable for the considered polynomial approach to the LQG controller design is of the form:

$$W(z) = \frac{y(z)}{u(z)} = \frac{0.2802z^{-1} + 0.0957z^{-2}}{1 - 0.7945z^{-1} + 0.208z^{-2}}, \quad W_0(z) = \frac{y(z)}{w(z)} = \frac{0.0591z^{-1} + 0.0348z^{-2}}{1 - 0.7945z^{-1} + 0.208z^{-2}}. \quad (45)$$

If the system is affected by the disturbance and measurement noise of the low intensity, defined with variances $R = 0.0001$, $Q_2 = 0.01$, cost function to be minimized and finally the discrete transfer function of the LQG controller [30] is determined, where cascade PI controller is also introduced to provide for the elimination of the steady-state error:

$$J = E\{Q_c y^2(t) + R_c u^2(t)\},\ Q_c = 1,\ R_c = 0.0001 \Rightarrow C_0(z) = \frac{0.59 - 0.265z^{-1}}{0.38 + 0.129z^{-1} + 0.0000115z^{-2}} \cdot \frac{1 - 0.1z^{-1}}{1 - z^{-1}}. \quad (46)$$

To the contrary to the LQG controller, for the synthesis of the fuzzy PI controller mathematical model of the controlled pneumatic servosystem is not required. Inputs of the fuzzy PI controller are error $e(k)$ and its difference $\Delta e(k)$ while the output is the increment of the control $\Delta u(k)$. The controller is of the discrete incremental type, and domains of the inputs and output are transformed to the normalized closed interval [-1,1] and vice versa. Fuzzy partitioning of the input/output variables is accomplished by choice of the 7 primary fuzzy sets, labeled with linguistic labels that appear in fuzzy control rules, and for which triangular membership functions are chosen. If singleton fuzzification is applied, and for 49 fuzzy control rules, Mamdani's minimum operational rule and COG defuzzification, controller is defined with:

$$\alpha_i = \mu_{\tilde{A}_{i1}}(E_0) \wedge \mu_{\tilde{A}_{i2}}(\Delta E_0), \quad \mu_{\tilde{C}_i'}(w) = \alpha_i \wedge \mu_{\tilde{C}_i}(w), \quad (47)$$

$$\mu_{\tilde{C}}(w) = \mu_{\tilde{C}_1'}(w) \vee \ldots \vee \mu_{\tilde{C}_{49}'}(w) = [\alpha_1 \wedge \mu_{\tilde{C}_1}] \vee \ldots \vee [\alpha_{49} \wedge \mu_{\tilde{C}_{49}}],\ \ u_0 = \int \mu_{\tilde{C}}(w) \cdot w dw \Big/ \int \mu_{\tilde{C}}(w) dw. \quad (48)$$

Step responses of the system controlled with LQG and fuzzy PI controller are shown in Fig. 8. In the absence of disturbances (Fig. 11(a)) both controllers perform well. When system is affected by disturbance and measurement noise (variances $R = 0.0001$ and $Q_2 = 0.01$, Fig. 8(b)), LQG optimal controller also provides very good control quality, which does not surprise considering that real system and disturbances exactly match the models that have been used for controller design. As the intensity of the disturbances is relatively weak, fuzzy PI controller in its basic form provides good quality of control, with the design parameters unchanged in comparison with previous experiment.

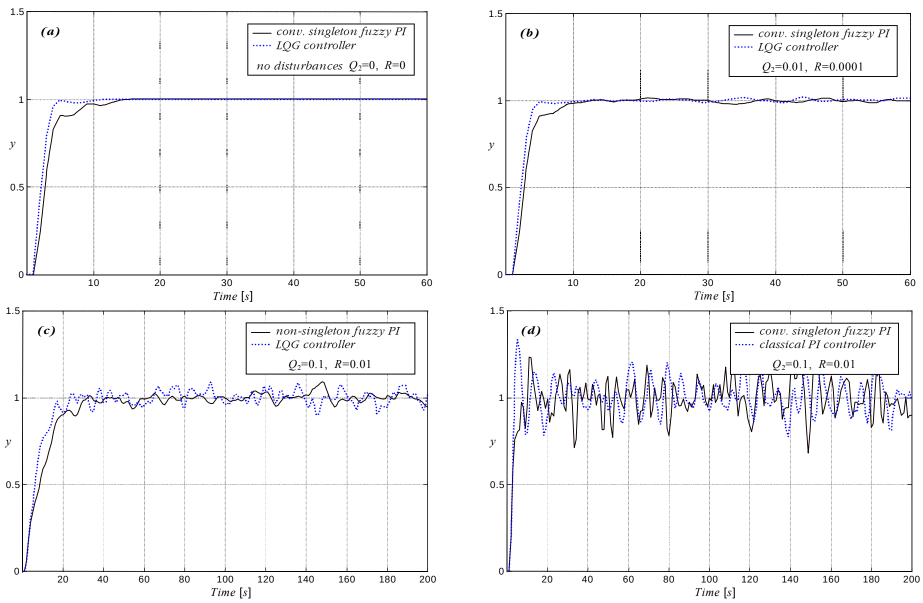

Fig. 8. System responses with various disturbances and controller realizations

When system is affected by the disturbances of the greater intensity, defined with variances $Q_2 = 0.1$ and $R = 0.01$, control systems have to be adapted according to the new requirements. For the optimal LQG controller it is:

$$J = E\{Q_c y^2(t) + R_c u^2(t)\},\quad Q_c = 100,\ R_c = 0.02.\ \Rightarrow\ C_0(z) = \frac{0.0956 - 0.0245z^{-1}}{0.2983 + 0.1004z^{-1} + 0.0001z^{-2}} \cdot \frac{1 - 0.1z^{-1}}{1 - z^{-1}}. \quad (49)$$

To improve filtration capabilities of the fuzzy PI controller, non-singleton fuzzification is applied. Gaussian membership functions have been adopted, which yields to the modifications described in the case of vehicle parking control case. Unit step response of the system affected with described disturbances ($Q_2 = 0.1$, $R = 0.01$) is shown in Fig. 4(c), when system is controlled with non-singleton fuzzy PI controller and LQG optimal controller . It can be concluded that non-singleton fuzzy controller maintains performance quite comparable with LQG optimal stochastic controller. For the sake of comparison, in Fig. 4(d) step response under the same disturbance conditions is shown, but when system is controlled with singleton fuzzy PI controller and with classical PI controller, which proves that both non-singleton fuzzy PI and LQG controller provide good filtration capabilities in comparison with their basic configurations.

In Figure 9 unit step responses of the system controlled with non-singleton fuzzy PI controller and LQG controller are shown, when system is affected by the disturbance with variance $Q_2 = 0.1$ and by measurement noise with variance $R = 0.1$. Parameters have been kept the same as in the previous experiment, which means that synthesis of the controllers have been done with wrong priors indicating 10 times smaller variance of the measurement noise $R = 0.01$.

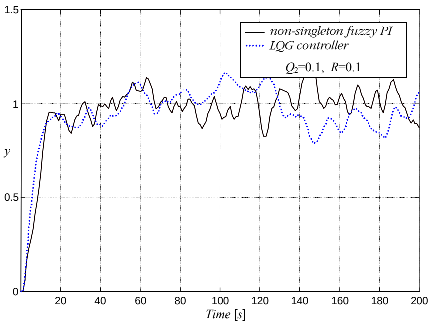

Fig. 9. Responses with inaccurate priors

It is obvious that when divergence of disturbances from the prior information which is used for controllers design and tuning exists, deterioration of performance of both controllers occurs. Results indicate that fuzzy controller has somewhat better robustness. Bearing in mind that need for reliable priors represents one of the main drawbacks of the stochastic controllers, robustness of the fuzzy control systems with respect to the disturbances' effect is highly significant, especially because some results indicate that fuzzy controllers can often be adopted as robust and numerically simpler alternative to conventional stochastic controllers [24]. Robustness of the both control systems in case of inaccurate priors can be significantly improved by introducing adaptation mechanisms [32] [28] [27].

7. CONCLUSION

In this paper attention has been paid to the choice of optimal control for stochastic systems. Starting from well known results from deterministic and stochastic optimal control theory, two simple but highly applicable algorithms have been considered. One of them has been used for optimal control design for asynchronous motor and those results have also been presented in the paper.

It is well known that random structure of stochastic processes in control systems raises a range of new tasks that need to be solved in the modern stochastic theory of dynamic systems. In this paper author has presented only part of results of his research in this field, that consider state estimation for nonlinear systems with random structure, as well as optimal control choice for these systems, by using methods of the direct and inverse continuos-discrete dynamic programming.

Although quality of the control that can be obtained with stochastic controllers when reliable priors and object model exist is very high, possession of those information is extremely rare in practice. Conversely, as it has been considered in the paper, robustness of fuzzy controllers indicate that in many cases they can be adopted as numerically simpler alternative to conventional stochastic systems. Also, significant results in the field of noise filtration have lately been obtained with use of soft computing techniques and especially neural nets [24].

REFERENCES

1. Hyland D.C., Bernstein D.S. (1984), *The optimal projection equations for fixed-order dynamic compensation*, IEEE Trans. Autom. Control, AC-29, 1034-1037.
2. Mielczarski W. (1980), *Stabilization of angular speed of asynchconous motors*, Archiwum Automatyki i Telemechaniki, XXV, 1, 75-84.
3. Szkalarski L., Jaracz K. (1985), *Influence of random load disturbances on the driving motors of mining machines*, Archiwum Gornictva, 30, 1, 41-51.
4. Nikolić V., Stokić D., Antić D. (1993), *Hierarchical procedure for choosing optimal control of a continual large nonlinear mechanical system*, Int. J. Nonlinear Vibration Problems, v. 25, pp. 325-333, Poland.
5. Kumar P.R. (1985), *A survey of some results in stochastic adaptive control*, SIAM J.Control and Optim., 23, 329-380.
6. Kumar P.R. (1983), *Optimal adaptive control of linear-quadratic Gaussian systems*, SIAM J. Control and Optimiz., 21, 163-178.
7. Iserman R. (1992), *Estimation of physical parameters for dynamic processes with application to an industrial robot*, Int. J. of Control, Vol. 55, No. 6, 1287-1299.
8. Yaesh I., Shaked U. (1992), *Game theory approach to state estimation of linear discrete-time processes and its relation to H_∞-optimal estimation*, Int. J. of Control, Vol. 55, No. 6, 1443-1453.
9. Yonezawa K. (1980), Reduced-order Kalman filtering with incomplete observability, J. Guidance and Control, V. 3, No. 3, 280-282.
10. I.D. Siluànova (1980), *Ocenivanie sostoània i parametrov nelineïnìh sistem pri pomehe i nablòdeniàh, otliånoï ot belogo ãuma*, Avtomatika i Telemehanika, No 10.
11. Nikolić V., Ćojbašić Ž., Pajović D. (1993), *Proposal for an algorithm for increasing the accuracy of an optimal estimation of the generalized Kalman discrete filter for non-linear systems*, Teorijska i primenjena mehanika, 19, pp. 99-106.
12. Kalman R.E. (1960), *A new approach to linear filtering and prediction problems*, Trans. ASME, Series D, Vol. 84, 35-45.
13. Jazwinski A.H. (1969), *Adaptive filtering*, Automatica, Vol. 5, No. 4.
14. Nikolić V., Ćojbašić Ž., Pajović D. (1994), *Choice of the optimal control with the stochastic systems with the random structure by applying inverse and direct algorithm of the continious-discrete dynamic programming*, Bulletins for Applied Mathematics, BAM 1011/94, 223-232.
15. Nikolić, V. (1987), *Chice of the optimal control for discrete systems with random influences by using global optimality criterion and optimal state estimation*, VIII Yugoslav Congress of Aerocosmonautics, pp. 62-69, Mostar (in Serbian).
16. Nikolić, V., Petrušić, Z. (1990), *Adaptive algorithm for optimal scalar estimation of the state vector components for the discrete model of the mine elevator*, 10 International Conference ICAMC 90, pp. 359-364, Ostrava, Checoslovakia.
17. Miter K. S., Moro A. (1982), *Non linear filtering and stohastic control*, Springer-Verlag, Lecture Notes

in Math 972, Berlin.
18. Nikolić V. (1987), *Choice of the optimal control for stochastic nonlinear systems by using indirect continuous-discrete algorithm of dynammic programming*, Kibernetika - Sistemi, automatsko upravljanje i merenje, pp. 5-17, Beograd (in Serbian).
19. Nikolić V. (1986), *Choice of the optimal control for stochastic nonlinear systems by using direct continuous-discrete algorithm of dynammic programming*, Proceedings of the XVII YUCTAM, A-23, pp. 135-143, Zadar (in Serbian).
20. Kazakov I.E. (1979), *Ocenivanie i identifikacija v sistemah peremennoj strukturi*, Automatika i telemehanika, No. 11, pp. 11-14.
21. Grimble M.J., Johnson M.A. (1987), *Optimal control and stochastic estimation-Theory and Applications, Volume 1 and Volume 2*, John Wiley & Sons, New York.
22. Zadeh L. A., Fuzzy Logic (1994), *Neural Networks and Soft Computing*, Communications of the ACM, Vol. 37, No. 3, pp. 77-84.
23. Schwartz D. G., George J. K., Harold W. L., Yoshinori E. (1994), *Applications of Fuzzy Sets and Approximate Reasoning*, Proceedings of the IEEE, Vol. 82., No. 4, pp. 482-497.
24. Kosko B. (1992), *Neural Networks and Fuzzy Systems - A Dynamical Systems Approach to Machine Intelligence*, Prentice Hall, Englewood Cliffs.
25. Lee C. C. (1990), *FuzzyLogic in Control Systems: Fuzzy Logic Controller - Parts I and II*, IEEE Transactions on Systems, Man and Cybernetics, Vol 20., No. 2, pp. 404-435.
26. Munakata T., Jani Y. (1994), *Fuzzy Systems: An Overview, Communications of the ACM*, Vol. 37, No. 3, pp. 69-76.
27. Cox E. (1993), *Adaptive fuzzy systems*, IEEE Spectrum, Vol. 30, No. 2, pp. 27-31.
28. Grimble J. (1994*), Robust Industrial Control*, Prentice-Hall, Hemel Hempstead.
29. Nedić N., Pršić D., Rakonjac V. (1993), *Analytical-experimental determination of the mathematical model of pneumatic servo-cylinder*, HIPNEF '93, pp. 115-120 (in Serbian).
30. Nikolić V., Pajović D., Ćojbašić Ž. (1997), *Polynomial solution of the LQG optimal regulation problem for a pneumatic system*, Bulletins for Applied Mathematics, Budapest.
31. Pršić D., Nedić N., (1993), *Pneumatic servo-cylinder control by using natural tracking law*, HIPNEF '93, pp. 127-132.
32. Pršić D., Nedić N. (1995), *Pneumatic servo-cylinder control by using state variables*, Proceedings of the 5th SAUM Conference, pp. 281-285, Novi Sad (in Serbian).
33. Ćojbašić Ž., Nikolić V. (1997), *Comparison of Fuzzy PI and Stochastic LQG Controller for a Pneumatic Servosystem Exposed to Random Disturbances*, Bulletins for Applied Mathematics, Budapest.

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NEKI ALGORITMI ZA OPTIMALNU OCENU VELIČINA STANJA I IZBOR OPTIMALNOG UPRAVLJANJA KOD STOHASTIČKIH SISTEMA

Vlastimir Nikolić

U ovom radu su razmatrani neki pristupi upravljanju stohastičkim sistemima. Prezentirani su algoritmi za optimalnu estimaciju stanja i parametara kod stohastičkih sistema, sa konstantnom i promenljivom strukturom. Za sisteme sa promenljivom strukturom pretpostavljena je slučajna promena strukture i da predstavlja Markovljev proces. Na osnovu algoritama za optimalnu estimaciju stanja i parametara formirani su odgovarajući algoritmi za izbor optimalnog upravljanja. Izazov za istraživanje u ovoj oblasti je pronalaženje adaptivnih algoritama za povećanje tačnosti estimacije, a na taj način i upravljanja, u slučajevima kada apriorne informacije o stohastičkim poremećajima nisu dovoljno tačne. U takvim slučajevima mogu uspešno biti primenjene alternative tehnike zasnovane na primeni fazi logike u upravljačkim sistemima. Performanse i robusnost fazi PI kontrolera i stohastičkog LQG optimalnog kontrolera upoređene *su za jedan pneumatski servosistem, u slučaju kada na sistem deluju slučajni poremećaji.*

Ključne reči: *upravljanje, stohastički sistemi, estimacija stanja, optimalno upravljanje, fazi upravljanje, filtracija, estimacija, LQG upravljanje*