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IS DYNAMIC CONTROL NEEDED FOR ROBOTS INTERACTING WITH DYNAMIC ENVIRONMENT?

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Abstract. *The paper addresses the role of dynamics in the control of robots interacting with a dynamic environment. The objective is to define is it necessary to include compensation for the different dynamic factor in the position/force control law. Starting from the complete dynamic model of the robot and of the environment, the effects of different dynamic factors upon position and force control of the robot are analysed. The control law based on complete inverse dynamic model and the decentralised control law are compared. The dynamics effects are first qualitatively analysed, and then an approach to explore these effects based on analysis of the practical stability of manipulation robots in the constrained motion control tasks is presented. The elaborated stability test can be used to check for each particular robot structure and environment and for different control tasks which dynamic effects have to be taken into account.*

1. INTRODUCTION

The problem of effects of dynamics upon the control of manipulation robots has been a subject of a number of studies. Different dynamic control laws (i.e. control laws which include explicit compensation for effects of dynamics upon the robot performance and tracking of desired trajectories) have been proposed. In our previous books and papers [1-3] we have analysed the role of the dynamics in the free motion control of robots. The conclusion has been that the effects of dynamics have to be explicitly compensated by the control law if accurate tracking of the fast nominal trajectories has to be achieved, otherwise more simple (and more robust, less sensitive to model and parameter uncertainties) decentrilised control laws can be effectively applied.

In this paper we shall address the effects of dynamics of the robot and of the environment in a control of the robot interacting with the dynamic environment. The

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synthesis of the control of robots in the tasks in which manipulation robots are coming into a contact to the environment, attracts high attention in last 15 years. A number of control laws have been proposed which include explicit compensation for different dynamic effects. Numerous papers have considered the stability aspects assuming different approximate models of the robot and the environment [4-7] It has been recognised [7] that the dynamics of robots is playing very important role in this class of the manipulation robot tasks. In [4-6] the control laws stabilising simultaneously the robot motion and its interaction forces with a dynamic environment have been synthesised, ensuring exponential stability of the closed loop systems. It is obvious that the compensation of dynamic effect may contribute to improve robot tracking of the desired trajectories. However, one of the main problems in a synthesis of dynamic control laws represent uncertainties in the dynamic models of the robot and, specially, of the environment. A complete analysis of the necessity to explicitly compensate for different dynamic effects (both robot dynamics and the environment dynamics) has not been carried out up to now.

The objective of the paper is to establish a procedure to test which dynamic effects have to be compensated for by a control law under given conditions (manipulation robot structure, environment dynamics, required perfomance, i.e. speed, accuracy, etc.). This procedure has to give an answer to which extend a dynamic control laws are needed in this special class of robotic tasks. We shall start from the complete dynamic model of the robot and the environment, and from the most general formulation of the control task. Then, two control laws will be considered: one which completely compensates for all dynamic effects, and the one (so-called decentralised) control law without any explicit compensation for these effects. A qualitative analysis of the effects of dynamics will be carried out by comparing these two laws. Next, a procedure to test the efects of the dynamics will be presented based on the practical stability analysis. The conditions for the practical stability of the robot interacting with the dynamic environment have been derived in our previous papers [8,9]. In this paper we shall apply a new, improved stability test, derived based on the decomposition-aggregation principle, which enables better to study the dynamic effects upon the robotic system performance.

2. Mathematical Model

We shall consider a robot in a contact a with dynamic environment, assuming that the contact is permanently maintained. An important problem of keeping of contact and impact between the robot and environment (i.e. discontinuity of the model since the robot can only push and not pull the environment) is out of the scope of this paper (and will be a subject of our future work). The complete dynamic model of the robot with n ($n \leq 6$) degrees of freedom and the dynamic model of the environment (described by the second-order differential equations) are considered in Cartesian space. The model of dynamics of the mechanical part of the robot (in Cartesian space) can be written in the form:

$$\Lambda(p,d)\ddot{p} + \rho(p,\dot{p},d) = J^{-T}(p,d)\tau + F \tag{2.1}$$

where $p = p(q)$ is the $nx1$ vector of the robot Cartesian coordinates, while q is the $n\times 1$ vector of the robot internal coordinates, $L(p,d)$ is the nxn inertia matrix, $r(p,\dot{p},d)$ is the

$n\times 1$ nonlinear vector function of Coriolis, centrifugal and gravity moments, d is the 1x1 vector of parameters which belongs to the constrained set D, $J(p,d)$ is the $n\times n$ Jacobian matrix, t is the $n\times 1$ vector of driving torques[1] (inputs), F is the $m\times 1$ vector of Cartesian forces, generalised interaction forces (forces and moments) acting upon the end-effector of the robot. In this paper we shall consider the case $n = m$. For the sake of simplicity we shall write functions without arguments.

The model of dynamic environment can be writte in the form:

$$M(p,d)\ddot{p} + L(p,\dot{p},d) = -SF \tag{2.2}$$

where M is the $n\times n$ matrix, L is the $n\times 1$ nonlinear vector function, and S is the $n\times n$ matrix expressed by Cartesian coordinates, with a rank equal[2] $m = n$. We chall assume that $S = I$. It is assumed that all mentioned matrices and vectors are continuous functions of their arguments. The model of the robot in the state space can be defined in the following form:

$$\dot{x} = f(x,d) + B(x,d)\tau + G(x,d)F \tag{2.3}$$

where $x = (p^T, \dot{p}^T)^T$ is $2n\times 1$ state vector, $f(x,d)$ is $2n\times 1$ vector function, $B(x,d)$ is $2n\times n$ matrix, $G(x,d)$ is $2n\times m$ matrix.

3. DEFINITION OF CONTROL TASK

Let us assume that in m_1 directions ($m_1 < m$) desired force trajectories $F^{01}(t)$ are specified, where $F^{01}(t)$ is $m_1\times 1$ vector, while in n_1 directions desired trajectory $x^1(t)$ is specified, where $x^1(t)$ is $n_1\times 1$ and where $n_1 + m_1 = n$. Note that the meaning is following: in some directions only forces are specified, in some directions only desired position trajectories are specified. Let us introduce the following notations: $p^0(t)=(p^{01T}(t),p^{02T}(t))^T$, where $p^{02T}(t)$ is $(n-n_1)\times x$ vector of the nominal trajectories of the Cartesian coordinates in the directions in which force trajectories are specified. Note that the trajectories $p^{02T}(t)$ are not specified in advance, but have to be determined based on the model of the environment. Similarly the vector of desired force trajectories can be denoted as $F^0(t)=(F^{01T}(t),F^{02T}(t))^T$,where $F^{02T}(t)$ is $(m-m_1)\times 1$ vector of the nominal force trajectories in directions in which forces are acting upon the robot, but the nominal trajectories of the Cartesian space coordinates are specified. The force trajectories ($F^{02T}(t)$) are not specified in advance, but have to be calculated based on the dynamic model of environment. The nominal trajectories of the forces and of the Cartesian coordinates must satisfy the model of the environment i.e.:

$$M(p^0(t))\ddot{p}^0(t) + L(p^0(t),\dot{p}^0(t)) = -F^0(t) \tag{3.1}$$

Let us introduce the following notations:

[1] For the sake of simplicity we shall consider the second order models of actuators, which are assumed to be included in the robot dynamic model.

[2] This is the property of the environment dynamic model (2.2) that its solution is unique with respect to the force F.

$$M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \text{ and } L = \begin{bmatrix} L_1^T, & L_2^T \end{bmatrix}^T \tag{3.2}$$

where dimensions of matrix M_{11} are $m_1 \times n_1$, of M_{12} are $m_1 \times (n - n_1)$, of M_{21} are $(m - m_1) \times n_1$, M_{22} are $(m - m_1) \times (n - n_1)$, while L_1 and L_2 are vectors of dimensions $m_1 \times n_1$ and $(m - m_1) \times 1$ respectively. The desired trajectory in the state space is denoted by $x^0(t) = (p^{0T}(t), \dot{p}^{0T}(t))^T$.

Now, the control task is specified in the following form as a task of practical stability of the robot around the nominal trajectory $x^0(t)$: The control of robot has to ensure that $\forall x(0) \varepsilon \mathbf{X^I}$ and $\forall d \varepsilon \mathbf{D}$ imply $x(t) \varepsilon \mathbf{X^t}(t)$, where $\mathbf{X^I}$ and $\mathbf{X^t}(t)$ are the finite regions in the state space around the prescribed nominal trajectory $x^0(t)$ and $\mathbf{T} = (t, t\varepsilon(0, t_1))$, t_1 is the predefined time period. It is assumed that $x^0(0) \varepsilon \mathbf{X^I}$, $x^0(t) \varepsilon \mathbf{X^t}(t)$, $\forall t \varepsilon \mathbf{T}$, $\mathbf{X^t}(0) \subset \mathbf{X^I}$.

This formulation of the control task can be interpreted as presented in [8,9]. Due to (3.1), and as shown in [8,9], a fulfilment of the specified control task also guarantees tracking of the desired force trajectories $F^0(t)$, i.e. it guarantees that $F(0) \varepsilon \mathbf{F^I}$ and $\forall d \varepsilon \mathbf{D}$ imply $F \varepsilon \mathbf{F^t}(t)$, $\forall t \varepsilon \mathbf{T}$, where $\mathbf{F^I}$ and $\mathbf{F^t}(t)$ are regions in the $m \times 1$ space around the nominal trajectory $F^0(t)$. Note that regions $\mathbf{F^I}$ and $\mathbf{F^t}(t)$ must correspond to the regions $\mathbf{X^I}$ and $\mathbf{X^t}(t)$, respectively. In order to simplify stability analysis let us consider specific forms of the finite regions $\mathbf{X^I}$ and $\mathbf{X^t}(t)$: $\mathbf{X^I} = \{x(0) : \|\Delta x(0)\| < \underline{X}^I\}$, $\mathbf{X^t}(t) = \{x(t) : \|\Delta x(t)\| < \underline{X}^t \exp(-\alpha t)\}$, $\forall t \varepsilon \mathbf{T}$, $\mathbf{F^I} = \{F(0) : \|\Delta F(0)\| < \underline{F}^I\}$ $\mathbf{F^t}(t) = \{F(t) : \|\Delta F(t)\| < \underline{F}^t \exp(-\beta t)\}$, $\forall t \varepsilon \mathbf{T}$, where $\underline{\mathbf{X}}^{\mathbf{t}} > \underline{\mathbf{X}}^{\mathbf{I}} > 0$, $\underline{\mathbf{F}}^{\mathbf{t}} > \underline{\mathbf{F}}^{\mathbf{I}} > 0$, $\alpha > 0$, $\beta > 0$. Here $\underline{X}^t, \underline{X}^I, \underline{F}^t, \underline{F}^I$, α, β, denote real-valued positive numbers, $\|\cdot\|$ denotes Euclidean norm of the corresponding vector, and Δx is $2n \times 1$ vector of the state deviation around the desired nominal trajectory $x^0(t)$, i.e. $\Delta x(t) = x(t) - x^0(t) = (\Delta p^T(t), \Delta \dot{p}^T(T))^T$ while $\Delta F(t)$ is $n \times 1$ vector of the force deviation around the desired force trajectory.

4. Control Law

We shall consider two control laws:

First, **dynamic position/force control law** (see Fig. 1) is considered [4,5]:

$$\tau = U^*(p, \dot{p}, \ddot{p}_c, F) \tag{4.1}$$

where

$$U^*(p, \dot{p}, \ddot{p}, F) = J^{*T}(\Lambda^* \ \ddot{p} + \rho^* - F) \tag{4.2}$$

where J^*, Λ^*, ρ^* denote matrices and vector corresponding to J, Λ, ρ from the model (2.1) but with the assumed parameters values $d = d_0 \varepsilon \mathbf{D}$. This means that we assume that the structure of the model is known, while the parameters values are not accurately known. $\ddot{p}_c$ in (4.1) stands for

$$\ddot{p}_c = \begin{bmatrix} \ddot{p}^{01} + P_1(\Delta p^1, \Delta \dot{p}^1) \\ W^*(p, \dot{p}, \ddot{p}^{01} + P_1(\Delta p^1, \Delta \dot{p}^1), F^{01} + \Delta F^1) \end{bmatrix}$$

where

$$W^*(p,\dot{p},\ddot{p}^{01},F^{01}) = M_{12}^{*-1}(-F^{01} - M_{11}^*\ddot{p}^{01} - L_1^*)$$

$$\Delta F^1 = K^{1F}\int(F^1(t) - F^{01}(t))dt$$

$$P_1(\Delta p^1,\Delta\dot{p}^1) = K_1^1\Delta p^1 + K_2^1\Delta\dot{p}^1$$

where M_{11}^*, M_{11}^*, M_{11}^* denote matrices and vector corresponding to M_{11}, M_{12} and L_l from the model (3.1) but with the assumed parameters values d_0. F^1 is $m_1\times1$ sub-vector of force vector F for which the nominal trajectories are specified in advance, $F = (F^{1T},F^{2T})^T$, K^{1F} is $m_1\times m_1$ matrix of force feedback gains, K_1^1 and K_2^1 denote $n_1\times n_1$ matrices of position and velocity feed-back gains (for the sake of simplicity we shall assume that the both matrices are diagonal), respectively, $\Delta p^1 = p^1(t)-p^{01}(t)$ is $n_1\times1$ vector. This control law takes into account complete dynamic models of the robot and environment as well as interaction among directions in which position is controlled and directions in which force is controlled. This means that both position and force feedback loops are used in all directions. The control law may be considered in a more general form, i.e. P_1 and ΔF^1 can be defined in a more general form.

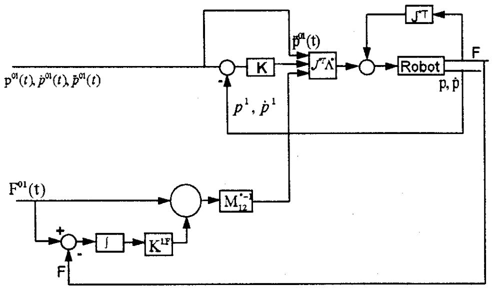

Fig.1. Dynamic position/force control law

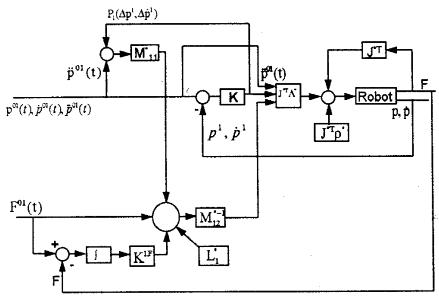

Fig. 2. Decentralised (non=dynamic) position/force control law

The second control law to be considered (see Fig.2), so-called **decentralised position/force control law**, can be derived from the first one by taking $M_{11}^* = 0$, $L_1^* = 0$, $\rho^* = 0$, $M_{12}^* = diag(m^{ii^*})$, $\Lambda^* = diag(\Lambda^{ii^*})$, where m^{ii^*} and Λ^{ii^*} are constants. This control law ignores practically all dynamic effects. It actually represents the so-called classical hybrid control scheme [10], although selectivity matrix explicitly is not presented. Assuming that K_1^1 and K_2^1 as well as K^{1F} are diagonal matrices, this control law stabilises each Cartesian d.o.f. independently from the others (i.e. the dynamic coupling between d.o.f. is ignored both for position and for force controlled directions).

5. Qualitative Analysis of Dynamic Effect

In this section a brief qualitative analysis of the effects of the different dynamic factors will be presented. By comparing the two proposed control laws, we may analyse what can be the influences of the different dynamic effects which are ignored in the second control law.

Position control part: The effects of the robot dynamics upon the position controlled part are essentially the same as in the free motion control. This means that ignoring of the Coriolis forces $\rho^* = 0$, non-diagonal elements in the matrix of inertia $(\Lambda^* = diag(\Lambda^{ii^*}))$, may lead to errors in a tracking of the (fast) desired trajectories, but not in errors in final positioning of the robot [3]. Gravity moments have to be compensated in order to ensure accurate positioning of the robot. Influence of the force controlled directions upon the position control part via non-diagonal elements in the matrix Λ (2.1) may lead to errors in the trajectory tracking if excessive accelerations (forces) appear and if these non-diagonal elements are large. Effects of the interaction forces in the position controlled directions can be easily compensated by direct force feedback (if delay in feedback loop is acceptable and Jacobean J is accurately calculated) as presented in Fig. 2.

Force control part: The main problems of the stability of the force controlled directions (and of the over-all system) are deeply related to the characteristics (i.e. of the character of the vector L in the model (2.2)) of the environment dynamics as shown in [4-6]. However, once the environment dynamics fulfils stability requirements, the influence of the dynamic effects upon the force controlled directions have also to be analysed. Ignoring of the dynamic effects of the position controlled directions upon the force controlled directions $(M_{11}^* = 0)$, the dynamic coupling between different directions $(\Lambda^* = diag(\Lambda^{ii^*}))$, as well as ignoring of the interactions among the force controlling directions themselves $(M_{12}^* = diag(m^{ii^*}))$, may lead to high errors in the force controlled directions since these force control loops may be more sensitive to perturbations than the position controlled directions. Obviously, these influences depend on the robot structure (matrix of inertia Λ in the model (2.1)), characteristics of the environment (matrices of the moments of inertia of the environment M_{11} and M_{12} in the model (3.2)), and the imposed control task (characteristics of the desired force trajectories, desired accuracy, i.e. the defined regions of the practical stability).

6. PRACTICAL STABILITY ANALYSIS AS A PROCEDURE TO EXPLORE EFFECTS OF DYNAMICS

In this section we shall establish a procedure for analysis of practical stability of the robots interacting with the dynamic environment, which can be used to explore which dynamic effects can be ignored and which have to be taken into account in order to fulfil the stated control task. The closed loop model of the robotic system (model of deviation around the desired nominal trajectory $x^0(t)$ in the state space is obtained by combining the robot model in the state space (2.3) and the corresponding control law (4.1):

$$\Delta\dot{x} = \Delta f(\Delta x, x^{0^*}, d) + \Delta G(\Delta x, d)F \tag{6.1}$$

where $\Delta f(\Delta x, x^{0^*}, d)$ is $2n\times 1$ vector and $\Delta G(\Delta x,d)$ is $2n\times n$ matrix. In the previous papers [4-6] it has been shown that application of the control law (4.1) ensures desired tracking of the prescribed nominal force trajectory (i.e. desired transient behaviour of the force $F(t)$, if an ideal model of the system used in the control law is assumed. Since the model of the system (robot + environment) in (4.1) is not ideal (due to parameters uncertainties), the desired force transient process cannot be perfectly achieved. However, it can be shown that by appropriate selection of the force feedback gains, and under assumption of the limited deviations of the assumed model parameters from the actual values, the control law (4.1) can guarantee that the force transient process satisfies the conditions of the practical stability, i.e.

$$\|\Delta F(t)\| < \underline{F}^t \exp(-\beta t) \tag{6.2}$$

Starting from the assumption that the force transient process satisfies (6.2), it has to be examined whether the control law (4.1) can ensure the practical stability of the overall system. This means that the conditions, under which the proposed control law fulfils the specified control task, i.e. the conditions of the practical stability of the robotic system have to be derived. It can be shown [11, 12]. that the system is practically stable with respect to $(\mathbf{X}^{\mathbf{I}},\mathbf{X}^{\mathbf{t}},\mathbf{T})$ as defined above, if there exist a real valued continuously differentiable function $v(t,x)$ and a real valued function of time $\Psi(t)$ which is integrable over the time interval T such that

$$\dot{v}(t,x) \le \Psi(t)\,,\ \forall x\varepsilon\widetilde{X}^t(t)\,,\ \forall t\varepsilon T \tag{6.3}$$

$$\int_0^t \Psi(t')dt' < v_m^{\partial X^t}(t) - v_M^{\partial X^I}(0)\,,\ \forall t\varepsilon T \tag{6.4}$$

$\partial\mathbf{X}(t)$ denotes the boundary of the corresponding region and $\widetilde{X}^t(t) = \mathbf{X}^{\mathbf{t}}(t) - \underline{X}^I \exp(-\alpha t)$. In (6.3) $\dot{v}$ denotes the time derivative of the function $v(t,x)$ along the solution of the closed-loop system. v_m and v_M denote minimum and maximum values of v at the corresponding boundaries of the regions, respectively. Note that the conditions (6.3) and (6.4) are sufficient but not necessary conditions for the practical stability of the system. The proof of the above stated method for the testing of practical stability is provided in previous papers [11, 12].

Following the aggregation/decomposition approach to system stability analysis we may decompose the system (6.1) into a set of n 'subsystems': the position controlled part subsystems $i = 1,...,n_1$ and the force controlled part subsystems ($i = n_1+1,...,n$):

$$\Delta\dot{x}^i = A_{ii}\Delta x^i + \Delta f_i(\Delta x, x^{0^*}, \Delta F, F^0, d) + \Delta G_i(\Delta x, d)F\,,\quad i = 1,...,n \tag{6.5}$$

where $\Delta x^i(t) = x^i(t) - x^{0i}(t) = (\Delta p^i(t), \Delta\dot{p}^i(t))^T$, $i = 1,...,n$ and Δf_i and ΔG_i are vectors and matrices of the appropriate dimensions, while the 2×2 matrices A_{ii} are given by:

$$A_{ii} = \begin{bmatrix} 0 & 1 \\ \overline{Q}_{11}^{ii} K_1^{1ii} & \overline{Q}_{11}^{ii} K_2^{1ii} \end{bmatrix},\ i = 1,...,n_1\,,$$

$$A_{ii} = \begin{bmatrix} 0 & 1 \\ \overline{Q}_{22}^{ii} L_1^{Pi} & \overline{Q}_{22}^{ii} L_1^{Vii} \end{bmatrix},\ i = n_1 + 1,...,n\,,$$

where $\overline{Q}_{11}^{ii}$ and $\overline{Q}_{22}^{ii}$ diagonal (constant) elements of the $n_1\times n_1$ and $n_2\times n_2$ matrices representing estimates of $Q_{11}=\Lambda_1^{-1}J^{-T}J^*\Lambda_1^*$ and $Q_{22}=\Lambda_2^{-1}J^{-T}J^*\Lambda_2^*$ (where Λ_1^{-1}, Λ_1^{-1}, Λ_1^*, Λ_2^* are appropriate submatrices of the matrices Λ^{-1} and Λ^*. L_1^{Pi} and L_1^{Vi} represent elements of the vectors L_1^P and L_1^V estimating stability factors in the dynamic model of the environment, i.e $M_{12}^{*-1}L_1^*-M_{12}^{0-1}L_1^0=L_1^P\Delta p^2+L_1^V\Delta\dot p^2+L_1^R$.

Let us consider the practical stability of the decoupled subsystems $\Delta\dot x^i = A_{ii}\Delta x^i$. We may assume that the regions of practical stability can be presented in the form $X^I=X^{I(1)}\times\ldots\times X^{I(n)}$ and $X^t=X^{t(1)}\times\ldots\times X^{t(n)}$ where $\mathbf{X}^{\mathbf{I}(i)}=\{x^i(0):\|\Delta x^i(0)\|<\underline{X}^{I(i)}\}$, $\mathbf{X}^{\mathbf{t}(i)}=\{x^i(t):\|\Delta x^i(t)\|<\underline{X}^{t(i)}\exp(-\alpha_i t)\}$, $\forall t\varepsilon T$, where $\underline{X}^{t(i)}>\underline{X}^{I(i)}>0$, $\alpha_i>0$. Let us select the functions $v_i(t,x)$ $(i=1,\ldots,n)$ in the form: $v_i(t,x)=(\Delta x^{iT}H_i\Delta x^i)^{1/2}$, $(i=1,\ldots,n)$ where H_i are the positive definite matrices of the appropriate dimensions. The derivative of the functions v_i along the solutions of the decoupled subsystems can be written as:

$$\dot v_i(t,x)=(grad\,v_i)^T A_{ii}\Delta x^i\le-\gamma_i v_i\le-\gamma_i'\,\|\Delta x^i\| \tag{6.6}$$

where $\gamma_i=-\min|\lambda(A_{ii})|$, $\lambda(A_{ii})$ denotes eigenvalues of the corresponding matrix, $\gamma_i'=\gamma_i\lambda_m(H_i)$. The conditions (6.6) are valid under assumption that the matrices H_i are selected to satisfy $H_iA_{ii}+A_{ii}^TH_i\le-2\gamma_iH_i$. The coupling members in the subsystems (6.5) can be estimated in the following form (sums indices are omitted for the sake of simplicity):

$$\begin{gathered}(grad\,v_i)^T\Delta f_i(\Delta x,x^{01},\Delta F,F^{01},d)<\\ \sum\xi_{ij}^1\,|x_j^{01}(t)|+\sum\xi_{ij}^2\,|F_j^{01}(t)|+\sum\xi_{ij}^3\,|\dot x_j^{01}(t)|+\sum\xi_{ij}^4\,|\Delta x_j|+\sum\xi_{ij}^5\,|\Delta F_j|\end{gathered}\tag{6.7}$$

$$\begin{gathered}(grad\,v_i)^T\Delta G_i(\Delta x,d)F<\sum\xi_{ij}^6\,|\Delta x_j|+\\ \sum\xi_{ij}^7(|F_j^0(t)|+|\Delta f_j|),\quad i=1,\ldots,n\end{gathered}\tag{6.8}$$

where ξ_{ij}^k, $k=1,2,\ldots,n_k$ (n_k denotes appropriate number corresponding to k) are the real numbers (note that these numbers may be also negative). Inequalities (6.7), (6.8) have to be valid for $\forall x\varepsilon\mathbf{X}^{\mathbf{t}}(t)$, $\forall t\varepsilon T$, and $\forall d\varepsilon D$. The practical stability conditions of the over-all system can be established by considering the derivative of the function v_i along the solutions of the coupled subsystems (6.5) (aggregation principle). Based on (6.6) - (6.8), candidates for the functions $\psi_i(t)$ for each subsystem i can be obtained:

$$\begin{aligned}&\Psi_i(t)=-\gamma_i\underline{X}^{I(i)}\exp(-\alpha_i t)+\sum(\xi_{ij}^4+\xi_{ij}^6)\underline{X}^t\\ &\exp(-\alpha_i t)+\sum\xi_{ij}^1\,|x_j^{01}(t)|+\sum\xi_{ij}^3\,|\dot x_j^{01}(t)|+\\ &+\sum\xi_{ij}^2\,|F_j^{01}(t)|+\sum\xi_{ij}^5\,|\underline{F}_j^t\exp(-\beta t)|dt+\sum\xi_{ij}^7\,|F_j^0+\underline{F}_j^t\exp(-\beta t)|,\ \ i=1,2\end{aligned}\tag{6.9}$$

By substituting (6.9) into (6.4) we obtain the practical stability test for the robot interacting with the dynamic environment when the control law (4.1) is applied. If we ignore all dynamic compensation elements in the control law, we obtain stability test for the decentralised control law (fig. 2).

6. CONCLUSION

The obtained test may be used to analyse which dynamic elements may be ignored but still to ensure fulfilment of the desired control task. This means, that this test may serve as a procedure for identification of the 'minimal' dynamic control law which may practically stabilise the robot interacting with the dynamic environment.

Note that the practical stability analysis is the only appropriate way to study these effects taking into account uncertainties in the dynamic models of robots and, specially, of the environment.

Such procedure is of high importance in the synthesis of the appropriate control laws for the robots in contact to the dynamic environment, since, on one hand, ignoring of the dynamics (both of the robot and of the environment) may lead to inappropriate performance (and even instability) of the robot, while, on the other hand, an introduction of all dynamic factors in the control law may lead to a high complexity and an insufficient robustness of the control system, due to high uncertainties in modelling of the environment. Therefore, for each specific task (or, classes of tasks) and for each specific robot structure and environment, it should be carefully studyed which of the dynamic factors have to be \`directly\` compensated for by the control law. Although it is obvious that the role of the system dynamics in the constrained motion control tasks is more important than for the free motion control tasks, it is recommendable to constrain the application of the relatively complex dynamic control laws to the tasks where they are needed and bringing improvements in the system performance.

The derived practical stability test has to be further elaborated (e.g. better estimates of the soupling elements among the subsystems have to be elaborated, etc.), but it certainly exhibits certain advantage (less conservatism) over the previously derived test [8,9], specially for the decentralised control laws (Fig. 2).

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DA LI JE DINAMIČKO UPRAVLJANJE POTREBNO ZA ROBOTE KOJI SU U INTERAKCIJI SA DINAMIČKIM OKRUŽENJEM?

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Rad se odnosi na ulogu dinamike u upravljanju robotima koji su u interakciji sa dinamičkim okruženjem. Cilj rada je da se definiše da li je potrebno uključiti kompenzaciju različitih dinamičkih faktora u upravljački zakon pozicija / sila. Počevši od potpunog dinamičkog modela robota i okruženja, analizirani su uticaji različitih dinamičkih faktora na upravljanje položajem i silom robota. Upoređen je upravljački zakon zasnovan na potpunom inverznom dinamičkom modelu i decentralizovani upravljački zakon. Dinamički efekti su najpre kvalitativno analizirani, a zatim je prikazan pristup za ispitivanje ovih efekata zasnovan na analizi praktične stabilnosti manipulacionih robota u upravljačkim zadacima ograničenog kretanja. Priloženi test stabilnosti može se koristiti za proveru svake pojedinačne strukture robota i okruženja i različitih upravljačkih zadataka, čiji se dinamički efekti moraju uzeti u obzir.