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ON GREEN & LAWS' ENTROPY PRINCIPLE FOR A MATERIAL INTERFACE

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Abstract. *In this paper the linear theory of heat conduction in a material interface is considered. Under the assumption introduced by I-Shih Liu, and using the entropy production inequality of the type of Green & Laws', it has been shown how Fourrier Law of heat conduction in a material interface can be obtained.*

1. INTRODUCTION

The problem of heat conduction in a material interface has been considered in the continuum mechanics with the classical assumption that the temperature gradient is the driving force of heat conduction [1], [2], [3]. Abandoning this assumption, in [4], the problem of heat conduction in a fluid has been considered, starting with the assumption that among the independent constitutive variables, instead of temperature, the internal energy appears. Then, it has been shown that in this, more general case, Fourier's law of heat conduction in the fluid under consideration can be derived. In this paper we consider the linear theory of heat conduction in a material interface. The assumption of I-Shih Liu that the temperature of the interface is not an independent constitutive variable represents the basis of our investigation. The thermodynamic analysis has been carried out upon the entropy production inequality, which has the form of Green & Law's inequality for a three dimensional body.

2. BULK MATERIAL

2.1 The balance equations of the bulk material

Let us consider a three dimensional continuum of the volume $v(t)$, which is divided

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into two parts by the material interface $s(t)$. The interface $s(t)$ is the common boundary of both parts, so that $v(t) = v^+(t) \cup s(t) \cup v^-(t)$ holds. The bulk material covers the parts of the continuum under consideration having the volume $v^{\pm}(t)$ and which are on both sides of the interface $s(t)$.

The balance equations of the mass, momentum, moment of momentum and energy of the bulk material can be written in the well known from [1]:

$$\dot{\rho} + \rho \dot{\xi}^k_{,k} = 0 \; ; \tag{1}$$

$$\rho \ddot{\xi}^i - t^{ik}_{,k} - \rho b^i = 0 \; ; \tag{2}$$

$$\varepsilon_{ijk} t^{jk} = 0 \; ; \tag{3}$$

$$\rho \dot{e} - t^{ik} \dot{\xi}_{i,k} + q^k_{,k} - \rho h = 0 \tag{4}$$

where $\dot{\xi}^i$ is the bulk material particle velocity, ρ -the mass density, t^{ik}-the stress tensor, b^i - the body force; ε_{ijk} is alternator tensor, e - the specific internal energy, q^k- the heat flux vector, h -the heat supply; "," denotes the covariant derivative, while "." denotes the material time derivative.

2.2 The constitutive equations of the bulk material

It has been supposed that the bulk material is a heat conducting fluid which has the constitutive equation of the form [3]

$$\eta = \eta(\rho; e) \, , \tag{5}$$

$$q_i = k_1 \rho_{,i} + k_2 e_{,i} \, , \tag{6}$$

$$t_{ij} = -p \delta_{ij} \, , \tag{7}$$

where η is the specific entropy, and where the pressure p and the coefficients k_1 and k_2 are the functions of density ρ and internal energy e.

3. INTERFACE

3.1 The geometry and kinematics of the interface

The interface $s(t)$ under consideration can be defined with respect to the Cartesian coordinates x^i in the following analytical form

$$x^i = x^i(u^\alpha; t) \, , \quad \text{rankdet}\left(\frac{\partial x^i}{\partial u^\alpha} \right) = 2 \, , \quad (i = 1, 2, 3) \, , \quad (\alpha = 1, 2) \, , \tag{8}$$

where t is the time and where u^α are coordinates of a two-dimensional space containing the interface $s(t)$. The equation (8) represents one of the possible parametrisations of the interface $s(t)$. From all of these, we choose the normal parametrisation for which

$$\left.\frac{\partial x^i}{\partial t}\right|_{u^\alpha} = \underset{v}{U} v^i , \tag{9}$$

holds. Here $\underset{v}{U}$ is the normal speed of the interface and v^i is the outward normal unit vector of $s(t)$. Thus,

$$v^i v^i = 1, \quad x^i_{,\alpha} v^i = 0 . \tag{10}$$

The first fundamental metric tensor of the interface is

$$g_{\alpha\beta} \equiv x^i_{,\alpha} x^i_{,\beta} . \tag{11}$$

Denoting by $b_{\alpha\beta}$ the tensor of the second metric form of the interface, its mean curvature can be expressed in the form

$$K_M \equiv \frac{1}{2} b^\alpha_\alpha , \tag{12}$$

Gaussian curvature of the interface is given by

$$K_G \equiv \det(b^\alpha_\beta) \tag{13}$$

It has been assumed that the interface is impermeable for the bulk material, i.e.,

$$\dot{\xi}^+_v = \underset{v}{U} = \dot{\xi}^-_v , \tag{14}$$

where $\dot{\xi}^+_v$ ($\dot{\xi}^-_v$) denotes the normal component of the velocity of the bulk material particle which, in a given moment, has the position on the positive (negative) side of the interface.

3.2 The balance equations of the interface

The balance equations of the mass, momentum, moment of momentum and energy of the interface can be written as in [1]:

$$\dot{\gamma} + \gamma(V^\alpha_{,\alpha} - 2U_v K_M) = 0 ; \tag{15}$$

$$\gamma \ddot{x}^k - S^{k\alpha}_{,\alpha} - \gamma B^k - [| t^{ki} v_i |] = 0 ; \tag{16}$$

$$\varepsilon_{ijk} x^j_{,\alpha} S^{k\alpha} = 0 ; \tag{17}$$

$$\gamma\dot{\varepsilon} + Q^\alpha_{,\alpha} - S^{k\alpha}\dot{x}_{k,\alpha} - \gamma H + [| q^k v_k |] - [| t^{ki}(\dot{\xi}^i - \dot{x}^i) v_i |] = 0 , \tag{18}$$

where γ is the surface mass density, $S^{k\alpha}$- the surface stress, B^i- the body force of the material surface, ε - the specific internal surface energy, Q^α - the surface heat flux vector, and H - the heat surface supply; $\dot{x}^i$ is the velocity of the particle of the interface which can be expressed in the form

$$\dot{x}^i = \underset{v}{U} v^i + V^\alpha x^i_{,\alpha} , \tag{19}$$

Here, V^{α} is the tangential velocity of the particle of the interface. In the previous balance equations of the material interface, the jump of the quantity ψ of the bulk material on the interface has been denoted by $[|\psi|] = \psi^{+} - \psi^{-}$.

If the stress decomposition is performed in the following way

$$S^{k\alpha} = S^{\alpha\beta} x^{k}_{,\beta} + S^{\alpha} v^{k} , \tag{20}$$

it can be shown that (17) is equivalent to

$$S^{\alpha} = 0 , \quad S^{\alpha\beta} = S^{\beta\alpha} . \tag{21}$$

3.3 Entropy production inequality of the Green & Laws' type for the material interface

For thermodynamic considerations, the entropy production inequality of the Green & Law type has been adopted for the material interface [5]

$$\gamma\Phi\dot{N} + S^{k\alpha}\dot{x}_{k,\alpha} - \gamma\dot{\varepsilon} - \frac{Q^{\alpha}\Phi_{,\alpha}}{\Phi} + \left[\left| q^{j}\left(\frac{\Phi}{\varphi} - 1\right)\right|\right] v_{i} + [|\, t^{ki}(\dot{\xi}^{i} - \dot{x}^{i})\,|] v_{k} \geq 0 , \tag{22}$$

where $\Phi(>0)$ is the specific entropy of the interface, N is the constitutive quantity of the interface to be determined, and where $\varphi(>0)$ is the constitutive quantity of the bulk material.

3.4 Thermodynamic properties of the material interface

The subject of the present study is a fluid heat conducting interface the thermodynamic properties of which are characterized by the following set of independent constitutive variables

$$c_{o} = \{\gamma; \gamma_{,\alpha}; x^{i}_{,\alpha}; \varepsilon; \varepsilon_{,\alpha}; v^{i}; v^{i}_{,\alpha}\} , \tag{23}$$

The constitutive equations for symmetric stress tensor $S^{\alpha\beta}$, heat flux vector Q^{α}, specific entropy N, and constitutive quantity Φ now take the form

$$\omega_{o} = \{S^{\alpha,\beta}; Q^{\alpha}; N; \Phi\}(c_{o}) . \tag{24}$$

The form of each of these function depend on universal physical principles, such as the principle of material indifference and the entropy principle, inposed on the material interface.

4. Restrictions of constitutive functions of the material interface

The principle of the material indifference imposes the condition that the constitutive functions characterizing the physical properties of the material interface connot depend upon the observer.

By using the principle of material indifference in the space and the surface [1], it can be shown that the constitutive quantities of the interface $S^{\alpha\beta}$, Q^{α}, N and Φ must be functions of the form

$$\omega = \{S^{\alpha,\beta}; Q^{\alpha}; N; \Phi\}(c) , \tag{25}$$

where

$$c = \{\gamma; \gamma_{,\alpha}; \varepsilon; \varepsilon_{,\alpha}; g_{\alpha,\beta}; b_{\alpha\beta}\} . \tag{26}$$

Further restrictions to the form of the constitutive functions are imposed by the entropy production inequality of the Green & Laws' type which, for the material interface has the form (22).

By using (15), (25) and (26) in (22) we obtain the following entropy inequality,

$$\gamma\left(\Phi\frac{\partial N}{\partial\varepsilon}-1\right)\dot{\varepsilon}+\gamma\Phi\frac{\partial N}{\partial\varepsilon_{,\alpha}}\frac{\partial\varepsilon_{,\alpha}}{\partial t}+ +\left(\gamma\Phi\frac{\partial N}{\partial\varepsilon_{,\alpha}}V^{\beta}-\frac{Q^{\beta}}{\Phi}\frac{\partial\Phi}{\partial\varepsilon_{,\alpha}}\right)\varepsilon_{,\alpha\beta}+$$

$$+\left[2\gamma^{2}\Phi K_{M}\frac{\partial N}{\partial\gamma}-\gamma\Phi\frac{\partial N}{\partial g_{\alpha\beta}}2b_{\alpha\beta}+\gamma\Phi\frac{\partial N}{\partial b_{\alpha\beta}}(K_{G}g_{\alpha\beta}-2K_{M}b_{\alpha\beta})-S^{\alpha\beta}b_{\alpha\beta}\right]U_{\nu}+$$

$$+\gamma\Phi\frac{\partial N}{\partial\varepsilon_{,\alpha}}\frac{\partial\varepsilon_{,\alpha}}{\partial t}+\left(\gamma\Phi\frac{\partial N}{\partial b_{\alpha\beta}}V^{\gamma}-\frac{Q^{\gamma}}{\Phi}\frac{\partial\Phi}{\partial b_{\alpha\beta}}\right)b_{\alpha\beta,\gamma}+\gamma\Phi\frac{\partial N}{\partial\gamma_{,\alpha}}\frac{\partial\gamma_{,\alpha}}{\partial t}+$$

$$+\left(\gamma\Phi\frac{\partial N}{\partial\gamma b_{,\alpha}}V^{\beta}-\frac{Q^{\beta}}{\Phi}\frac{\partial\Phi}{\partial\gamma b_{,\alpha}}\right)\gamma_{,\beta\alpha}+\left(-\gamma^{2}\Phi\frac{\partial N}{\partial\gamma}\delta^{\alpha}_{\beta}+S^{\beta}_{\alpha}\right)V^{\beta}_{,\alpha}-$$

$$-\frac{Q^{\alpha}}{\varphi_{s}}\left(\frac{\partial\Phi}{\partial\gamma}\gamma_{,\alpha}+\frac{\partial\Phi}{\partial\varepsilon}\varepsilon_{,\alpha}\right)+\left[\left|q^{i}\left(\frac{\Phi}{\varphi}-1\right)\right|\right]v_{i}+[|\,t^{ki}(\dot{\xi}_{i}-\dot{x}_{i})\,|]v_{k}\geq 0 .$$

Obviously this inequality is linear with respect to the following set of quantities:

$$\left\{\dot{\varepsilon};\frac{\partial\gamma_{,\alpha}}{\partial t};\frac{\partial\varepsilon_{,\alpha}}{\partial t};\underset{v}{U}_{,\alpha\beta};\gamma_{,\alpha\beta};\varepsilon_{,\alpha\beta};b_{\alpha\beta,\gamma};\underset{v}{U};V^{\beta}_{,\alpha}\right\} . \tag{27}$$

This means their corresponding coefficients in that inequality must vanish, i.e.,

$$\frac{\partial N}{\partial\varepsilon}=\frac{1}{\Phi} ; \tag{28}$$

$$\frac{\partial N}{\partial\gamma_{,\alpha}}=0 ; \tag{29}$$

$$\frac{\partial N}{\partial\varepsilon_{,\alpha}}=0 ; \tag{30}$$

$$\frac{\partial N}{\partial b_{\alpha\beta}}=0 ; \tag{31}$$

$$\left(\gamma\Phi\frac{\partial N}{\partial\varepsilon_{,\alpha}}V^{\beta}-\frac{Q^{\beta}}{\Phi}\frac{\partial\Phi}{\partial\varepsilon_{,\alpha}}\right)_{(\alpha,\beta)}=0 ; \tag{32}$$

$$\left(\gamma\Phi\frac{\partial N}{\partial\gamma_{,\alpha}}V^{\beta}-\frac{Q^{\beta}}{\Phi}\frac{\partial\Phi}{\partial\gamma_{,\alpha}}\right)_{(\alpha,\beta)}=0\,; \tag{33}$$

$$\left(\gamma\Phi\frac{\partial N}{\partial b_{\alpha\beta}}V^{\gamma}-\frac{Q^{\gamma}}{\Phi}\frac{\partial\Phi}{\partial b_{\alpha\beta}}\right)_{(\alpha,\beta,\gamma)}=0\,; \tag{34}$$

$$2\gamma^{2}\Phi K_{M}\frac{\partial N}{\partial\gamma}-\gamma\Phi\frac{\partial N}{\partial g_{\alpha\beta}}2b_{\alpha\beta}+\gamma\Phi\frac{\partial N}{\partial b_{\alpha\beta}}(K_{G}g_{\alpha\beta}-2K_{M}b_{\alpha\beta})=S^{\alpha\beta}b_{\alpha\beta}\,; \tag{35}$$

$$-\gamma^{2}\Phi\frac{\partial N}{\partial\gamma}\delta^{\alpha}_{\beta}+S^{\beta}_{\alpha}=0\,. \tag{36}$$

In this way, the residual inequality is obtained in the following form

$$\frac{Q^{\alpha}}{\varphi_{s}}\left(\frac{\partial\Phi}{\partial\gamma}\gamma_{,\alpha}+\frac{\partial\Phi}{\partial\varepsilon}\varepsilon_{,\alpha}\right)+\left[\left|q^{i}\left(\frac{\Phi}{\varphi}-1\right)\right|\right]v_{i}+[|\,t^{ki}(\dot{\xi}_{i}-\dot{x}_{i})\,|]v_{k}\geq 0\,. \tag{37}$$

We proceed analyzing each of conditions (28)-(36) and inequality (37). It is easy to see that, from (25), (26), (29), (30) and (31) we conclude that the specific entropy is the function of the form

$$N=N(\gamma;\varepsilon;g_{\alpha\beta})\,. \tag{38}$$

Next, using (28) and (38), we obtain

$$\Phi=\Phi(\gamma;\varepsilon;g_{\alpha\beta})\,. \tag{39}$$

Substituting (28) in (35) we get

$$S^{\alpha\beta}b_{\alpha\beta}=2\gamma^{2}\Phi K_{M}\frac{\partial N}{\partial\gamma}-2\gamma\Phi\frac{\partial N}{\partial g_{\alpha\beta}}b_{\alpha\beta}\,. \tag{40}$$

By the further analyses of the entropy inequality, from (12) and (36), we obtain

$$S^{\alpha\beta}b_{\alpha\beta}=2\gamma^{2}\Phi K_{M}\frac{\partial N}{\partial\gamma}\,. \tag{41}$$

Then, from (40) and (41) we simply conclude

$$\frac{\partial N}{\partial g_{\alpha\beta}}=0\,, \tag{42}$$

i.e.,

$$\frac{\partial\Phi}{\partial g_{\alpha\beta}}=0\,, \tag{43}$$

By using (42) and (43) in (38) and (39), we have

$$\begin{aligned}N&=N(\gamma;\varepsilon),\\ \Phi&=\Phi(\gamma;\varepsilon).\end{aligned} \tag{44}$$

Obviousely, from (36), we have

$$S^{\alpha\beta} = -\sigma_o g^{\alpha\beta} . \tag{45}$$

Here

$$\sigma_o = -\gamma^2 \Phi \frac{\partial N}{\partial \gamma} , \tag{46}$$

i.e.

$$\sigma_o = \sigma_o(\gamma, \varepsilon) . \tag{47}$$

Further, from $(44)_1$ and (28), we get

$$dN = \frac{1}{\Phi} d\varepsilon + \frac{\partial N}{\partial \gamma} d\gamma . \tag{48}$$

We compare it with Gibb's equation, which reads

$$dN = \frac{1}{T_s} d\varepsilon + \frac{p}{T_s} d\left(\frac{1}{\gamma}\right), \tag{49}$$

where T_s is the absolute temperature of the interface, and see that the constitutive quantity Φ can be identified with the absolute temperature, i.e.,

$$\Phi = T_s . \tag{50}$$

Then the residual entropy inequality may be written as

$$\frac{Q^\alpha}{T_s}\left(\frac{\partial T_s}{\partial \gamma}\gamma_{,\alpha} - \frac{\partial T_s}{\partial \varepsilon}\varepsilon_{,\alpha}\right) + \left[\left| q^i \left(\frac{T_s}{T} - 1\right)\right|\right] v_i + [|\, t^{ki} (\dot{\xi}_i - \dot{x}_i) \,|] v_k \geq 0 . \tag{51}$$

Assuming that the absolute temperatures of the bulk material and the interface are equal, e.i. $\varphi = T = T_s$, the inequality (51) becomes

$$-\frac{Q^\alpha}{T_s}\left(\frac{\partial T_s}{\partial \gamma}\gamma_{,\alpha} + \frac{\partial T_s}{\partial \varepsilon}\varepsilon_{,\alpha}\right) + [|\, t^{ki} (\dot{\xi}_i - \dot{x}_i) \,|] v_k \geq 0 . \tag{52}$$

For a further usage of the inequality (52), the particle velocity of the bulk material is expressed in the way as it has been done for the interface particle (Eq.19). Thus,

$$\dot{\xi}^i = \underset{v}{\dot{\xi}}\, v_i + \underset{\tau}{\dot{\xi}^\alpha}\, x^i_{,\alpha} ,$$

where $\underset{\tau}{\dot{\xi}^\alpha}$ is the tangential component of the velocity of the bulk material particle which, at a given moment, has the position on the interface, and $\underset{v}{\dot{\xi}}$ is the normal component of $\dot{\xi}^i$.

It can easily be shown that the following holds:

$$\dot{\xi}_i - \dot{x}_i = \left(\underset{v}{\dot{\xi}} - \underset{v}{U}\right) v_i + \left(\underset{\tau}{\dot{\xi}^\alpha} - v^\alpha\right) x_{i,\alpha} , \tag{53}$$

Taking into account (14) in (53), it follows

$$\dot{\xi}_i - \dot{x}_i = W^\alpha x_{i,\alpha} \,, \tag{54}$$

where

$$W^\alpha \equiv \underset{\tau}{\dot{\xi}}{}^\alpha - v^\alpha \,. \tag{55}$$

By using (54) in (52), we obtain

$$-\frac{Q^\alpha}{T_s}\left(\frac{\partial T_s}{\partial \gamma}\gamma_{,\alpha} + \frac{\partial T_s}{\partial \varepsilon}\varepsilon_{,\alpha}\right) + [| \, t^i W^\alpha x_{i,\alpha} \, |] \geq 0 \,, \tag{56}$$

where $t^i \equiv t^{ki} v_k$.

Since the quantities $W^{+\alpha}$ and $W^{-\alpha}$ can be chosen arbitrarily at the point on the interface at the initial moment, it follows that inequality (57) is linear in $W^{+\alpha}$ and $W^{-\alpha}$ wherefrom we obtain

$$t^{+i} x_{i,\alpha} = 0 \quad \text{and} \quad t^{+i} x_{i,\alpha} = 0 \,. \tag{57}$$

Inequality (56) then becomes

$$-\frac{Q^\alpha}{T_s}\left(\frac{\partial T_s}{\partial \gamma}\gamma_{,\alpha} + \frac{\partial T_s}{\partial \varepsilon}\varepsilon_{,\alpha}\right) \geq 0 \tag{58}$$

Generally, the constitutive equation for heat flux vector of the interface Q^α is of the form

$$Q^\alpha = g^{\alpha\beta}(K_1 \gamma_{,\beta} + K_2 \varepsilon_{,\beta}) + b^{\alpha\beta}(L_1 \gamma_{,\beta} + L_2 \varepsilon_{,\beta}) \,, \tag{59}$$

where the coefficient K_1, K_2, L_1, and L_2 are functions of

$$\gamma, \varepsilon,$$
$$\gamma_{,\alpha}\gamma^{,\alpha}, \varepsilon_{,\alpha}\varepsilon^{,\alpha}, \gamma_{,\alpha}\varepsilon^{,\alpha},$$
$$K_M, K_G,$$
$$b^{\alpha\beta}\gamma_{,\alpha}\gamma_{,\beta}, b^{\alpha\beta}\varepsilon_{,\alpha}\varepsilon_{,\beta}, b^{\alpha\beta}\gamma_{,\alpha}\varepsilon_{,\beta}$$

In a case of linear heat conducting material interface K_1, K_2, L_1, and L_2 are function of

$$\gamma, \varepsilon, K_M, K_G \,.$$

Substituting (59) in (58), the residual inequality gets the form

$$\sigma \equiv -\frac{1}{T_s}\left(\frac{\partial T_s}{\partial \gamma}\gamma_{,\alpha} + \frac{\partial T_s}{\partial \varepsilon}\varepsilon_{,\alpha}\right)[g^{\alpha\beta}(K_1 \gamma_{,\beta} + K_2 \varepsilon_{,\beta}) + b^{\alpha\beta}(L_1 \gamma_{,\beta} + L_2 \varepsilon_{,\beta})] \geq 0 \,, \tag{60}$$

The left-hand side of the inequality (60) is denoted by σ for further considerations. Notice that the σ is particularly a function of the following set of the independent variables

$$\{X_A\} = \{\gamma_{,\alpha};\, \varepsilon_{,\alpha}\} \,. \tag{61}$$

5. EQUILIBRIUM PROCESSES

Equilibrium process E of the material interface can be defined as a time independent and uniform thermodynamic process. Then the following holds

$$E: \quad \gamma_{,\alpha} = \varepsilon_{,\alpha} = 0 \,.$$

In equilibrium σ has its minimum, i.e., $\sigma|_E = 0$, and the necessary and sufficient conditions for it are

$$\left.\frac{\partial \sigma}{\partial X_A}\right|_E = 0\,; \tag{62}$$

$$\left\|\frac{\partial^2 \sigma}{\partial X_A \partial X_B}\right\|_E \text{ is positive semi-definite.} \tag{63}$$

We are investigating the case when the coeficient K_1, K_2, L_1, and L_2 do not depend on K_M and K_G. Than, $L_1 = L_2 = 0$ since (60) has to hold for arbitrary $b^{\alpha\beta}$.

By using (60) it can easily be shown that the conditions (62) are identically satisfied, while the matrix (63) takes the form

$$\left\|\frac{\partial^2 \sigma}{\partial X_A \partial X_B}\right\|_E = -\frac{1}{T_s}\begin{Vmatrix} 2K_1 a_1 \mathrm{I} & (K_1 a_2 + K_2 a_1)\mathrm{I} \\ * & 2K_2 a_2 \mathrm{I} \end{Vmatrix} \tag{64}$$

where **I** is (3 × 3) unit matrix and

$$a_1 = \frac{\partial T_s}{\partial \gamma}\,; \qquad a_2 = \frac{\partial T_s}{\partial \varepsilon} \tag{65}$$

In its expanded form, matrix (64) reads

$$\left\|\frac{\partial^2 \sigma}{\partial X_A \partial X_B}\right\|_E = -\frac{1}{T_s}\times$$

$$\begin{Vmatrix} 2a_1K_1 & 0 & 0 & a_1K_2 + a_2K_1 & 0 & 0 \\ 0 & 2a_1K_1 & 0 & 0 & a_1K_2 + a_2K_1 & 0 \\ 0 & 0 & 2a_1K_1 & 0 & 0 & a_1K_2 + a_2K_1 \\ a_1K_2 + a_2K_1 & 0 & 0 & 2a_2K_2 & 0 & 0 \\ 0 & a_1K_2 + a_2K_1 & 0 & 0 & 2a_2K_2 & 0 \\ 0 & 0 & a_1K_2 + a_2K_1 & 0 & 0 & 2a_2K_2 \end{Vmatrix} \tag{66}$$

From the condition that matrix (66) is positive semidefinite, it follows that

$$a_1K_1 \le 0\,;\; a_2K_2 \le 0\,, \tag{67}$$

$$-(a_1K_2 - a_2K_1)^2 \ge 0\,, \tag{68}$$

must hold. Then, from (68) it immediately follows that

$$a_1K_2 = a_2K_1 \tag{69}$$

6. FOURIER'S LAW OF HEAT CONDUCTION

By using the results from the previous chapter, we are able to write the explicit form of the heat flux vector (59). Taking into account (69), it can easily be demonstrated that

$$K_1\gamma_{,\alpha} + K_2\varepsilon_{,\alpha} = \frac{K_2}{a_2}(a_1\gamma_{,\alpha} + a_2\varepsilon_{,\alpha}) \,. \tag{70}$$

On the other hand, taking into consideration $(44)_2$ and (50), we conclude

$$T_s = T_s(\gamma;\, \varepsilon)\,, \tag{71}$$

so that, by using (65), we can write

$$\frac{\partial T_s}{\partial u^\beta} = a_1\gamma_{,\beta} + a_2\varepsilon_{,\beta} \,. \tag{72}$$

Then, from (70) and (72), it follows that

$$K_1\gamma_{,\beta} + K_2\varepsilon_{,\beta} = -\kappa T_{s,\beta}\,, \tag{73}$$

where

$$\kappa \equiv -\frac{K_2}{K_1} \,. \tag{74}$$

Thereby, regarding $(67)_2$, the following must hold

$$\kappa \geq 0 \,. \tag{75}$$

Finally, from (59) and (73), the law of heat conduction is obtained in the form

$$Q^\alpha = -\kappa a^{\alpha\beta} T_{s,\beta} \,. \tag{76}$$

7. CONCLUSION

The thermodynamic analyses carried out has been based upon the entropy production inequality of the Green and Laws' form. It has been shown that the constitutive quantity Φ can be identified with the absolute temperature of the interface.

The basic result of the present paper given by expression (76) represents Fourier's law of heat conduction in a case which appears offen in practice. From this result it can be seen that the driving force for heat conduction in a material interface is only the temperature gradient although we started tracing the idea of I-Shih Liu that the heat flux vector of a material interface is a linear combination of the gradients of temperature and internal energy. The more general case when K_1, K_2, L_1, and L_2 depend on $b_{\alpha\beta}$ is very complicated and will be the object of our further investigation.

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O GRIN - LOOVOM PRINCIPU ENTROPIJE ZA DODIRNU POVRŠINU MATERIJALA

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U ovom radu razmatra se linearna teorija provođenja toplote na dodirnoj površini materijala. Pod pretpostavkom koju je uveo I-Shih Liu i primenom nejednakosti (stvaranja) entropije tipa Grin - Loa, pokazano je kako se može dobiti Furijeov zakon provođenja toplote za dodirnu površinu materijala.