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THE BEHAVIOR OF THE SYSTEM WITH ABSORBER

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Abstract. *In this paper the motion of the nonlinear system with one-degree-of-freedom and a linear absorber is considered. The conditions for suppressing chaos in the system with strong cubic nonlinearity applying the absorber are analysed. Also, the conditions which have to satisfy the absorber for eliminating motion are discussed.*

1. INTRODUCTION

Recently, many investigations dealing with the problem of oscillations of nonlinear one degree of freedom systems. As it is shown in papers [1]-[3] for the case of strong cubic nonlinearity the motion is described with a differential equation of Duffing type

$$\ddot{y}_1 - \alpha_1 y_1 + \alpha_3 y_1^3 + \beta \dot{y}_1 = F(t) \tag{1}$$

where $(\cdot)=d/dt$, y_1 is the deflection, $F(t)$ is a periodic function, α_1, α_3 are coefficients of the linear and non-linear term and β is the damping coefficient. Analyzing the solutions of the equation (1) it is concluded that beside the periodical solutions also aperiodic solutions exist. It means that beside the regular motion irregular motion exists. As it is discussed in the papers [4]-[6] it is evident that the chaotic motion is harmful. The aim of this paper is to develope a system for eliminating of the unwilling motion.

In praxis usually an absorber which contains a mass and a spring is applied for eliminating the vibrations in the linear systems. In the papers [7]-[12] the motion of the nonlinear system with an absorber are analyzed. In the papers [8] and [9] it is assumed that the primary system and the absorber are weak nonlinear. The stability of motion of the non-linear vibration absorber is discussed in the paper [7]. The absorbers are applied for controlling transient motion [12]. For all af these investigations it is comon that they consider the system with weak nonlinearity.

In this paper a linear absorber is applied for suppressing the unwilling motion in a strong nonlinear system.

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2. MATHEMATICAL MODEL

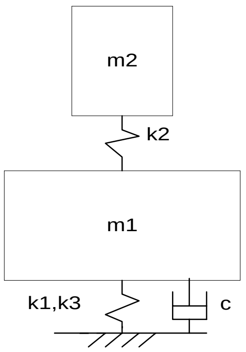

Fig.1. Model of the system

The model of the system is plotted in Fig.1. The mathematical model of the system is

$$\ddot{y}_1 - \alpha_1 y_1 + \alpha_3 y_1^3 + m\alpha_a (y_1 - y_3) + \beta\, \dot{y}_1 = F(t)\,, \tag{2}$$

$$\ddot{y}_3 + \alpha_a (y_3 - y_1) = 0\,, \tag{3}$$

where the dimensionless parameters are

$$\alpha_a = \frac{k_2}{m_2}\,, m = \frac{m_2}{m_1}\,, \alpha_1 = \frac{k_1}{m_1}\,, \alpha_3 = \frac{k_3}{m_1}\,, \beta = \frac{c}{m_1}\,. \tag{4}$$

It is a system of two degrees o freedom. Eliminating the function y_3 from the equation (3)

$$y_3 = \frac{1}{m\alpha_a}[\ddot{y}_1 - \alpha_a y_1 + \alpha_3 y_1^3 + m\alpha_a y_1 + \beta\dot{y}_1 - F(t)]\,, \tag{5}$$

and substituting it into the equation (2) a fourth order nonlinear differential equation is obtained

$$y_1^{IV} - [\alpha_1 - (m+1)\alpha_a]\ddot{y}_1 + 3y_1\alpha_3(2\dot{y}_1^2 + y_1\ddot{y}_1) + \beta\,\dddot{y}_1$$
$$-\alpha_1\alpha_a y_1 - [\ddot{F}(t) + \alpha_a F(t)] + \alpha_a\alpha_3 y_1^3 + \beta\alpha_a \dot{y}_1 = 0\,. \tag{6}$$

To solve the equation (6) it is necessary to know the form of the periodical function. Two types of periodical forces will be considered: first, the excitation force is described with elliptic function and second, it is described with circular function.

The main aim of the absorber is to eliminate the vibrations of the primary system. It means that $y_1 = 0$. Substituting this relation into (6) it can be concluded that it is satisfied only if the parameter of the absorber is

$$\alpha_a = -\frac{\ddot{F}(t)}{F(t)} = const\,. \tag{7}$$

This relation is not satisfied for all of the periodical functions.

3. THE MOTION OF THE SYSTEM WHERE THE EXCITATION HAS THE FORM OF ELLIPTIC FUNCTION

Let us consider the case when the excitation force has the form of the elliptic function

$$F(t) = F_0 cn(\Omega t, k^2)\,, \tag{8}$$

where F_0 and Ω are the amplitude and the frequency of the excitation force, respectively, and k is the modulus of the elliptic function [13]. For this periodical function the relation (7) can not be satisfied. The aim of the absorber is to suppress the chaotic motion and to transform it to periodical.

For (8) the differential equation (6) is

$$y_1^{IV} - [\alpha_1 - (m+1)\alpha_a]\ddot{y}_1 + 3y_1\alpha_3(2\dot{y}_1^2 + y_1\ddot{y}_1) + \beta\,\dddot{y}_1$$
$$-\alpha_1\alpha_a y_1 - F_0\{[\alpha_a - \Omega^2(1-2k^2)]cn - 2k^2\Omega^2 cn^3\} - \alpha_a\alpha_1 y_1^3 + \beta\alpha_a \dot{y}_1 = 0\,. \tag{9}$$

For the case when $\beta = 0$ the solution of the equation is assumed in the form

$$y_1 = Acn(\Omega t, k^2), \tag{10}$$

where A is the amplitude of vibrations. Substituting (10) into (9) and separating the terms with cn, cn^3, cn^5 a system of three algebraic equations is obtained

$$\begin{aligned} &A\Omega^4(1-2k^2)^2 - 12k^2 A\Omega^4(1-2k^2) - A\Omega^2(1-2k^2)[\alpha_a(1+m)-\alpha_1] \\ &+ 6\alpha_3 A^3\Omega^2(1-k^2) - \alpha_1\alpha_a A + F_0\Omega^2(1-2k^2) - \alpha_a F_0 = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} &20A\Omega^4(1-2k^2)k^2 - 2k^2 A\Omega^2[\alpha_a(1+m)-\alpha_1] + \alpha_a\alpha_3 A^3 - \\ &- 9\alpha_3 A^3\Omega^2(1-k^2) + 2k^2 F_0\Omega^2 = 0 \end{aligned} \tag{12}$$

$$2\Omega^2 k^2 - \alpha_3 A = 0. \tag{13}$$

Solving these equations the conditions for periodical motion are obtained.

The motion of the mass 2 is also periodic

$$y_3 = \frac{1}{m\alpha_a}\{[(m\alpha_a - m\alpha_1)A - A\Omega^2(1-2k^2) - F_0]\, cn + (\alpha_3 A^3 - 2k^2 A\Omega^2)\, cn^3\}. \tag{14}$$

For the linear system when

$$\alpha_3 = 0, \tag{15}$$

according to (13) the modulus of elliptic function is $k = 0$ the elliptic force transforms to circular function and the excitation force is

$$F = F_0 \cos\Omega t. \tag{16}$$

According to (10) the motion is oscillatory

$$y_1 = A\cos\Omega t, \tag{17}$$

where

$$A = \frac{-8F_0}{8\Omega^2 + 9\Omega^2 m + 8\alpha_1}, \tag{18}$$

for

$$\alpha_a = 9\Omega^2. \tag{19}$$

The motion of the absorber is

$$y_3 = \frac{1}{m\alpha_a}\left[-A\Omega^2 - \alpha_1 A + 9\Omega^2 mA - F_0\right]\cos\Omega t. \tag{20}$$

4. THE MOTION OF THE SYSTEM WHERE THE EXCITATION HAS THE FORM OF CIRCULAR FUNCTION

For the case when the periodic force (16) acts the differential equation (6) has the form

$$y_1^{IV} - [\alpha_1 - (m+1)\alpha_a]\ddot{y}_1 + 3y_1\alpha_3(2\dot{y}_1^2 + y_1\ddot{y}_1) + \beta\,\dddot{y}_1$$

$$-\alpha_1\alpha_a y_1 - F_0(\Omega^2 - \alpha_a)\cos\Omega t + \alpha_a\alpha_3 y_1^3 + \beta\alpha_a\dot{y}_1 = 0\,. \tag{21}$$

We assume the solution in the form

$$y_1 = A\cos\Psi\,, \tag{22}$$

where

$$\Psi = \Omega t + \gamma\,, \tag{23}$$

A is the amplitude of vibrations, γ is the phase angle. Substituting (22) into (21) and separating the terms with cos ψ, sin ψ and $\cos^3\psi$, a system of three algebraic equations is obtained

$$A\Omega^4 + A\{\Omega^2[\alpha_1 - (m+1)\alpha_a] - \alpha_1\alpha_a\} + 6\alpha_3\Omega^2 A^3 = -F_0(\Omega^2 - \alpha_a)\cos\gamma\,, \tag{24}$$

$$\beta A(\Omega^3 - \alpha_a\Omega) = -F_0(\Omega^2 - \alpha_a)\sin\gamma\,, \tag{25}$$

$$9\Omega^2 - \alpha_a = 0\,. \tag{26}$$

Eliminating A and γ from the first two equations the amplitude and phase angle of vibrations are obtained

$$F_0^2(\Omega^2 - \alpha_a)^2 = A^2\{\Omega^4 + \Omega^2[\alpha_1 - (m+1)\alpha_a] - \alpha_1\alpha_a + 6\alpha_3\Omega^2\}^2 + A^2\beta^2\Omega^2(\Omega^2 - \alpha_a)^2 \tag{27}$$

$$\tan\gamma = -\frac{\beta\,\Omega(\Omega^2 - \alpha_a)}{\Omega^4 + \Omega^2[\alpha_1 - (m+1)\alpha_a] - \alpha_1\alpha_a + 6\alpha_3\Omega^2 A^2}\,, \tag{28}$$

for

$$\alpha_a = 9\Omega^2 \tag{29}$$

Then the absorber vibrates as

$$y_3 = \frac{1}{m\alpha_a}\left\{\left[\left(m\alpha_a - \Omega^2 - \alpha_1 + \frac{3}{4}\alpha_3 A^2\right)A - F_0\right]\cos\Psi + \frac{1}{4}\alpha_3 A^3\cos 3\Psi - \beta A\Omega\sin\Psi\right\}. \tag{30}$$

It is evident that for the case when the parameter of absorber satisfies the condition (29) the motion is always harmonical - there are no possibilities for chaotic motion. The amplitude of vibrations depends on the mass ratio, amplitude and frequency of excitation force, rigidity and damping properties of the system.

5. Steady State Positions

Substituting the function (16) into (7) we conclude that for

$$\alpha_a = \Omega^2 \tag{31}$$

the absorber eliminates the motion of the primary structure. The mass m_1 is settled in one of three equilibrium positions

$$y_{1_1} = 0\,, \tag{32}$$

$$y_{1_{2,3}} = \pm\sqrt{\frac{\alpha_1}{\alpha_3}} = const. \tag{33}$$

The positions (33) are stable and for that case the absorber oscillates according to

$$y_3 = \pm\sqrt{\frac{\alpha_1}{\alpha_3}} - \frac{F_0}{m\alpha_a}\cos\Omega t \tag{34}$$

The unstable steady state position is reached if the mass m_2 vibrates as

$$y_3 = -\frac{F_0\cos\Omega t}{m\Omega^2}. \tag{35}$$

6. EXAMPLE

Let us assume an oscillator with parameters $\alpha_1 = 1$, $\alpha_3 = 1$, $\beta = 0{,}25$, $F_0 = 0{,}3$, $\Omega = 1$. Various values of parameter α_a are considered. In Fig.2, 3 and 4 the time history diagrams and phase portraits are plotted. In Fig. 2 the case without absorber is considered ($\alpha_a = 0$). In Fig.3, it is assumed that $\alpha_a = 9\Omega^2 = 9$, and in Fig.4 $\alpha_a = \Omega^2 = 1$. It can be concluded that for $\alpha_a = 0$ the motion is chaotic. If the parameter of absorber is $\alpha_a = 1$ the motion of the primary system stops and it drops to an equilibrium position. For $\alpha_a = 9$ the absorber transforms the chaotic motion to steady state harmonic motion.

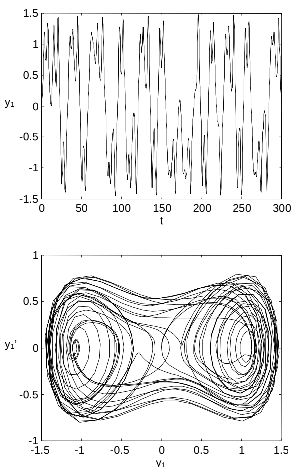

Fig.2. The time history and phase plane diagrams for $\alpha_a = 0$

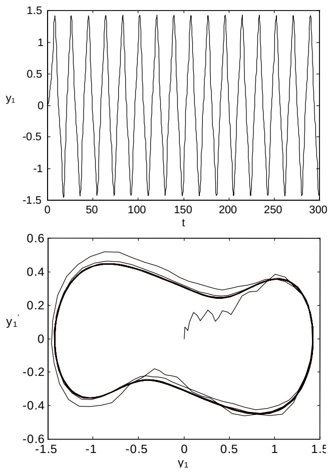

Fig.3. The time history and phase plane diagrams for $\alpha_a = 9$

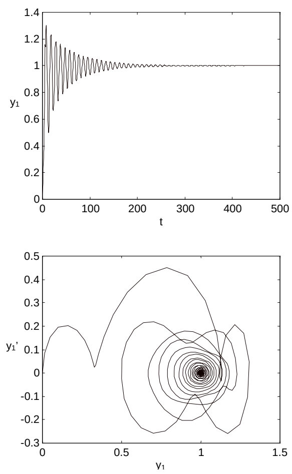

Fig.4. The time history and phase plane diagrams for $\alpha_a = 1$

7. CONCLUSION

In the nonlinear system (1) beside the periodic motion, chaotic motion may appear. If a linear system, which consists of a mass and a spring, is fixed on the primary system, its motion can be transformed.

1. The chaotic motion transforms to periodical motion. For the case when the excitation force is elliptic the parameters of absorber satisfy the relations (11)-(13), and the relation (29) if the excitation is with circular force.
2. The absorber eliminates the motion of the primary system if the relation (31) is fulfilled. It means, that for the system excited with circular function the primary system reaches one of the three equilibrium positions.

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DINAMIČKO PONAŠANJE SISTEMA SA APSORBEROM

Livija Cvetičanin

U radu se razmatra kretanje nelinearnog sistema sa jednim stepenom slobode i linearnim prigušivačem. Analizirani su uslovi sprečavanja haosa upotrebom prugušivača u sistemima sa jakom kubnom nelinearnošću. Takođe, diskutovani su uslovi koje mora da zadovolji prigušivač pri eliminaciji kretanja.