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PARAMETRIC METHOD IN UNSTEADY MHD BOUNDARY LAYER THEORY OF FLUID WITH VARIABLE ELECTROCONDUCTIVITY

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Abstract. *In this paper is considered unsteady plane MHD boundary layer. The fluid is incompressible and its electroconductivity is the function of the velocity ratios. By introducing the new variables and also two groups of parameters, the universal equation has been obtained for all electroconductivities which depend on the velocity ratio. From in the such way obtained universal equation, it can be obtained the universal equations for particular electroconductivities.*

There are different problems of magneto-hydrodynamic boundary layer in technical practice, and not only steady but also unsteady cases. The problems with variable electroconductivity have been considered [1] and also the problems with variable electroconductivity [2,3]. For solving of this problem, in recent time, the method of "universality" of boundary layer equations have been used very often and in very different versions [4,5,6]. This method have been created for steady boundary layer, problems, and later has been extended to the different unsteady boundary layer problems [6,7,8]. The universal results obtained by this method enable us to bring more general conclusions about the boundary layer development and these results can be used for calculations of particular problems. The general conclusions are independent of the velocity distribution at the edge of boundary layer, and if it is MHD boundary layer, they are independent of external magnetic field.

In order to extend this theory, in this paper is investigated the unsteady plane MHD boundary layer on the body. The existing external magnetic field is homogenous and perpendicular to the body which the fluid flows around, and it is stationary with respect to the body. The fluid is incompressible and its electroconductivity varies according to

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the following law:

$$\sigma = \sigma_0 S\left(\frac{u}{U}\right) \tag{1}$$

in which the following notations have been accepted: u - streamwise velocity in boundary layer, U -the velocity at the edge of boundary layer, S (u/U) - dimensionless differentiable functions of velocity ratio, σ and σ_0 - electroconductivity.

The mathematical model of the described flow is presented by the following equations:

$$\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^3\psi}{\partial y^3}+NU\left[S(1)-\frac{1}{U}\frac{\partial\psi}{\partial y}S\left(\frac{1}{U}\frac{\partial\psi}{\partial y}\right)\right] \tag{2}$$

with the boundary and initial conditions:

$$\begin{gathered}\psi=0,\ \frac{\partial\psi}{\partial y}=0 \quad for\ y=0; \quad \frac{\partial\psi}{\partial y}\to U(x,t) \quad for\ y\to\infty;\\ \frac{\partial\psi}{\partial y}=u_1(x,y) \quad for \quad t=t_0; \quad \frac{\partial\psi}{\partial y}=u_0(y,t) \quad for\ x=x_0.\end{gathered} \tag{3}$$

In the equation (2) and in the boundary and initial conditions (3) the following notations have been used: ψ - stream function, x - streamwise coordinate, y - cross-stream coordinate in boundary layer, t - time, ν - coefficient of cinematic viscosity; $N=B^2\sigma_0/\rho$, where is B- magnetic induction, ρ - fluid density, $u(x,y)_1$ - velocity distribution in boundary layer at the moment $t=t_0$; $u_0(t,y)$ - velocity distribution at the distance $x = x_0$.

Following the method of "universality" the following variables are introduced:

$$x=x,\ \ t=t,\ \eta=\frac{y}{K(x,t)},\quad \Phi(x,\eta,t)=\frac{\psi(x,y,t)}{U(x,t)K(x,t)} \tag{4}$$

where is

$$K(x,t)=\left(a_0\nu U^{-b_0}\int_0^x U^{b_0-1}dx\right)^{\frac{1}{2}}$$

and the constants a_0 and b_0 have got the values a_0=0.44 and b_0=5.35 . In that case the derivation operation at the old variables is substituted by the new ones:

$$\begin{gathered}\frac{\partial}{\partial x}=\left(\frac{\partial}{\partial x}\right)+\frac{\partial}{\partial\eta}\frac{\partial\eta}{\partial x}=\left(\frac{\partial}{\partial x}\right)-\frac{\eta}{K}\frac{\partial K}{\partial x}\frac{\partial}{\partial\eta},\\ \frac{\partial}{\partial t}=\left(\frac{\partial}{\partial t}\right)+\frac{\partial}{\partial\eta}\frac{\partial\eta}{\partial t}=\left(\frac{\partial}{\partial t}\right)-\frac{\eta}{K}\frac{\partial K}{\partial t}\frac{\partial}{\partial\eta},\\ \frac{\partial}{\partial y}=\frac{1}{K}\frac{\partial}{\partial\eta},\ \frac{\partial^2}{\partial y^2}=\frac{1}{K}\frac{\partial}{\partial\eta},\ \frac{\partial^3}{\partial vy^3}=\frac{1}{K^3}\frac{\partial^3}{\partial\eta^3}.\end{gathered} \tag{5}$$

Further, introducing in consideration the momentum thickness

$$\delta^{**}(x,t)=\int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)dy = K(x,t)B(x,t), \tag{6}$$

where is

$$B(x,t)=\int_0^\infty \frac{\partial\Phi}{\partial\eta}\left(1-\frac{\partial\Phi}{\partial\eta}\right)d\eta, \tag{7}$$

and the value

$$z=\frac{\delta^{**2}}{\nu}=\frac{K^2B^2}{\nu} \tag{8}$$

the equation (2) is transformed to the equation

$$\begin{aligned}&\frac{\partial^3\Phi}{\partial\eta^3}+\frac{K^2}{\nu}\frac{\partial U}{\partial x}\left[1-\left(\frac{\partial\Phi}{\partial\eta}\right)^2+\left(1-\frac{b_0}{2}\right)\Phi\frac{\partial^2\Phi}{\partial\eta^2}\right]+\frac{a_0}{2}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+\frac{K^2}{\nu U}\frac{\partial U}{\partial t}\left(1-\frac{\partial\Phi}{\partial\eta}\right)+\\ &+N\frac{K^2}{\nu}\left[S(1)-\frac{\partial\Phi}{\partial\eta}S\left(\frac{\partial\Phi}{\partial\eta}\right)\right]+\frac{\eta}{2B^2}T\frac{\partial^2\Phi}{\partial\eta^2}=\frac{K^2}{\nu}\frac{\partial^2\Phi}{\partial t\partial\eta}+\eta\frac{z}{B^3}\frac{\partial B}{\partial t}\frac{\partial^2\Phi}{\partial\eta^2}+U\frac{K^2}{\nu}X(\eta;x)\end{aligned} \tag{9}$$

in which the denotes used, are

$$T=\frac{\partial z}{\partial t},\quad X(x_1;x_2)=\frac{\partial\Phi}{\partial x_1}\frac{\partial^2\Phi}{\partial\eta\partial x_2}-\frac{\partial\Phi}{\partial x_2}\frac{\partial^2\Phi}{\partial x_1\partial\eta}. \tag{10}$$

The boundary conditions, written in first row of conditions, have the form

$$\Phi=0,\frac{\partial\Phi}{\partial\eta}=0\quad\text{for}\quad\eta=0;\quad\frac{\partial\Phi}{\partial\eta}\to 1\quad\text{for}\quad\eta\to\infty. \tag{11}$$

but the conditions, written in the second row of conditions, are used for the calculation of particular problems and they do not have significance at the derivation of universal equation.

Assuming that $U(x,t)$ and $N(x,t)$ are the differential functions, the following group of parameters are introduced

$$f_{k,n}=U^{k-1}\frac{\partial^{k+n}U}{\partial x^k\partial t^n}z^{k+n}\quad(k,n=0,1,2,...;\ \ k\vee n\neq 0);$$

$$g_{k,n}=U^{k-1}\frac{\partial^{k-1+n}N}{\partial x^{k-1}\partial t^n}z^{k+n}\quad(k,n=0,1,2,...;\ \ k\neq 0) \tag{12}$$

where can be noticed that the first parameters have the form

$$f_{1,0}=z\frac{\partial U}{\partial x}=\frac{B^2K^2}{\nu}\frac{\partial U}{\partial x};\ f_{0,1}=\frac{z}{U}\frac{\partial U}{\partial t}=\frac{B^2K^2}{\nu U}\frac{\partial U}{\partial t};\ g_{1,0}=Nz=\frac{B^2K^2}{\nu}N. \tag{13}$$

In this way chosen parameters, which substitute streamwise coordinate x and time t, express the influence on the characteristics of MHD boundary layer, on the velocity at the edge of boundary layer and on the magnetic field, and on the flow prehistory. These parameters are mutual independent and in the next paragraphs we hold them for independent variables.

For further transformations of the equation (9) the following operators are used.

$$\begin{aligned}
\frac{\partial}{\partial x} &= \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty} \frac{\partial f_{k,n}}{\partial x}\frac{\partial}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty} \frac{\partial g_{k,n}}{\partial x}\frac{\partial}{\partial g_{k,n}}\\
\frac{\partial}{\partial t} &= \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty} \frac{\partial f_{k,n}}{\partial t}\frac{\partial}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty} \frac{\partial g_{k,n}}{\partial t}\frac{\partial}{\partial g_{k,n}}
\end{aligned} \tag{14}$$

where the derivations of parameters with respect to x and t are determined by immediate differentiation of the expression (12) and they have the forms:

$$\begin{aligned}
\frac{\partial f_{k,n}}{\partial x} &= \frac{1}{Uz}\{[(k-1)f_{1,0}+(k+n)F]f_{k,n}+f_{k+1,n}\} = \frac{A_{k,n}}{Uz};\quad F = U\frac{\partial z}{\partial x};\\
\frac{\partial g_{k,n}}{\partial x} &= \frac{1}{Uz}\{[(k-1)f_{1,0}+(k+n)F]g_{k,n}+g_{k+1,n}\} = \frac{B_{k,n}}{Uz};\\
\frac{\partial f_{k,n}}{\partial t} &= \frac{1}{z}\{[(k-1)f_{1,0}+(k+n)T]f_{k,n}+f_{k+1,n}\} = \frac{C_{k,n}}{z};\\
\frac{\partial g_{k,n}}{\partial t} &= \frac{1}{z}\{[(k-1)f_{1,0}+(k+n)T]g_{k,n}+g_{k+1,n}\} = \frac{D_{k,n}}{z}.
\end{aligned} \tag{15}$$

By using the operators (14) and the derivations (15), the equation (9) is transformed to the equation

$$\begin{aligned}
&\frac{\partial^3\Phi}{\partial\eta^3} + \frac{1}{2B^2}[a_0B^2+(2-b_0)f_{1,0}]\Phi\frac{\partial^2\Phi}{\partial\eta^2} + \frac{f_{1,0}}{B^2}\left[1-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right] + \frac{f_{0,1}}{B^2}\left(1-\frac{\partial\Phi}{\partial\eta}\right)+\\
&\frac{g_{1,0}}{B^2}\left[S(1)-\frac{\partial\Phi}{\partial\eta}S\left(\frac{\partial\Phi}{\partial\eta}\right)\right] + \frac{1}{2}\eta\frac{T}{B^2}\frac{\partial^2\Phi}{\partial\eta^2} = \eta\frac{1}{B^3}\left(\sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty} C_{k,n}\frac{\partial B}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty} D_{k,n}\frac{\partial B}{\partial g_{k,n}}\right)+\\
&+\frac{1}{B^2}\left\{\sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\left[C_{k,n}\frac{\partial^2\Phi}{\partial f_{k,n}\partial\eta} + A_{k,n}X(\eta;f_{k,n})\right] + \sum_{\substack{k=1\\ n=0}}^{\infty}\left[D_{k,n}\frac{\partial^2\Phi}{\partial g_{k,n}\partial\eta} + B_{k,n}X(\eta;g_{k,n})\right]\right\}.
\end{aligned} \tag{16}$$

From the equation (16) can be seen, that for the chosen particular electroconductivity variation, the characteristics of external flow dominate by the means of functions F and T. In order the equation (16) to be independent of the outer flow characteristics i.e. to be universal, it is necessary to show the existence of the equality

$$F = F[(f_{k,n}),(g_{k,n})],\qquad T = T[(f_{k,n}),(g_{k,n})]. \tag{17}$$

So as to show the existence of such equalities, we start from the momentum equation

$$\frac{\partial}{\partial t}(U\delta^*) + \frac{\partial}{\partial x}(U^2\delta^{**}) + U\delta^*\frac{\partial U}{\partial x} + NU\tilde{\delta}^* - \frac{\tau_w}{\rho} = 0 \tag{18}$$

and energy equation

$$\frac{\partial}{\partial t}(U^2\delta^{**}) + U^3\frac{\partial\delta_1^{**}}{\partial x} + U^2\left(\frac{\partial\delta^*}{\partial t} + 3\delta_1^{**}\frac{\partial U}{\partial x} + 2N\tilde{\delta}^{**} - 2\nu e\right) = 0 \qquad (19)$$

of the problem considered where the following denotes have been used:

$$\delta^*(x,t) = \int_0^\infty \left(1 - \frac{u}{U}\right)dy = K\int_0^\infty \left(1 - \frac{\partial\Phi}{\partial\eta}\right)d\eta\,;$$

$$\tilde{\delta}^*(x,t) = \int_0^\infty \left[S(1) - \frac{u}{U}S\left(\frac{u}{U}\right)\right]dy = K\int_0^\infty \left[S(1) - \frac{\partial\Phi}{\partial\eta}S\left(\frac{\partial\Phi}{\partial\eta}\right)\right]d\eta$$

$$\tau_w(x,t) = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu\frac{U}{K}\left.\frac{\partial^2\Phi}{\partial\eta^2}\right|_{\eta=0} \qquad (19a)$$

$$\delta_1^{**}(x,t) = \int_0^\infty \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right)dy = K\int_0^\infty \frac{\partial\Phi}{\partial\eta}\left[1 - \left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]d\eta\,;$$

$$\tilde{\delta}^{**}(x,t) = \int_0^\infty \frac{u}{U}\left[S(1) - \frac{u}{U}S\left(\frac{u}{U}\right)\right]dy = K\int_0^\infty \frac{\partial\Phi}{\partial\eta}\left[S(1) - \frac{\partial\Phi}{\partial\eta}S\left(\frac{\partial\Phi}{\partial\eta}\right)\right]d\eta\,;$$

$$e(x,t) = \int_0^\infty \left[\frac{\partial\left(\frac{u}{U}\right)}{\partial y}\right]^2 dy = \frac{1}{K}\int_0^\infty \left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^2 d\eta\,,$$ μ - coeficient of dynamic viscosity.

First, we write the derivations with respect to x and t in equation (18) in the developed form and we substitute τ_w with the expression (19a), and afterwards we introduce the values

$$H = \frac{\delta^*}{\delta^{**}} = \frac{1}{B}\int_0^\infty \left(1 - \frac{\partial\Phi}{\partial\eta}\right)d\eta\,; \quad \tilde{H} = \frac{\tilde{\delta}^*}{\delta^{**}} = \frac{1}{B}\int_0^\infty \left[S(1) - \frac{\partial\Phi}{\partial\eta}S\left(\frac{\partial\Phi}{\partial\eta}\right)\right]d\eta$$

$$\zeta = \frac{\tau_w\delta^{**}}{\mu U} = B\left.\frac{\partial^2\Phi}{\partial\eta^2}\right|_{\eta=0} \qquad (20)$$

and in this way we obtain the equation from which it is obtained the following expression

$$F = 2\left[\zeta - 2f_{1,0} - H\left(f_{1,0} + f_{0,1} + \tfrac{1}{2}T\right) - \tilde{H}g_{1,0} - z\frac{\partial H}{\partial t}\right]. \qquad (21)$$

From the expression (20) it can be seen, that the value H is the function only of the parameter (12), so that its derivation with respect to t determines by mediate derivation the value H with respect to the parameters, and afterwards the parameters with respect to *t*. After this determination of derivation of the value *H* with respect to *t*, the expression (21) obtains the form

$$F = 2\left[\zeta - 2f_{1,0} - H\left(f_{1,0} + f_{0,1} + \tfrac{1}{2}T\right) - \widetilde{H}g_{1,0} - \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty} C_{k,n}\frac{\partial H}{\partial f_{k,n}} - \sum_{\substack{k=1\\ n=0}}^{\infty} D_{k,n}\frac{\partial H}{\partial g_{k,n}}\right]. \quad (22)$$

As the values ζ, H, $\widetilde{H}$,depend only on parameter (12), than the function *F*, given by the expression (22), depends only on the same parameters and on the value *T*, which appears in it explicitly and implicitly by the means of values $C_{k,n}$ and $D_{k,n}$. So for the evidence of the existence of the first equality (17), it is needed and sufficient to prove also the existence of the second equality (17). For the evidence of the second equality existence (17) let's write first in developed form the derivations with respect to *x* and *t* in the energy equation (19) and let's introduce the values

$$\begin{aligned} H_1 &= \frac{\delta_1^{**}}{\delta^{**}} = \frac{1}{B}\int_0^{\infty}\frac{\partial\Phi}{\partial\eta}\left[1-\left(\frac{\partial\Phi}{\partial\eta}\right)^2\right]d\eta \\ \mathrm{e}\delta^{**} &= B\int_0^{\infty}\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)d\eta = \alpha\left[(f_{k,n});(g_{k,n})\right] \end{aligned} \quad (23)$$

than the energy equation obtains the form

$$z\frac{\partial H}{\partial t} + \tfrac{1}{2}(H+1)T + 2f_{0,1} + Uz\frac{\partial H_1}{\partial x} + \left(\tfrac{1}{2}F + 3f_{1,0}\right)H_1 + 2\widetilde{H}_1 g_{1,0} - 2\alpha = 0 \quad (24)$$

going to, in the last equation, to the new variables - parameters, substituting function *F* by the expression (22) and solving in such way obtained equation with respect to *T*, it is obtained that is

$$\begin{aligned} T = \{&2(2M+H_1)[\zeta - 2f_{1,0} - H(f_{1,0}+f_{0,1}) - \widetilde{H}g_{1,0} - M_1^*] + 2(M_1 + M_1^* + 2f_{0,1}) + \\ &+ 6H_1 f_{1,0} + 4\widetilde{H}_1 g_{1,0} - 4\alpha\} \big/ [(H+2M^*)(2M+H_1-1)-1] \end{aligned} \quad (25)$$

where the denotes are introduced

$$\begin{aligned} M &= \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}(k+n)f_{k,n}\frac{\partial H_1}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty}(k+n)g_{k,n}\frac{\partial H_1}{\partial g_{k,n}}, \\ M_1 &= \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\theta_{k,n}\frac{\partial H_1}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty}\theta^*_{k,n}\frac{\partial H_1}{\partial g_{k,n}}, \\ \theta_{\mathrm{k,n}} &= (k-1)f_{1,0}f_{k,n} + f_{k+1,n}; \quad \theta^*_{\mathrm{k,n}} = (k-1)f_{1,0}g_{k,n} + g_{k+1,n}; \\ M^* &= \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}(k+n)f_{k,n}\frac{\partial H}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty}(k+n)g_{k,n}\frac{\partial H}{\partial g_{k,n}}, \\ M_1^* &= \sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\Lambda_{k,n}\frac{\partial H}{\partial f_{k,n}} + \sum_{\substack{k=1\\ n=0}}^{\infty}\Lambda^*_{k,n}\frac{\partial H}{\partial g_{k,n}}, \end{aligned} \quad (26)$$

$$\Lambda_{\mathrm{k,n}}=(k-1)f_{0,1}f_{k,n}+f_{k,n+1};\quad \Lambda^{*}_{\mathrm{k,n}}=(k-1)f_{0,1}g_{k,n}+g_{k,n+1}.$$

As the values M, H_1, ζ, H, $\widetilde{H}$, M_1^*, M_1, $\widetilde{H}_1$, α, M^* which exists in the expression (25) depends only on parameters ($f_{k,n}$) and ($g_{k,n}$), so the same expression is also the expression (25), and in that way the existence of the second equality (17) has been confirmed, and much more, it explicit form has been determined. As it has been shown till now, the value of T depends only on parameters, and than on the basis of the expression (22) it is shown that also the value F is only the function of parameters.

Now we conclude that the equation (16), for the chosen electroconductivity variation, do not depend explicitly on characteristics of outer flow and outer magnetic field, and in that sense we can consider it as universal equation of the problem investigated. The boundary conditions, also universal that correspond to the equation (16) have the form:

$$\begin{gathered}\Phi=0,\quad \frac{\partial\Phi}{\partial\eta}=0\ \text{ for }\ \eta=0;\quad \frac{\partial\Phi}{\partial\eta}\to 1\ \text{ for }\ \eta\to\infty;\\ \Phi=\Phi_0(\eta)\quad\text{for}\quad \begin{cases}\mathrm{f}_{\mathrm{k,n}}=0 & (k,n=0,1,2,\ldots;\ k\vee n\neq 0)\\ g_{k,n}=0 & (k,n=0,1,2,\ldots;\ k\neq 0)\end{cases}\end{gathered}\tag{27}$$

where is $\Phi_0(\eta)$ the Blausius solution for steady boundary layer on the flat plate. The independence of the equation (16) and boundary conditions (27) of particular velocity distribution at the edge of boundary layer and of outer magnetic field enable the integration of this equation, for the chosen electroconductivity variation of the form (1), once for all. In the procedure of integration we determine the velocity dimensionless profiles in cross-stream sections of boundary layer Φ, friction coefficient ζ, and also the characteristic functions H_1, H, $\widetilde{H}$, α, F and T in the dependence of parameters ($f_{k,n}$) and ($g_{k,n}$).

That obtained so called universal solutions should, in appropriate manner, be kept and be used not only for drawing the general conclusions about the unsteady MHD boundary layer development, but also for the calculation of the particular problems. Surely, the equation (16) can be used only in the some corresponding approximation which assumes the finite number of terms on the right-hand side of that equation.

From the equation (16), for the particular variation of electroconductivity of the form (1), it can be obtained the corresponding universal equations for this particular electroconductivity variations. So for the case of the inconductive fluid or in the absence of the outer magnetic field we obtain the equation

$$\mathrm{L}=\eta\frac{1}{B^3}\sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}C_{k,n}\frac{\partial B}{\partial f_{k,n}}+\frac{1}{B^2}\sum_{\substack{k,n=0\\ k\vee n\neq 0}}^{\infty}\left[C_{k,n}\frac{\partial^2\phi}{\partial\eta\partial f_{k,n}}+A_{k,n}X(\eta;f_{k,n})\right]\tag{28}$$

where is

$$\mathrm{L}=\mathrm{L}^*-\frac{g_{1,0}}{B^2}\left[S(1)-\frac{\partial\Phi}{\partial\eta}S\left(\frac{\partial\Phi}{\partial\eta}\right)\right]\tag{29}$$

and with the L^* is denoted the left-hand side of the equation (16). The function F and the function T, which correspond to the equation (29) have the form

$$F = 2\Big[\zeta - 2f_{1,0} - H\big(f_{1,0} + f_{0,1} + \tfrac{1}{2}T\big) - \sum_{\substack{k,n=0 \\ k\vee n\neq 0}}^{\infty} C_{k,n}\frac{\partial H}{\partial f_{k,n}}\ \Big], \tag{30}$$

$$\begin{aligned} T = \{&2(2M + H_1)[\zeta - 2f_{1,0} - H(f_{1,0} + f_{0,1}) - M_1^*] + 2(M_1 + M_1^* + 2f_{0,1}) +\} \\ &+ 6H_1 f_{1,0} - 4\alpha \big/ [(H + 2M^*)(2M + H_1 - 1) - 1] \end{aligned} \tag{31}$$

in which the values M, M_1^*, M_1, M^* are given by the expressions (26) without other summation which contain the derivations with respect to the parameters $g_{k,n}$. The universal boundary conditions which correspond to the equation (28) are obtained from the condition (27) and they have the form

$$\begin{aligned} &\Phi = 0\,, \quad \frac{\partial\Phi}{\partial\eta} = 0 \quad \text{for} \quad \eta = 0\,; \quad \frac{\partial\Phi}{\partial\eta} \to 1 \quad \text{for} \quad \eta \to \infty\,; \\ &\Phi = \Phi_0(\eta) \quad \text{for} \quad f_{k,n} = 0 \quad \big(k,n = 0,1,2,...;\ \ k \vee n \neq 0\big) \end{aligned} \tag{32}$$

For the case of constant electroconductivity (σ=cons.) from the equation (16) the corresponding equation in the following form is obtained

$$\mathrm{L} + \frac{g_{1,0}}{B^2}\left(1 - \frac{\partial\Phi}{\partial\eta}\right) = \mathrm{R} \tag{33}$$

where with R is denoted the right-hand side of the equation (16). The function F and function T, which correspond to the equation (33), have got the form (22) and (25) respectively, but in these expressions we have now $\widetilde{H} = H$. The boundary conditions which correspond to the equation (33) have the form (27). For the case of the electroconductivity variation in the Rossov's form [2]

$$\sigma = \sigma_0\left(1 - \frac{u}{U}\right) \tag{34}$$

the equation (16) obtains the form

$$\mathrm{L} - \frac{g_{1,0}}{B^2}\frac{\partial\Phi}{\partial\eta}\left(1 - \frac{\partial\Phi}{\partial\eta}\right) = \mathrm{R} \tag{35}$$

and the functions F and T are given by the expressions (22) and (25) respectively in which it is now $\widetilde{H} = -1$. The boundary conditions which correspond to the equation (35) have the form (27). In the same way it can be obtained from the equation (16) the corresponding universal equation and also for the other electroconductivity variation of the form (1).

With this the investigation of the problem could be finished. In spite of this in the paper will be given, as illustration, the part of universal results obtained by solving the equation (33) in three-parametric once localised approximation. In the mentioned approximation it has been kept the influence of the parameters $f_{1,0}$; $f_{0,1}$ and $g_{1,0}$, but the influence of the all others parameters and its derivations have been neglected, and also the derivation with respect to the parameter $f_{0,1}$. In that approximation the equation (33) have the form

$$
\begin{aligned}
&\mathrm{L}+\frac{g_{1,0}}{B^2}\left(1-\frac{\partial\Phi}{\partial\eta}\right)=\eta\frac{1}{B^3}T\left(f_{1,0}\frac{\partial B}{\partial f_{1,0}}+g_{1,0}\frac{\partial B}{\partial g_{1,0}}\right)\frac{\partial^2\Phi}{\partial\eta^2}+\\
&+\frac{1}{B^2}\left\{T\left(f_{1,0}\frac{\partial^2\Phi}{\partial f_{1,0}\partial\eta}+g_{1,0}\frac{\partial^2\Phi}{\partial g_{1,0}\partial\eta}\right)+F f_{1,0}X(\eta;f_{1,0})+F g_{1,0}X(\eta;g_{1,0})\right\}
\end{aligned}
\tag{36}
$$

and the corresponding boundary conditions are

$$
\begin{aligned}
&\Phi=0\,,\quad \frac{\partial\Phi}{\partial\eta}=0 \quad\text{for}\quad \eta=0\,;\quad \frac{\partial\Phi}{\partial\eta}\to 1 \quad\text{for}\quad \eta\to\infty\,;\\
&\Phi=\Phi_0(\eta)\quad\text{for}\quad f_{1,0}=0\,;\quad f_{0,1}=0\,;\quad g_{1,0}=0\ .
\end{aligned}
\tag{37}
$$

The function F which correspond to this problem in the same approximation have the form

$$
F=2\left[\zeta-2f_{1,0}-H(f_{1,0}+f_{0,1}+g_{1,0}+\tfrac{1}{2}T)-T\left(f_{1,0}\frac{\partial H}{\partial f_{1,0}}+g_{1,0}\frac{\partial H}{\partial g_{1,0}}\right)\right]
\tag{38}
$$

and the function T have the form (25) in which the values M^* and M are now

$$
\begin{aligned}
&M=f_{1,0}\frac{\partial H_1}{\partial f_{1,0}}+g_{1,0}\frac{\partial H_1}{\partial g_{1,0}}\,;\quad M_1=0\,;\\
&M^*=f_{1,0}\frac{\partial H}{\partial f_{1,0}}+g_{1,0}\frac{\partial H}{\partial g_{1,0}}\,;\quad M_1^*=0.
\end{aligned}
\tag{39}
$$

The part of the mentioned results, which refer to the unsteady, plane problem of MHD boundary layer of incompressible fluid with constant electroconductivity, have been shown in the figure 1.

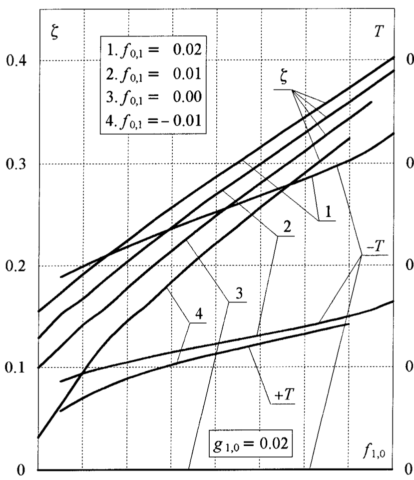

Figure 1.

On the basis of these results can be drawn some general conclusions about the boundary layer development without solving particular problems. In that sense, for example, it can be concluded that the accelerated flow in the main stream postpones the arise of flow separation but the decelerated flow favours the arise of boundary layer separation. Also it can be concluded that the magnetic field postpones the arise of the boundary layer separation.

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PARAMETARSKA METODA U TEORIJI NESTACIONARNOG MHD GRANIČNOG SLOJA FLUIDA PROMENLJIVE ELEKTROPROVODNOSTI

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U radu se razmatra nestacionarni ravanski MHD granični sloj. Fluid je nestišljiv a njegova elektroprovodnost je funkcija odnosa brzina. Uvođenjem novih promenljivih a zatim i dva skupa parametara dobijena je univerzalna jednačina za sve elektroprovodnosti koje zavise od odnosa brzina. Iz tako dobijene univerzalne jednačine dobijaju se univerzalne jednačine za konkretne promene elektroprovodnosti.