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VARIABLE STRUCTURE SYSTEMS OF QUASI RELAY TYPE WITH PROPORTIONAL-INTEGRAL ACTION

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Abstract*. Three variants of using the proportional-integral action in the variable structure systems of quasi relay type are considered in this paper. Their characteristics in respect to reaching and sliding mode conditions, accuracy in the steady-state, forms of produced control signals, its influence to chattering generation as well as capabilities of load disturbance rejection are analyzed. Some examples which satisfy theoretical conclusions are given.*
Key Words: *Automatic control, nonlinear systems, variable structure systems, sliding mode, proportional-integral action.*

1. INTRODUCTION

In the modern variable structure systems (VSS) theory with sliding mode of motion two control strategies are in use: the quasi relay [1] and relay [2]. The first control law is characterized with high frequency control signal which amplitude is modulated by the signal proportional to the system error. The amplitude of control signal is exponentially decreasing to a value determined by the steady-state error of the control system with the approach to the steady-state. If the control object does not contain pure integrators, the steady-state error is not zero and VSS go out of the sliding mode. Then, in the given class of systems, a selfoscillation with small amplitude may accur. This phenomenon was observed earlier [2] and to eliminate it the low level control signal of the relay type is introduced, parallel with quasi relay action. In the same way, the third - combination type of control algorithm was introduced. Control strategy of the relay type gives a high frequency control signal with a constant amplitude without regard whether the control system is in steady-state or not. Such control strategy gives better sliding mode conditions, high steady-state accuracy (similar to the systems with unlimited gain). However, its main disadvantage is the generation of high level, high frequency control signal in the steady-state regime. This high level signal, often, excites unmodelled dynamics of control object causing chattering and losses in switching devises. From this point of view, quasi

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relay control law has advantage if steady-state accuracy is to be improved.

Improving of the steady-state accuracy may be obtained by increasing the control loop gain and/or by introducing integrators in the control unit. The first approach decreases systems error similarly as in conventional linear control systems. Since the VSS are stable for wide range of loop gain, such approach is possible. However, the analysis shows that this approach may be valid in case if no control loop limitations. If a nonlinearity of limiter type is present (which is natural for the real systems) in the control system , the quality of motion decreases and the system stability may be lost. In these conditions the relay control law stays stable, and with the same quality of motion.

Introducing the integral action in the system can reduce steady-state error to zero, for some type of perturbation, enabling systems action with lower gain and eliminating problems of nonlinearities and their influence to the system stability. Such approach was used in introducing integral action immediately after error detector (before switching element of VSS regulator) [6, 9, 11]. The integral (I) or proportional-integral (PI) action may be introduced in this way. In both cases, especially with I-terms, the control signals will be like those in the relay type of VSS, which is in opportunity with using quasi relay control law. Except this, some authors, who use this approach, increase in design process the order of equivalent control object by one and make the VSS controller more complex.

The second approach, which may be used, introduces PI action parallel to the VSS controller. This is a simpler case to implement VSS controller in the existing conventional control system with PI action. In this way, the dynamic characteristics of the system will be improved. However, in this paper it will be shown, this control algorithm has poor sliding mode existence conditions.

The third approach, considered in [3, 4, 5, 7[1]], and up to now less known, places the PI action after VSS controller, before control object. The main idea, which brought this approach, was taken from the conventional method of controller design using, so-called, "method of magnitude optimum". This method is based on the cancellation of the bigger time constant of controlled object and the introduction of the pure integrator. In this way, the equivalent control object, from VSS controller point of view, retains the same order. Even if the cancellation is not complete, thanks to the known robustness of VSS in respect to the unmodelled dynamics, the system will stay in working regime. Such approach was analyzed in detail in [4]. The basic conclusion, which may be taken from the previous explanation is that the order of the equivalent control object is not changed and PI action may be incorporated in the control algorithm as its integral part. In this paper it will be shown that the sliding mode existence conditions, for this approach, are better then in the first two approaches and that they improve with time.

On the other side, since the VSS is a nonlinear system (in which, very often, we operate with linear mutual commutated structures), the first approach can be observed, from the control loop point of view, as introducing the PI term to cancellation of undesired object dynamics, too, and do the process of the VSS controller design without increasing the equivalent object order so that the PI term may be included in the VSS control algorithm. However, analysis, given in this paper, will show that this approach may lead to degradation of the sliding mode existence condition if the load perturbation is present and, therefore, must be used design procedure given in [6, 8, 9].

[1] In [7] PI action is introduced after conventional VSS controller and together in parallel with P action.

The intention of this paper is to analyze features ofthe VSS with PI action, using given approaches in the class of the quasi relay control algorithm without increasing design order of the eqvivalent control object. On the basis of this analysis, recommendations for practical design of new VSS controller, or reconstruction of present PI controllers will be given.

This work, except being a theoretical analysis, has clear practical importance: research of the best approach to implementation of the VSS control algorithm in the existing control systems with the PI controller (which are dominant in the conventional control systems design) in order to improve their performances: rise-time without overshoot, robustness on the parametric and load disturbances etc.

2. PROBLEM FORMULATION

Let the mathematical model of a linear single input - single output control object of n-*th* order, without finite zeros,be given in the controllable canonical form :

$$\dot{\mathbf{x}}(\mathrm{t}) = \mathbf{A}\mathbf{x} + \mathbf{b}u \qquad (1)$$
$$c = x_1$$

For the output variable *c(t)* regulation, the VS regulator with PI action will be used, taking one of the following three control algorithms:

(i)
$$u = \Psi_1 y + \sum_{i=2}^{k} \Psi_i e_i,\ 2 \le k \le n\text{-}1, \quad \mathrm{y} = \mathrm{e}_1 + k_i \int_{-\infty}^{t} e_1 dt, \qquad (2)$$

(ii)
$$u = e_1 + k_i \int_{-\infty}^{t} e_1 dt + \Psi_1 e_1 + \sum_{i=2}^{k} \Psi_i e_i \qquad (4)$$

(iii)
$$u = z + k_i \int_{-\infty}^{t} z dt + \sum_{i=2}^{k} \Psi_i e_i; \quad 2 \le k \le n-1; \quad z = \Psi_1 e_1, \qquad (3)$$

where:

$$\Psi_i = \begin{cases} +\alpha_i & \text{for } ge_i > 0 \\ -\alpha_i & \text{for } ge_i < 0 \end{cases}, \qquad (5)$$

$$g = \mathbf{c}^T \mathbf{e}; \ \ \mathbf{c} \in R^n; \quad c_i = const > 0; \quad i = 1,2,...,n-1; c_n = 1. \qquad (6)$$

$$\mathbf{e} \in R^n, e_1 = r - c, e_i = x_i, i = 2,...,n. \qquad (7)$$

From (7) it may be seen that the system of regulator type is considered (*r*=const). The assignment of the controller is to provide *c*=*r* after short time, In order to satisfy this requirement, the sliding mode of motion will be organized on the hyperplane *g*=0. The process of regulation flows in two phases: *reaching* phase and *sliding mode* phase. In the reaching phase the system state, from any initial condition, must be taken on the state *g*=0, when second phase begins - the sliding mode in which the system state stays on the given hyperplane *g*=0, and slides on it. The first phase occurs in short time, but the second - one should take place continuously.

Therefore, the dominant motion in the system is the sliding mode, which is described by the relation g=0. Thanks to the chosen systems state (controllable canonical form) this relation is, de facto, differential equation of n-1-*th* order. The system motion in this phase is independent (invariant) from its parameters and depends only on the VSS controller parameters, given by vector c.

3. Steady-State Error of the System

We will show first, that the considered system, without integral term in (2), (3) or (4), has steady-state error, if the control object (1) is of self regulation type. In the steady-state it is $\dot{\mathbf{x}} = 0$ and the system model will be:

$$-a_1 x_1 + b\Psi_1 e_1 = 0,$$

-for the systems with controls in form (2) and (4), and

$$-a_1 x_1 + b e_1 + b\Psi_1 e_1 = 0,$$

-for the systems with control (3).

In the steady-state, $g(\infty)=c_1 e_1$, and, therefore, $ge_1>0$, $\Psi_1=\alpha_1$. Taking in account $e_1=r-x_1$, we have

$$e_1(\infty) = \frac{r}{1+\dfrac{b(q+\alpha_1)}{a_1}}, \tag{8}$$

where $q=0$ for the systems with control (2) or (4) and q=1 - with the control (3).

Relation (8) shows that the steady-state error for the considered systems has a value like as in the conventional linear systems with only one difference: the VSS can work with a higher gain then the conventional systems and, therefore, may obtained better steady-state accuracy. It is clear from (8), if the object contains a pure integrator, then $a_1=0$, and the system steady -state error will be zero.

It may be shown that the steady-state error in the considered class of the system is a consequence of the sliding mode loss in the neighborhood of equilibrium state. If the sliding mode was to occurr continuously - the steady - state error would be zero.

If the PI type of control is introduced, with zero initial conditions, the steady-state of the considered system will be given by relations:

$$-a_1(r-e_1) + b\alpha_1\left(e_1 + k_i \int_0^{t\to\infty} e_1 dt\right) = 0\ ,$$

-for the control in form (2) or (4), and

$$-a_1(r-e_1) + b\left(e_1 + \alpha_1 e_1 + k_i \int_0^{t} e_1 dt\right) = 0\ ,$$

-for the control (3).

Using Laplace transform method to the previous relations and solving for $E_1(s)$, we have, for the control (2) or (4) and (3), respectively, the following relations:

$$E_1(s) = \frac{R(s)}{1 + \dfrac{b\alpha_1(1+\dfrac{k_i}{s})}{a_1}}, \tag{9}$$

$$E_1(s) = \frac{R(s)}{1 + \dfrac{b(1+\alpha_1+\dfrac{k_i}{s})}{a_1}}, \tag{9a}$$

from which it is clear that the steady-state error will be zero if the reference signal is the step function.

For the third case (the control is (4)) the following approach may be used . The PI term is linear and it is on the input of controlled object which is linear, too. For the VSS controller of proportional type (relation (4) without integral addend) the equivalent object (series connection of the controlled object itself and the PI term) is with the pure integrator, therefore $a_1=0$, and from relation (8) it becomes $e_1(\infty)=0$. Since the relations of the steady - state for the first and third approaches are the same, and for the second approach is unessentially different, it may be concluded: *steady-state error in the VSS with proportional-integral action will be zero without consideration where the PI-action is placed: before, parallel or after the VSS controller.* From this point of view, the three considered systems are equivalent.

4. THE SLIDING MODE EXISTENCE CONDITIONS

Now, we start to discuss conditions of the sliding mode existence in the considered class of VSS, assuming that the initial conditions of PI-term integrators (in relations: (2), (3), (4)) are zero. Later, in section 5, non zero initial conditions will be taken in consideration.

Starting from the well known general sliding mode condition [2]

$$\frac{d}{dt}(\frac{1}{2}g^2) = g\dot{g} < 0\ , \tag{10}$$

which provides both: the reaching and the sliding mode conditions, and taking in account that the system is of regulator type, therefore, it may be analyzed as autonomous system; in sliding mode

$$g=0$$

or

$$g = \left[\mathbf{c}_0^T \;\vdots\; 1\right]\begin{bmatrix}\mathbf{x}_0 \\ \hdashline x_n\end{bmatrix} = 0 \Rightarrow x_n = -\mathbf{c}_0^T\mathbf{x}_0\ ;$$

the system may be described by relations

$$\dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} = \left[\begin{array}{c:c}\mathbf{D} & \mathbf{d}_1 \\ \hdashline -\mathbf{d}_2^T & -a_n\end{array}\right]\begin{bmatrix}\mathbf{x}_0 \\ \hdashline -\mathbf{c}_0^T\mathbf{x}_0\end{bmatrix}; \quad \mathbf{x}_0^T = [x_1, x_2, \ldots, x_{n-1}], \tag{11}$$

$$\mathbf{c}_0^T = [c_1, c_2, \ldots, c_{n-1}],$$

$$d_2^T = [a_1 + b\Psi_1(x_1), a_2 + b\Psi_2, ... a_r + b\Psi_r, a_{r+1}, ..., a_{n-1}];\ \mathrm{b} > 0;$$

$$d_1 = [0,0,...,0,1]^T,$$

$$\mathbf{D}_{(n-1)x(n-1)} = \left[\begin{array}{c|c} \mathbf{0}_{(n-2)x1} & \mathbf{I}_{(n-2)x(n-2)} \\ \hline 0 & \mathbf{0}_{1x(n-2)} \end{array}\right],$$

where: $\mathbf{I}$ - is the unity matrix, $\Psi_1(x_1)$ - is symbolic annotation of the following operations, for the control (2), (3) and (4), respectively:

$$\Psi_1(x_1) = \Psi_1(\bullet + k_i \int_0^t \bullet\, dt)\ ;\quad \Psi_1(x_1) = \bullet + k_i \int_0^t \bullet dt + \Psi_1 \bullet\ ;\quad \Psi_1(x_1) = \Psi_1 \bullet + k_i \int_0^t \Psi_1 \bullet dt\ .$$

In previous relations $\bullet$ must be replaced by x_1, after the mentioned operations have been taken.

By differentiating the expression for sliding hyperplane (6), and replacing **e=x** for the autonomous system, the following expression it will be obtained, after replacing (1), (2) or (3) or (4) and simple transformation:

$$\dot{g} = c^T \dot{x} = \sum_{i=1}^{n-1} c_{i-1} x_i - c_{n-1} \sum_{i=1}^{n-1} c_i x_i + a_n \sum_{i=1}^{n-1} c_i x_i - \sum_{i=1}^{r} (a_i + b\Psi_i) x_i - \sum_{i=r+1}^{n-1} a_i x_i \qquad (12)$$

$$\Psi_i = \Psi_1(x_1), \Psi_j; j = 2,3,...,k\ .$$

By multiplying (12) with g, and assuming $g>0$ $(g<0)$, then, to satisfy (10), it follows that $\dot{g} < 0$ $(\dot{g} > 0)$, and this will be true if the following relations are satisfied:

$$\sum_{i=r+1}^{n-1} (c_{i-1} - c_{n-1} c_i + a_n c_i - a_i) x_i = 0, \qquad (13)$$

$$\sum_{i=1}^{k} (c_{i-1} - c_i c_{n-1} + c_i a_n - a_i - b\Psi_i) x_i < 0, \mathrm{g} > 0, \qquad (14)$$

Condition (13) must be fulfilled for every $x_i \neq 0$, and, therefore, the following relations must be satisfied:

$$c_{i-1} - c_i c_{n-1} + c_i a_n - a_i = 0,\ i = k+1,...,n-1. \qquad (15)$$

Condition (14) will be reliably fulfilled if its every term of sum is less simultaneously then zero, i.e.:

$$\begin{aligned} &(-c_1 c_{n-1} + c_1 a_n - a_1) x_1 - b\Psi_1(x_1) x_1 < 0, \\ &(c_{i-1} - c_i c_{n-1} + c_i a_n - a_i) x_i - b\Psi_i x_i < 0, i = 2,3,...,k \end{aligned} \qquad (16)$$

The second condition in (16) may be expressed as a well known condition [1]

$$\alpha_i > \left| \frac{c_{i-1} - c_i c_{n-1} + c_i a_n - a_i}{b} \right|; i = 2,3,...,k. \qquad (17)$$

The first of conditions (16) depends on the control signal type or on the introducing place of PI action in the controller: before (i), parallel (ii) or after (iii) the VSS controller.

(i) For the case when PI action is before the VSS controller, it may be written:

$$(-c_1c_{n-1} + c_1a_n - a_1)x_1 - b\Psi_1(x_1 + k_i\int_0^t x_1dt) < 0,\ \mathrm{g} > 0.$$

Since the commutation function Ψ_i is symmetrical, this expression may be rewritten in the form

$$(-c_1c_{n-1} + c_1a_n - a_1)x_1 - b\alpha_1(x_1 + k_i\int_0^t x_1dt)sign(x_1) < 0, \mathrm{g} > 0,$$

or

$$\alpha_1 > \frac{(-c_1c_{n-1} + c_1a_n - a_1)x_1}{b(x_1 + k_i\int_0^t x_1dt)sign(x_1)} = \frac{(-c_1c_{n-1} + c_1a_n - a_1)sign(x_1)}{b(1 + \frac{k_i}{x_1}\int_0^t x_1dt)}, \mathrm{g} > 0$$

This condition will be reliably satisfied if the next relation is true

$$\alpha_1 > \frac{\left|-c_1c_{n-1} + c_1a_n - a_1\right|}{b\left|1 + \frac{k_i}{x_1}\int_0^t x_1dt\right|}. \tag{18}$$

(ii) If the PI action is placed in parallel with VSS controller, then it will be:

$$(-c_1c_{n-1} + c_1a_n - a_1)x_1 - bx_1 - bk_i\int_0^t x_1dt - b\Psi_1x_1 < 0, \mathrm{g} > 0.$$

or

$$(-c_1c_{n-1} + c_1a_n - a_1)x_1 - bx_1 - bk_i\int_0^t x_1dt - b\alpha_1\left|x_1\right|sign(g) < 0, \mathrm{g} > 0.$$

from which we get

$$\alpha_1 > \frac{(-c_1c_{n-1} + c_1a_n - a_1 - 1)sign(x_1) - k_i\frac{1}{\left|x_1\right|}\int_0^t \left|x_1\right|sign(x_1)dt}{b}, g > 0$$

or in the final form:

$$\alpha_1 > \left|\frac{(-c_1c_{n-1} + c_1a_n - a_1 - 1)sign(x_1) - k_i\frac{1}{\left|x_1\right|}\int_0^t \left|x_1\right|sign(x_1)dt}{b}\right| \tag{19}$$

(iii) If the PI term is placed after the VSS controller, then:

$$(-c_1c_{n-1} + c_1a_n - a_1)x_1 - b(\Psi_1x_1 + k_i\int_0^t \Psi_1x_1dt) < 0, \mathrm{g} > 0.$$

Since the commutation function Ψ_1 is symmetrical, this expression may be rewritten in the form

$$(-c_1c_{n-1}+c_1a_n-a_1)x_1-b\alpha_1|x_1|sign(g)+k_i\int_0^t|x_1|sign(g)dt<0, \mathrm{g}>0,$$

or

$$\alpha_1>\frac{(-c_1c_{n-1}+c_1a_n-a_1)\dfrac{x_1}{|x_1|}}{b(b(1+\dfrac{k_i}{|x_1|}\int_0^t|x_1|dt)}=\frac{(-c_1c_{n-1}+c_1a_n-a_1)sign(x_1)}{b(1+\dfrac{k_i}{|x_1|}\int_0^t|x_1|dt)},g>0.$$

and reliably will be satisfied if the next relation is fulfilled:

$$\alpha_1>\frac{|-c_1c_{n-1}+c_1a_n-a_1|}{b\left|1+\dfrac{k_i}{|x_1|}\int_0^t|x_1|dt\right|}. \tag{20}$$

Relations (17), (18), (19) and (20) are valid for g < 0, $\dot{g}>0$ too. It is easy to show. Therefore, relations (15), (17) and (18), or (15), (17) and (19) or (15), (17) and (20) give sufficient conditions for existence of the sliding mode on the given hyperplane g=0 if the initial state of the system is in the neighborhood of the hyperplane.

Before we analyze the reaching conditions, we will compare obtained relations (17), (18), (19) and (20) with those of the conventional VSS without PI action. Condition (17) is the same as for the conventional case. Conditions (18), (19) and (20) become identical with the conventional conditions, if the integral term is omitted from the denominator of relations (18) and (20) or in the nominator of (19). Observing relation (20) we may maintain that the second addend in the denominator is reliably positive. Therefore, for the same parameters, the sliding mode existence conditions in the system with PI action after VSS controller will be easier to satisfy then in the other two cases.

Consider now the reaching conditions, i.e. the sufficient conditions which must be fulfilled to guarantee falling into the state *g=0* from any initial state. As it was earlier said, fulfilling the relation (10), according to Lyapunov's stability theory, provides the reaching conditions. The derived sliding mode existence conditions will be valid for reaching mode conditions, too if an additional requirement is satisfied. Namely, we suppose *g=0*, for deriving the sliding mode existence conditions, which is not valid for the reaching conditions. Because of that, for the reaching conditions derivation, one should start from the relation

$$x_n=g-\mathbf{c}_0^T\mathbf{x}_0\,.$$

In order to carry out the similar derivations as the one above, it may be concluded that the reaching mode conditions will be satisfied if , along the conditions (15), (17), (18) or (15), (17), (19) or (15), (17), (20), the following condition is fulfilled

$$c_{n-1}-a_n\le 0. \tag{21}$$

The derived conditions for the sliding mode and reaching mode are only sufficient.

5. INFLUENCE OF THE INTEGRATOR NON ZERO INITIAL CONDITIONS

Analysis shows that the integrator non zero initial conditions have essential influence to the system behavior, especially for the second and the third algorithms, which may destroy the sliding mode. In the first algorithm this will not occur but the control high frequency signal reaches a higher amplitude.

Starting from relations (2), (3) and (4), if the initial conditions are introduced they may be written in the form:

(i)
$$u = \Psi_1 y + \sum_{i=2}^{k} \Psi_i e_i , 2 \le k \le n-1, \quad y = e_1 + k_i \int_0^t e_1 \, dt + p_0 , \tag{22}$$

(ii)
$$u = e_1 + k_i \int_0^t e_1 dt + p_0 + \Psi_1 e_1 + \sum_{i=2}^{k} \Psi_i e_i \tag{23}$$

(iii)
$$u = z + k_i \int_0^t z dt + p_0 + \sum_{i=2}^{k} \Psi_i e_i , \quad 1 \le k \le n-1, \; z = \Psi_1 e_1 , \tag{24}$$

where non zero initial conditions of the integrators are denoted with p_0.
From this relation the following can be observed :

When the PI term is placed before the VSS controller, an additional term $\Psi_1 p_0$, which may be written in equivalent form $\alpha_1|p_0|$sign(g), will be present in the control signal with the influence similar to the action of the relay type of control, introduced usually, in addition to the VSS quasi relay control algorithm to eliminate the undesired fluctuation in the steady-state.

In the algorithm with PI term placed after VSS controller (or algorithm with PI term parallel to the VSS controller) non-zero initial conditions directly influence the controlled object input and may be treated as a disturbance transformed on the object input. This disturbance may be significant and to distort the sliding mode conditions. For the elimination of this negative influence, the initial conditions must be set to zero before starting the system to work.

Simulation and experimental results show that these conclusions are true. The first algorithm generates a high level, high frequency switching signal on the controlled object input whith the amplitude dependent on the initial condition and the value of integral of error signal. From one side, this high level signal can give better robustness of the system to the parametric and load disturbances, but, from the other side, it may be a carrier of excitation of unmodelled dynamics causing chatter arising. The chattering has a very bad consecuences on the mechanical transmission elements.

The second and the third algorithms, in contrast to the first, generates a low level continual signal in the steady-state. In this way the chattering is eliminated, and system comes in the steady-state without error. Naturally, these systems have lower robustness to the load disturbances. This appearance of low level continual control signal in the steady-state is a consequence of integration of a nearly symmetrical switching signal with exponentially decreasing amplitude.

6. ROBUSTNESS TO THE PARAMETER VARIATION AND DISTURBANCES

The previous explanations did not include the variation of the plant parameters (a_i, b) and the external disturbances. Let the plant parameters variations be bounded in the ranges

$$a_{i\min} \le a_i \le a_{i\max}; b_{i\min} \le b \le b_{\max}.$$

Relation (15) with *k<n-1* is very difficult to satisfy. For a robust control in this case it needs *k* taken as *k=n-1* and relations (17), (18) (or (19) or (20)) slightly changed in this manner:

$$\alpha_i > \max_{a_i, b} [\bullet]$$

where $[\bullet]$ replaces the correspondence relations (17)-(20).

It can be seen that for the system without constraints these relations may be always satisfied and robustness to the parameter variation provided.

It is difficult to provide full robustness for the external disturbances without knowing the place of disturbances action, its characters and intensity. Since the linear plant is considered, any external disturbance can be easily transformed to the plant input. Let the summary equivalent disturbance be designed by f. This disturbance will have the same influence as the non zero initial conditions of the PI term integrator in the second and the third approaches. In the nominator of relations (18)-(20) an additional term $\frac{f}{|x_1|}$ will appear. The disturbance effect to the sliding mode existence conditions can be estimated by relations

$$\frac{f}{b\left|x_1 + k_i \int_0^t x_1 dt\right|}; \quad \frac{\mathrm{f}}{b|x_1|}; \quad \frac{f}{b(|x_1| + k_i \int_0^t |x_1| dt)};$$

respectively for the first, the second and the third approach.

From these relations it can be seen that the first approach may have the significant influence on the sliding mode existence conditions if

$$k_i \int_0^t x_1 dt \approx -|x_1|.$$

After that the system goes out of sliding mode and, may, after long time come back to the sliding regime. For the second approach it is clear that the sliding mode existence conditions are difficult to satisfy because $x_1 \to 0$. In this case, if the equivalent disturbance is a step function, because of the parallel action of the PI term, the system may have satisfactory disturbance rejection. With the third approach it is clear, from the given relation, that the sliding mode existence conditions are dependent on the absolute value of integral of the error signal and, therefore, this approach has the best sliding mode characteristics to the load disturbance. Unfortunately, since the integral term has a small value in the steady-state, its influence to the disturbance rejection is slightly better then in the second approach.

7. EXAMPLE

In an example of the controlled object of third order the design method will be shown for the all three types of VSS regulators previously analyzed, as well as effects of their work in different conditions with the same parameters of PI term and with conventionally designed VSS regulator without PI action. Let transfer function of the controlled object be:

$$W(s)=\frac{200}{(s^2+8s+7)(0{,}05s+1)}=\frac{200}{(s+1)(s+7)(0{,}05s+1))}$$

We will choose parameters of the PI term to cancellation the bigger time constant of object (T=1 s) and neglect small time constant (T=0.05 s). Therefore, the controlled object, for design purpose, is of the second order.

The control signal will be formed on the basis of the error signal commutation (k=1). Sufficient reaching and sliding mode conditions, for the system with conventional VSS controller (without PI term) will be satisfied if the following relations are fulfilled:

$$c_1 \le a_2,$$
$$\alpha > \left|\frac{-c_1^2+c_1a_2-a_1}{b}\right|.$$

We will choose c=5, α_1=1.

The results of digital simulation will be present in the next figures .

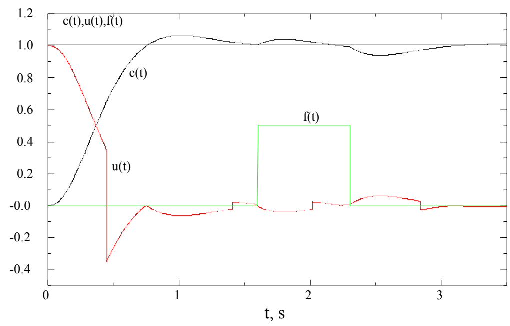

Fig. 1. *Output signal c(t) and control signal u(t) for the conventional linear system design with optimally tuned PI controller. Load disturbance f(t) is applied to the input of the last object integrator.*

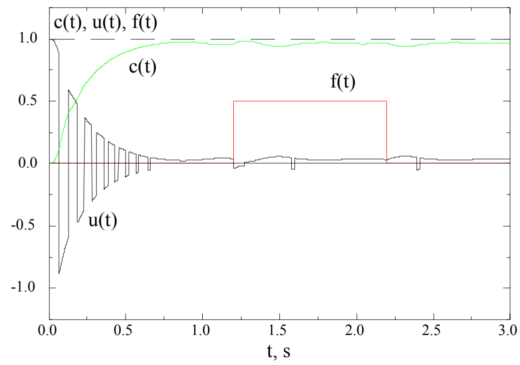

Fig. 2. S*ystem with conventional VSS controller (without PI term)*

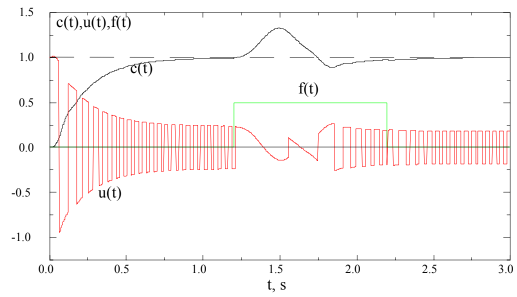

Fig. 3. *VSS controller with PI term before VSS controller*

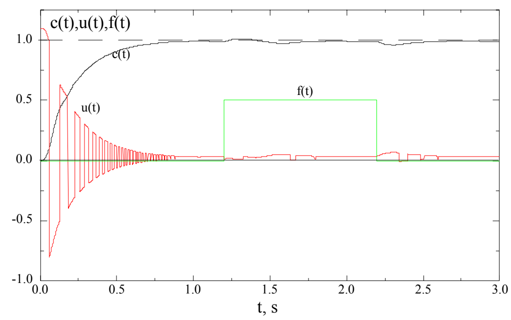

Fig. 4. *System with PI term parallel to the VSS controller*[2]

[2]The gains of the proportional and the integral action are 0.1. For the other VSS controllers with PI action they are 1.

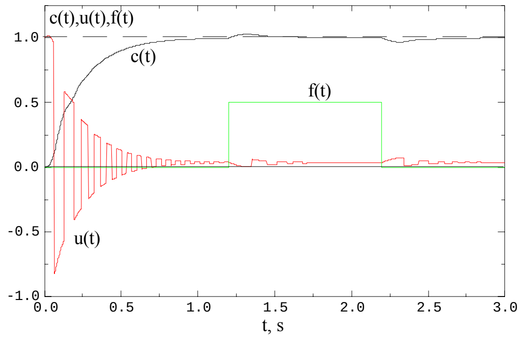

Fig. 5. *System with PI term after the VSS controller*

CONCLUSION

The problem of accuracy improvement in the steady-state for the VSS with quasi relay control algorithm, without the increase of its design complexity, is considered in this paper. The three approaches, which are characterized by the place of the introduced additional PI action i.e. before, parallel or after conventional quasi relay VSS controller.

The first approach generates a high level, high frequency control signal in the steady-state and, therefore, is like the control signal in the VSS with the relay control algorithm. From this point of view, better robustness may be expected to the parametric and load disturbances. Unfortunately, the system with such algorithm goes out from the sliding mode if an unmodelled dynamic and load disturbance are present. This is a direct consequence of the sliding mode existence conditions in which the integral term may change the sign. Non zero initial condition of the introduced PI term do not have a negative consequence. A high level steady-state control signal may cause chattering.

The second approach does not cause chattering in the steady-state, but is sensitive to the parameter variation of the PI term. The PI term integrator non zero conditions have negative influence to the system and must be set to zero before system starting, .

The third approach, excluding the negative influence of non zero initial conditions, similarly as in the second approach, has, on the whole, the best characteristics. In the steady-state it does not excite unmodelled dynamic (chattering), it is not sensitive to the PI term parameter variation, has a satisfactory insensitivity to the load disturbance and, finally, the sliding mode existence conditions improve with time.

REFERENCES

1. Emelynov, S. V., Utkin, V. I, et al., (1970), *Variable Structure Systems Theory*, Moskow: Naukha 1970. (in Russian).
2. Utkin, V. I., (1974), Sliding Modes and Their Application in Variable Structure Systems. Moscow: Naukha, (in Russian)
3. Milosavljević, Č., Antić, D., Djordjević, G. ,(1992), DC motor speed regulation using PI type variable structure regulator with identity observer, Facta Universitatis, Series Electronics and Energetics, 1., p. 69-78.

4. Antić, D., (1994), Variable Structure Systems with Proportional - Integral action, Ph. Dr Thesis, Faculty of Electronic Engineering, Niš, (in Serbian)
5. Potkonjak, V., Djordjević, G., Milosavljević, Č., Antić, D., Popović, D.,(1994), Variable structure systems for control of redundant robot, Elsevier: Robotics and Autonomous Systems 13, 13-24,
6. Yuan-Yih, H. Wah-Chun, Ch., (1984), *Optimal variable structure controller for DC motor speed control*, IEE Proc., Vol. 131, Pt. D, No 6, Nov. 1984, pp 233-237.
7. Rossi, C., Tonielli, A., (1994), *Robust control of permanent magnet motors: VSS techniques lead to simple hardware implementations*, IEEE Trans. Ind. Electron., Vol. 41, No 4, Aug. , pp 451-460.
8. Chern, T. L., Wu, Y. C., (1992), *An optimal variable structure control with integral compensation for elektrohydraulic position servo control systems*, IEEE Trans. on Ind. Electron., Vol. 39, No 5, Oct. ., pp. 460-463.
9. Bühler, H.: *Réglage par mode de glissement,* Presses Polytechniques Romandes, 1986. Lausanne.
10. Chan, W.-Ch., Hsu, Y.-Y., (1981), *Automatic generation control of interconected power systems using variable structure controllers*, IEE Proc. Vol. 128, Pt. C, No 5, Sept. , pp. 269-279.
11. Aggoune, M. E. et al., (1994), *Design of variable structure voltage regulator using pole assignment technique*, IEEE Trans. on Automatic Control, Vol. 39, No 10**,** Oct. , pp. 2106-2110.

SISTEMI PROMENLJIVE STRUKTURE KVAZI-RELEJNOG TIPA SA PROPORCIONALNO-INTEGRALNIM DELOVANJEM

Čedomir Milosavljević

Razmatraju se tri varijante primene proporcionalno-integralnog delovanja u sistemima promenljive strukture sa kvazi-relejnim zakonom upravljanja. Analiziraju se njihove karakteristike u pogledu uslova nastanka i održanja kliznog režima, tačnosti u stacionarnom stanju, oblika signala upravljanja koji generišu na ulazu objekta upravljanja, njegov uticaj na pojavu kliktanja, kao i mogućnosti rada u uslovima delovanja opterećenja. Daju se primeri koji potvrđuju teorijske zaključke.