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DC MOTOR POSITION CONTROL BY DISCRETE-TIME VARIABLE STRUCTURE CONTROLLERS

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Zoran Jovanović, Goran Golo

University of Niš, Faculty of Electronic Engineering, Niš, Yugoslavia

Abstract: *Discrete-time variable structure systems (DTVSS) with proportional, as well as with proportional-integral control algorithm have been presented. The properties of discrete time variable structure controller with proportional action (DTVSCP) and proportional - integral action (DTVSPI) for step response and external disturbances have been considered. The positional servo system (with DTVSP and DTVSPI) for ramp test signal has also been observed. The controllers were designed on the basis of the complete DC motor model. The object state variables were available for measurement. The design technique is illustrated by a simulated system.*

Keywords: *variable structure controller, PI controller, discrete position servo system, DC motor.*

1. INTRODUCTION

The variable structure controller was the subject of the investigation of great number of researches. For example, see [1-7]. Because of its simple construction, high reliability and other good performances (very fast response without overshoot and insensitivity to external disturbances and parameter variations) the VSC became more interesting for application in automatic control systems. One way to resolve the problem of the position control is the use the VS controller, as the simplest solution. The purpose of this paper is a new control algorithm, which is based on the combination of conventional PI and modern VS controllers. Discrete-time solution of this controller is different from others [4] which were made for continuos-time control. The completely steady state error is eliminated by the new discrete time VS controller with PI action (DTVSCPI).

Two DTVS control algorithms will be analyzed: the one with the proportional (DTVSP) as well as one with the proportional-integral (DTVSPI) action. The controllers will be designed on the basis of the complete DC motor model. The object state variables are available for measurement. The definition of quasi-sliding mode and the necessary conditions for the existence of a quasi-sliding mode for DTVSP and DTVSCPI are given

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in section 3. The conditions for the existence of a quasi -sliding mode for DTVSPI are less strict than those for the existence of a quasi-sliding mode for DTVSP. On the basis of the computer simulation the superiority of the new algorithm regarding the accuracy and load disturbances in steady state has been confirmed. Some recommendation for the design and experimental results obtained by computer simulation of the analyzed controllers are also given.

2. Object Model

The block diagram of the system with controller is shown in Fig. 1.

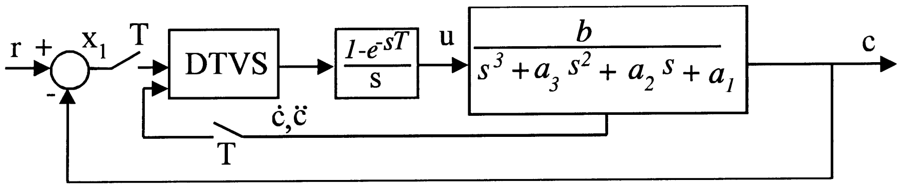

Fig. 1. Main block diagram

The object model in state space is described by:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_1 & -a_2 & -a_3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -b \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} f(t)$$
$$x_1(t) = r(t) - c(t) \tag{1}$$
$$f(t) = \dddot{r}(t) + a_3\ddot{r}(t) + a_2\dot{r}(t) + a_1 r(t)$$

The system (1) can be presented by difference equation [8]:

$$\begin{bmatrix} x_1((k+1)T) \\ x_2((k+1)T) \\ x_3((k+1)T) \end{bmatrix} = \mathbf{E}(T) \begin{bmatrix} x_1(kT) \\ x_2(kT) \\ x_3(kT) \end{bmatrix} + \mathbf{f}(T)u(kT) + \mathbf{m}(T)f(kT) \tag{2}$$

where:

$$\mathbf{E}(T) = \begin{bmatrix} e_{11}(T) & e_{12}(T) & e_{13}(T) \\ e_{21}(T) & e_{22}(T) & e_{23}(T) \\ e_{31}(T) & e_{32}(T) & e_{33}(T) \end{bmatrix}, \tag{3}$$

$$\mathbf{f}(T) = \left(\int_0^T e^{At} dt \right) b = \begin{bmatrix} f_1(T) \\ f_2(T) \\ f_3(T) \end{bmatrix}, \quad \mathbf{m}(T) = \left(\int_0^T e^{At} dt \right) d$$
$$\mathbf{b}^T = [0 \quad 0 \quad -b], \quad \mathbf{d}^T = [0 \quad 0 \quad 1] \tag{4}$$

3. DISCRETE-TIME VARIABLE STRUCTURE CONTROLLER WITH PROPORTIONAL ACTION (DTVSP)

By means of [7], the concept of the quasi-sliding mode can be defined, which is one of the basic motion in DTVS system. The desired state trajectory of a DTVS control should have the following attributes:

[A_1] *Starting from any initial state, the trajectory will move monotonically toward the switching plane and cross it in finite time.*

[A_2] *Any natural number m_o exist, thus:*

$$g(k)g(k+r)<0, \quad 1 \le r \le m_0{}^{*}$$

(*) **Remark:** If m_o=1 than the attribute A_2 reduced on the attribute proposed in [7].

Definition 1: *The motion of DTVS control satisfying attribute is called a quasi-sliding mode.*

Definition 2: *A DTVS control is said to satisfy a reaching condition if the resulting system possesses both attributes* A_1 *and* A_2.

The control, as well as commutation (sliding) plane are given by:

$$\begin{aligned} &u(kT) = \omega|x_1(kT)|\operatorname{sgn}(g(kT)), \\ &g(kT) = c_1 x_1(kT) + c_2 x_2(kT) + x_3(kT), \quad c_1, c_2 > 0, \end{aligned} \tag{5}$$

where $x_1(kT)$, $x_2(kT)$, and $x_3(kT)$, are error signal, the first derivation and the second derivation of the error signal in the sample time, respectively and c_1, c_2 are parameters of commutation (sliding) plane.

The block-diagram of the DTVSP controller is shown in Fig. 2.

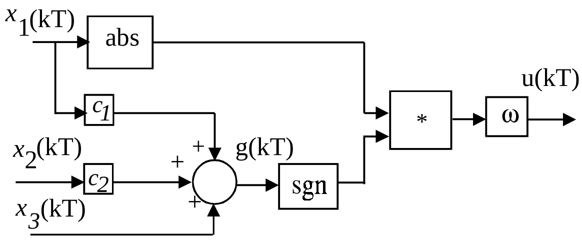

Fig. 2. Block-diagram of the DTVSP controller

The quasi-sliding mode conditions can be obtained from relation [1, 3]:

$$\begin{aligned} &g(kT)\Delta g(kT) < 0 \\ &\Delta g(kT) = g((k+1)T) - g(kT). \end{aligned} \tag{6}$$

By use the system (1) with control (5), the quasi-sliding mode conditions can be presented in following way:

$$c_1e_{12}+c_2e_{22}+e_{32}-c_1c_2e_{13}-c_2{}^2e_{23}-c_2e_{33}=0\,,$$
$$\omega>\left|\frac{c_1e_{11}+c_2e_{21}+e_{31}-c_1{}^2e_{13}-c_1c_2e_{23}-c_1e_{33}}{c_1f_1+c_2f_2+f_3}+\frac{f(kT)}{x_1(kT)}\right| \tag{7}$$

Proposition 1: *Let the system be described by (*1*), the control signal by (*5*), and input signal by* $r(t)=rh(t)$ ($f(t)=a_1r$)*.
a*) Necessary condition for the system to establish quasi-sliding mode is* $a_1=0$.
b*) If* $a_1\neq0$ *steady state is determined as:*

$$x_{1R}=\frac{a_1}{a_1+b\omega}r,\quad x_{2R}=0,\quad x_{3R}=0. \tag{8}$$

(*) **Remark:** Suppose the initial condition is equal zero. Function $f(t)$ has form: $f(t)=a_1r+a_2r\delta+a_3r\delta'+r\delta''$ (δ-Dirachle function) because the step signal is in action. It can be shown that the effect of the signal $f(t)=a_1r+a_2r\delta+a_3r\delta'+r\delta''$, with zero initial conditions is equivalent to the effect of the signal $f(t)=a_1r$ when initial conditions are not equal zero ($x_1(0)=r$, $x_2(0)=0$, $x_3(0)=0$).

Proof:

a) Suppose that $a_1\neq0$, from (7) it can be seen that for sufficiently small $x_1(kT)$ and fixed ω, the system can not stay in quasi-sliding mode. Since this is contradiction, than the initial supposition $a_1\neq0$ is wrong, i.e. $a_1=0$.
b) The value of $x_1(kT)$ for which the system is leaving quasi-sliding mode can be determined from (7), if the "<" is replaced by with "=". On the basis of this consideration, it is obviously that steady state is not in coordinate origin. It will be on x axes and can be determined as:

$$x_{iR}=\lim_{k\to\infty}x_i(kT)$$

Using this relation, a system of equations by which the steady state is described, can be written as:

$$x_R=\mathbf{E}(T)x_R+\mathbf{f}(T)u(\infty)+\mathbf{m}(T)a_1r$$
$$u(\infty)=\omega|x_{1R}|\operatorname{sgn}(g(\infty))$$

Let us suppose that, without loss of generality $r>0$, and $x_{1R}>0$. In this case steady-state is determined by relation (8).

Proposition 2: *Let the system be described by (*1*), the control signal by (*5*), and the input signal by* $r(t)=rth(t)$ ($f(t)=a_1rt+a_2r$ *and* $x_1(0)=0$, $x_2(0)=r$, $x_3(0)=0$).
a) The necessary condition for the system to establish quasi-sliding mode is $a_1=a_2=0$.
b) If $a_1=0$ *and* $a_2\neq0$ *steady-state is determined by*

$$x_{1R}=\frac{a_2}{b\omega}r\,,\quad x_{2R}=0\,,\quad x_{3R}=0\,. \tag{9}$$

c) If $a_1 \neq 0$ the system error gravitates toward the infinite.

$$x_{3R} = 0,$$
$$x_{2R} = \frac{a_1}{a_1 + b\omega}, \tag{10}$$
$$\lim_{k\to\infty} (x_1(kT) - kTx_{2R}) = \frac{a_2 b\omega}{(a_1 + b\omega)^2} r.$$

Proof:

a) Proof is the same as that from Proposition 1 (a).
b) The relations (9) can be obtained in similar way, as in Proposition 1 (b).
c) The solution of difference equations (2) is :

$$\mathbf{x}(kT) = \mathbf{x}_t(kT) + \mathbf{x}_p(kT),$$

where $\mathbf{x}_t$ (kT) is transitional mode vector, but $\mathbf{x}_p(kT)$ is vector due to external referent signal $f(kT)$. If the component of transitional mode disappears when $k\to\infty$, the system state is described by asymptotic values from (10).

For instance, the position servosystem with DTVSP controller can ensure zero steady-state error (existence of system of type 1). But, if positional servosystem with time variable input signal (ramp) is wanted, the error of the position exists. This controller type (DTVSP) can not eliminate error of the position. If type of system in the case of conventional controller is increased, the problems of stability appear. This property is not expression in the case of DTVS controller. On the basis of that fact, the combination of conventional discrete-time PI and discrete time VSS is proposed.

4. Discrete-Time Variable Structure Controller with Proportional-Integral Action (DTVSPI)

Block-diagram for DTVSPI controller is shown in Fig. 3.

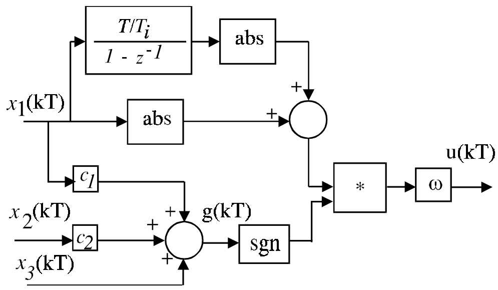

Fig. 3. Block-diagram of the DTVSPI controller

If control is formed in form:

$$u(kT) = \omega\left[|x_1(kT)| + \frac{T}{T_i}\sum_{i=0}^{k-1}|x_1(iT)|\right]\operatorname{sgn}(g(kT)) \tag{11}$$

the quasi-sliding regime appears, if relation

$$c_1e_{12}+c_2e_{22}+e_{32}-c_1c_2e_{13}-c_2^2e_{23}-c_2e_{33}=0,$$
$$\omega > \frac{1}{1+\frac{T}{T_i}\sum_{i=0}^{k-1}\left|\frac{x_1(iT)}{x_1(kT)}\right|}, \tag{12}$$
$$\left|\frac{c_1e_{12}+c_2e_{22}+e_{32}-c_1^2e_{13}-c_1c_2e_{23}-c_1e_{33}}{c_1f_1+c_2f_2+f_3}+\frac{f(kT)}{|x_1(kT)|}\right|,$$

is satisfied.

Proposition 3: *Let the system be described by* (1)*, the control signal by* (11)*, and the input signal by* $r(t)=rh(t)$ ($f(t)=a_1r$ *and* $x_1(0)=r$, $x_2(0)=0$, $x_3(0)=0$)*, steady state is in the coordinate origin.*

Proof : Let us suppose that steady state is not in the coordinate origin, but the point on axes x_1 (it means that $x_{1R}\neq 0$, without loss of generality, it can be proposed that $x_{1R}>0$). As

$$\lim_{k\to\infty} x_1(kT)=x_{1R}\ ,$$

then for each $\varepsilon>0$ and $\varepsilon<x_{1R}$, the value of $N_o=N_o(\varepsilon)$ ($-\varepsilon+x_{1R}<x_1(kT)<\varepsilon+x_{1R}$ for each $k>N_o$) exists. Let us consider the first equation of system (2) with $u(kT)$ given by (11).

$$\begin{aligned} x_1((k+1)T) &= e_{11}(T)x_1(kT)+e_{12}(T)x_2(kT)+e_{13}(T)x_3(kT)- \\ &- a_1rm_1(T)-\omega x_1(kT)\operatorname{sgn}(g(kT))f_1(T)- \\ &-\omega\frac{T}{T_i}\left|\sum_{i=0}^{k-1}x_1(iT)\right|\operatorname{sgn}(g(kT))f_1(T), \end{aligned} \tag{13}$$

as well as the same one on the right side

$$\begin{aligned} \left|\sum_{i=0}^{k-1}x_1(iT)\right| &= \left|\sum_{i=0}^{N_0}x_1(iT)+\sum_{i=N_0+1}^{k-1}x_1(iT)\right| > \left|\sum_{i=0}^{N_0}x_1(iT)+\sum_{i=N_0+1}^{k-1}x_1(x_{1R}-\varepsilon)\right| = \\ &= \left|\sum_{i=0}^{N_0}x_1(iT)+(x_{1R}-\varepsilon)(k-N_0-1)\right| = \left|\gamma(N_0,x_1(0))+(x_{1R}-\varepsilon)(k-N_0-1)\right|, \end{aligned}$$

if $k\to\propto$, the relation (13) is

$$\begin{aligned} x_{1R} &= e_{11}(T)x_{1R}-a_1rm_1(T)-\omega\left(x_{1R}+\lim_{k\to\infty}\frac{T}{T_1}\left|\sum_{i=0}^{k-1}x_1(iT)\right|\right)f_1(T) \\ x_{1R} &< e_{11}(T)x_{1R}-a_1rm_1(T)-\omega\left(x_{1R}+\lim_{k\to\infty}\frac{T}{T_1}\left|\gamma(N_0,x_1(0))+(x_{1R}-\varepsilon)(k-N_0-1)\right|\right)f_1(T). \end{aligned} \tag{14}$$

On the basis of relation (14) it can be seen that all terms except the last one, are finite. The last term gravitates toward $\propto$, because of that last relation is not valid (finite can not be infinite), i.e. x_{1R} must be *0*. In this way the Proposition 3 is proved.

Proposition 4: *Let the system be described by* (1)*, the control signal by* (11)*, and the input signal by* $r(t)=rth(t)$ ($f(t)=a_1rt+a_2r$)*.*

a) The necessary condition for the system to establish quasi-sliding mode is $a_1=0$.

b) If $a_1 \neq 0$, the system is not in quasi-sliding mode, i.e. steady-state is determined by

$$\begin{aligned} x_{1R} &= \frac{T_i a_1 r}{b}, \\ x_{2R} &= 0, \\ x_{3R} &= 0. \end{aligned} \tag{15}$$

Proof:

a) The following result can be used to prove the statement.

Cauchye Theorem 1: Let us consider the series $w_i, i=0, \infty$, that convergates to $W \in \Re$. Then:

$$\lim_{k\to\infty} \frac{\sum_{i=0}^{k} w_i}{k} = W$$

The proof of Theorem 1 can be found in [9].
Suppose that $a_1 \neq 0$, but the system is in quasi-sliding mode. Let us extract the following term from relation (12).

$$\lambda(k) = \frac{1}{1 + \frac{T}{T_i}\left|\sum_{i=0}^{k-1} \frac{x_1(iT)}{x_1(kT)}\right|} \frac{f(kT)}{|x_1(kT)|} = \frac{a_1 kT + a_2}{|x_1(kT)| + \frac{T}{T_i}\left|\sum_{i=0}^{k-1} x_1(iT)\right|} r = \frac{a_1 T + \frac{a_2}{k}}{\frac{|x_1(kT)|}{k} + \frac{T}{T_i}\frac{1}{k}\left|\sum_{i=0}^{k-1} x_1(iT)\right|} r. \tag{16}$$

Because $x_1(kT) \to 0$ for $k \to \propto$, than on the bases of Theorem 1:

$$\lim_{k\to\infty} \frac{1}{k}\left|\sum_{i=0}^{k-1} x_1(iT)\right| = x_{1R} = 0. \tag{17}$$

On the bases of relation (17) one can conclude that $\lambda(k) \to \propto$ when $k \to \propto$. That mean that for fixed ω, K_o exist, that is for each $k > K_o$ relation (12) is not fulfilled, i.e. the system is not in quasi-sliding mode which is contradictory to the initial statement.
b) The steady-state can be determined oh the following way. The relation for $x_1((k+1)T)$ must be divided with k ($k \to \infty$). Then the relation (15) can be obtained on the basis Theorem 1.

The conditions for the existence of a quasi-sliding mode for DTVSPI are less strict than those for the existence of a quasi-sliding mode for DTVSP. From (12), it can be concluded that the less gain is necessary for DTVSPI upon quasi-sliding mode conditions.

5. The Result of Design and Digital Simulation

Positional servosystems with DC motor are widely used in industry application (valve control, numeric control tools, radio locators, satellite antennae). These systems have been widely studied.

It is well known that the transfer function of the DC motor with separate excitation, in related to position, is described by [6]:

$$W_\theta(s) = \frac{K_1}{s(T_m T_r s^2 + T_m s + 1)},$$

where: $K_1 = \frac{1}{k_{me}}$, $T_m = \frac{R_r J}{k_{me} k_{em}}$, $T_r = \frac{L_r}{R_r}$,

R_r, L_r - total resistance and inductance in rotor circuit of DC motor,

k_{me}, k_{em} - constants of DC motor, J - rotor inertia of DC motor.

In the practice $T_m >> T_r$, according this fact, T_r can be ignored. Thus, the model of DC motor is simple. Meanwhile, in this paper, T_r is not ignored, i.e. the controller is designed using complete model of DC motor. System model in state space is described by (1), where: $a_1=0$, $a_2=k_{me}k_{em}/JL_r$, $a_s=R_r/L_r$, $b=k_{em}/JL_r$.

DC motor parameters:

k_{em}=0.33Vs/rad, k_{me}=0.33Nm/A, w_{max}=4000min^{-1}, R_r=4.2Ω,
L_r=14.28mH, J=0.000711kgm^2.

On the basis of the relation (7) and (9), DTVS parameter is chosen:
T =0.001s, c_1 =12000, c_2 = 200, w =20 and T_i =0.0001s. Ramp signal with slope a=2 is ordered. Results of digital simulation are shown in Fig. 4-8 (Appendix).

For both DTVSP and DTVSPI controllers, the step response is shown in Fig. 4. External disturbances p=0.3, on the positional servosystem in t=0.4 s are performed. The results of digital simulation confirm the superiority of the new controller with regard to the fast response without overshoot, zero steady state error, as well as external disturbances insensitivity.

6. Conclusion

The variable structure with quasi-sliding mode has been considered. Its application to the problem of positional servosystem position has also been illustrated. Two types of the DTVS have been observed. In contrast to DTVSP, DTVSPI zero error of the positional servosystem has been ensured. New control algorithm has better properties with regard to the accuracy and external disturbances in steady state. On the basis of given analysis, as well as the results of digital simulation, it can be concluded that DTVSPI control algorithm gives better performance than DTVSP control algorithm.

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APPENDIX

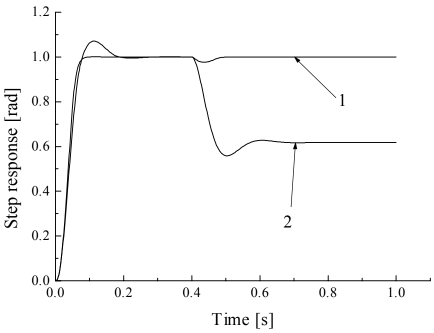

Fig. 4. Step response for 1) DTVSPI and 2) DTVSP

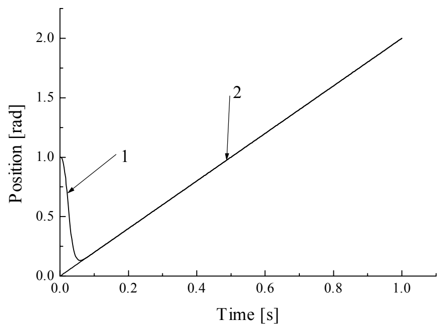

Fig. 5. Ramp response for 1) DTVSPI and 2) Reference signal.

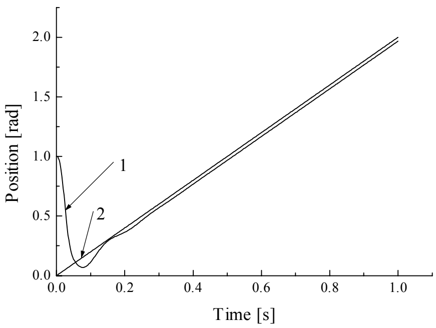

Fig. 6. Ramp response for 1) DTVSP and 2) Reference signal.

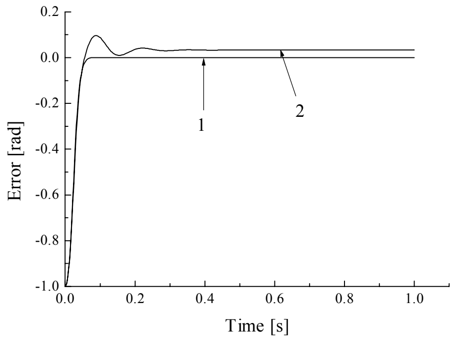

Fig. 7. Positioning error for ramp response 1) DTVSPI and 2)DTVSP.

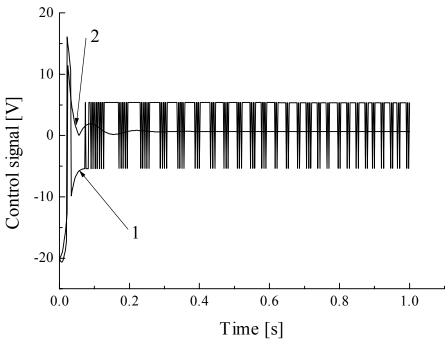

Fig. 8. Control for 1) DTVSPI and 2) DTVSCP.

DISKRETNI REGULATORI PROMENLJIVE STRUKTURE U UPRAVLJANJU POZICIJOM JEDNOSMERNOG MOTORA

Zoran Jovanović, Goran Golo

U radu su prikazani diskretni regulatori promenljive strukture sa proporcionalnim (DTVSCP) i proporcionalno-integralnim dejstvom (DTVSCPI). Razmatrane su osobine DTVSCP i DTVSCPI za odskočni odziv i spoljašnje poremećaje. Takodje, pozicioni servosistem (sa DTVSCP i DTVSCPI) se posmatra za nagibni ulazni signal. Diskretni regulatori su projektovani na osnovu potpunog modela motora jednosmerne struje. Sve promenljive stanja objekta bile su dostupne za merenje.