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POSITION CONTROL OF AN ELASTIC TWO-MASS DRIVING SYSTEM WITH BACKLASH AND FRICTION, USING A SLIDING MODE CONTROLLER

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Abstract. *The paper presents a sliding mode control structure for position control of an elastic two-mass system driven by a DC motor. The paper deals with the mathematical model of the controlled plant and with elements of design and digital simulation.*

1. INTRODUCTION

Electric driving systems with elastic coupling are widely used in various fields: robots, machine-tools, long shaft driving systems, s.o. The dynamic performances of speed and position controlled multi - mass driving system can be deteriorated especially due to the elastic coupling, non -linear friction and backlash. This is the reason why they look for new solutions for their position control.

Friction is an important aspect in many control systems, that can lead to tracking errors, limit cycles and undesired stick - slip motion. Most of the existing models use classical friction models, such as Coulomb friction and viscous friction; in applications with low velocity tracking, the classical model is not always satisfactory. Different authors [3] introduce new friction models, that complete Coulomb and viscous friction's with Striebeck effect.

Backlash appears as a characteristic with linear branches and dead band. Often backlash can be kept small by constructive measures but increase with abrasion.

In the paper are presented the (simplify) linear and non - linear mathematical models of an elastic two - mass system with friction and backlash. Based on the simplified linear model of the controlled plant a sliding mode control structure is developed. The performances ensured by the developed control system is analyzed by digital simulation, from which conclusion and recommendations are given.

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2. MATHEMATICAL MODELS

Linear model: The following simplified state space model of the elastic two - mass system driven by separately excited DC motor is obtained [1], when only viscous friction are taking into account, backlash is neglected and motor flux is kept constant:

$$\frac{\mathrm{d}i_a}{\mathrm{d}t} = -\frac{1}{T_a} i_a - \frac{k_e}{R_a T_a}\omega_m + \frac{1}{R_a T_a} u_a \tag{1}$$

$$\frac{\mathrm{d}\omega_m}{\mathrm{d}t} = -\frac{k_m}{T_m} i_a - \frac{\alpha_a}{T_m}\omega_m + \frac{\alpha_a}{T_m}\omega_1 - \frac{k_a}{T_m}\theta_m + \frac{k_a}{T_m}\theta_1 \tag{2}$$

$$\frac{\mathrm{d}\omega_1}{\mathrm{d}t} = \frac{\alpha_a}{T_1}\omega_m - \frac{\alpha_a}{T_1}\omega_1 + \frac{k_a}{T_1}\theta_m - \frac{k_a}{T_1}\theta_1 - \frac{1}{T_1} m_1 \tag{3}$$

$$\frac{\mathrm{d}\theta_m}{\mathrm{d}t} = \frac{1}{T_m}\omega_m \tag{4}$$

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}t} = \frac{1}{T_1}\omega_1 \tag{5}$$

The parameters from relations (1)-(5) have the following meaning [1], [2]: ω_m, ω_1 -the angular speed of DC motor and load, respectively; θ_m, θ_1 - the (angular) position of DC motor and load, respectively; -i_a, U_a armature current and voltage; m_1 - load torque; T_m [sec], T_1[sec] - mechanical time constant of DC motor and load; T_θ [sec] - time constant for the connection between angular position and speed; T_a [sec] - electrical time constant of DC motor; k_a - damping constant of the shaft; α_a - stiffens constant of the elastic coupling; R_a, L_a - total resistance and inductance of armature circuit; k_e - EMF constant; k_m - electromagnetic torque constant.

Non-linear model: A more detailed model, including Coulomb friction and backlash [4], has been used for system simulation. For simplicity Coulomb friction, acting on motor and load has been used:

$$m_{fi} = \begin{cases} -M_{f0i}\,\mathrm{sgn}(\omega_i), & \omega_i \neq 0 \\ -m_i, \omega_i = 0 \wedge |m_i| < M_{fi0} & \\ -M_{f0i}\,\mathrm{sgn}(m_i), & \omega_i = 0 \wedge |m_i| > M_{fi0} \end{cases} \qquad i = m, l$$

where M_{fi0} is magnitude of friction torque and m_i is total torque acting on the shaft. Generally speaking, there are three types of friction characteristic [2] (*a*, *b*, *c*, in fig.1); *a* - the idealized characteristic; *b* - is rolling bearing characteristic; *c* - is the worst case, because its negative slope can induce stick - slip motion. Other authors [3] use a more complicated friction model (necessary in applications with high precision), including Striebeck effect:

$$m_{f_i}(\omega_i) = m_i + (m_s - m_i)e^{-\left(\frac{\omega_i}{\omega_s}\right)}$$

Backlash is modeled as a linear characteristic with dead band:

$$m_t = \begin{cases} \alpha_a(\omega_m - \omega_1) + k_a\Delta\theta, & |\Delta\theta| > \varepsilon \\ \alpha_a(\omega_m - \omega_1), & |\Delta\theta| < \varepsilon \end{cases}$$

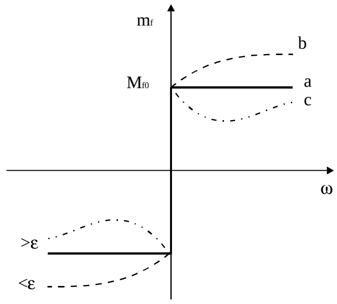

Fig. 1. Friction force

The block diagram of the controlled plant is presented in Fig. 2.

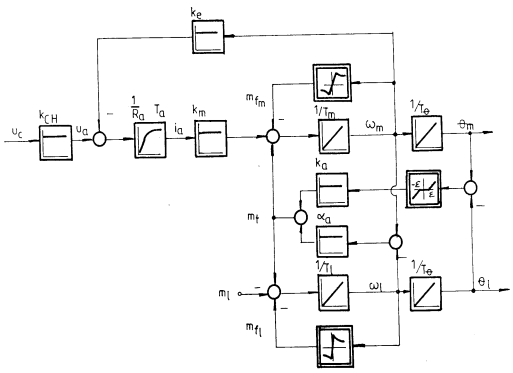

Fig. 2. Controlled plant block diagram

3. SLIDING MODE

Sliding mode control structures are interesting for driving system position control because their reduced sensitivity to controlled plant parameter modifications. The block diagram of the sliding mode position controller for the elastic two - mass system is presented in Fig. 3, in which: CB - state compensation block, ECB - error compensation block. Sliding mode control structure extended with ECB will ensure zero steady - state control error.

The control law is [1]:

$$u_c = U_o \text{sgn(s)}, \qquad U_o - const. > 0 \tag{6}$$

in which s is the switching variable of partial state feedback - type (it is an original approach to this application and it is justified as follows):

$$s = -k_c^T x + k_w w, \tag{7}$$

where $x = [i_a \quad \omega_m \quad \theta_m \quad x_R]^T$ is the new state vector and $k_c^T = [k_i \quad k_\omega \quad k_\theta \quad -k_R]$ is the partial state compensation block which has to be derived.

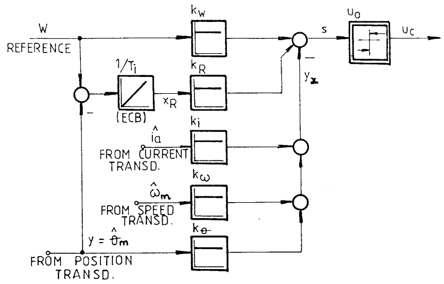

Fig. 3 Informational block diagram of the proposed sliding mode controller.

This partial state compensation block can be designed by different ways; the equivalent control method is used here [6], which is based on the fact that ideal sliding regime yields $\dot{s} = 0$. By differentiating s from relation (7) the equivalent control will be obtained. The equivalent control is substituted in the reduced state space model (1)-(5), result being the sliding mode state model of reduced system extended with ZEC (zero error controller):

$$\begin{aligned} \dot{x} &= A_{sm}x + b_{wsm}w + b_{\dot{w}sm}\dot{w} + b_{vsm}v \\ \theta_m &= c^T x \end{aligned} \tag{8}$$

$$A_{sm} = \begin{bmatrix} -\dfrac{k_m k_\omega}{k_i T_m} & -\dfrac{k_\theta}{T_\theta k_i} & -\dfrac{k_R}{T_i k_i} & 0 \\ \dfrac{k_m}{T_m} & 0 & 0 & 0 \\ 0 & \dfrac{1}{T_\theta} & 0 & 0 \\ 0 & 0 & -\dfrac{1}{T_i} & 0 \end{bmatrix}, \quad b_{wsm} = \begin{bmatrix} \dfrac{k_R}{k_i T_i} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad b_{vsm} = \begin{bmatrix} -\dfrac{k_R}{k_i J_a} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$c^T = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \quad b_{\dot{w}sm} = 0$$

The system has pole in the origin p_1=0, the others are chosen according to the following relation:

$$p_2 = -p, p_{3,4} = (-1 + j)p, \quad p > 0 \tag{9}$$

Identification the coefficients of the characteristic polynomial

$$\Delta(s) = \det(sI - A_m) = (s - p_1)(s - p_2)(s - p_3)(s - p_4)$$

the following expressions for state compensation block constants will result:

$$\begin{aligned} k_\omega &= 3pk_i \frac{T_m'}{k_m} \\ k_\theta &= 4p^2 k_i \frac{T_m' T_\theta}{k_m} \\ \frac{k_R}{T_i} &= 2p^3 k_i \frac{T_m' T_\theta}{k_m} \end{aligned} \tag{10}$$

where p is parameter that influences the control system settling time, and ($T_m' = T_m + T_1$).

There is a degree of freedom in the parameter design (k_i coefficient will be choice). Parameter k_θ is obtained from steady - state conditions, as:

$$w_\infty = \theta_{m\infty} .$$

Hence:

$$k_w = k_\theta \tag{11}$$

For the analysis of system stability and of its sliding motion on the switching hyperplane $\dot{s} = 0$, the following conditions have to be fulfilled:

- the condition for reaching the switching surfaces;
- the condition for sliding mode existence;
- the stability of sliding motion.

The domain of the state space for the stability of sliding motion is obtained from condition $-U_o < u_{eq} < U_o$:

$$U_o < \left| (1 - 3pT_a) R_a i_a + (k_e - 4p^2 R_a T_a T_m '/ km)\omega_m + 2p^3 R_a T_a T_m ' T_\theta (w - \theta_m) / k_m \right| \tag{12}$$

4. Application Example and Digital Simulation Results

In order to analyze the performances achieved by the developed control structure an example of DC drive was taken with the following technical data:
$R_a = 0.075\Omega, k_e = k_m = 1, k_a = 7.5, \alpha_a = 5.9, T_a = 43ms, T_a = 450ms, T_m = 300ms.$
The following values for the design parameters are chosen: $p = 9,\ U_o = 1,\ k_i = 1$. Taking into account the resulted design relations (10)-(12), the following parameters were obtained: $k_\omega = 20.25, k_\theta = 6.56, \dfrac{k_R}{T_i} = 29.25$. The digital simulation results corresponding to the position (of motor and load) response with respect to a limited ramp variation of reference (w) are presented in Fig. 4 (APPENDIX):

a) for the proposed sliding mode control system;
b) for a state feedback control system;
c) for a cascade control system designed according to Kessler [1], [7], [8].

5. Conclusion

The proposed sliding mode control structure ensures the best performance in comparison with other classical control structures in the case of an elastic two mass system. However, the author consider that some tuning problems (due to the nonlinear character of the plant) could appear when the implementation is going to be performed.

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APPENDIX

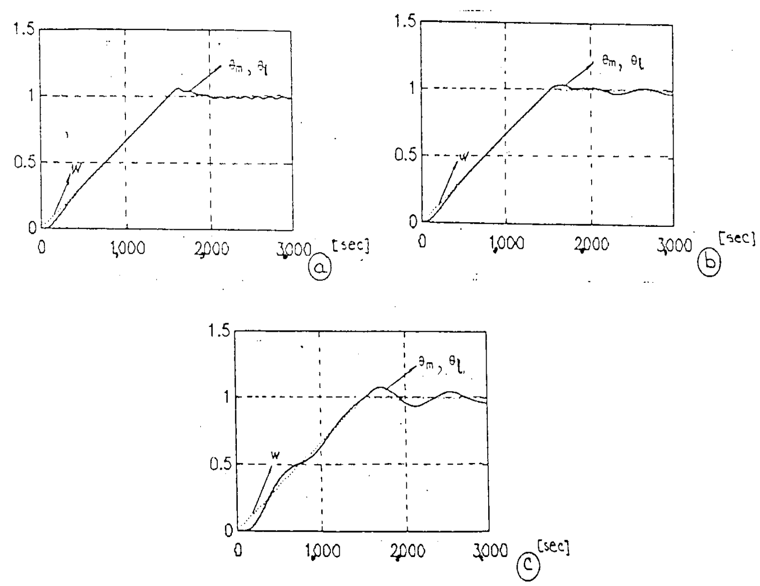

Fig. 4. Control system response with respect to the limited ramp modification of w for the proposed sliding mode control system (a), a state feedback control system (b), and a cascade control system (c).

PRIMENA KLIZNIH REŽIMA U UPRAVLJANJU POZICIJE MOTORNOG POGONA SA DVE MASE, TRENJEM I MRTVOM ZONOM

Angela Porumb

U ovom radu prezentovana je primena kliznih režima u upravljanju pozicije pogona sa dve mase pokretanog DC motorom. Rad sadrži matematički model upravljanog objekta, postupak sinteze upravljačkog signala i digitalnu simulaciju.