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LATERAL BUCKLING ANALYSIS OF I-BEAMS SUBJECTED TO FOLLOWER LOADING

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G. I. Ioannidis

Department of Civil Engineering, National Technical University of Athens,
42 Patission Street, Athens 106 82, Greece

Abstract. *In the present paper the lateral buckling of I-beams under follower loading is examined. A simply supported beam, subjected to a concentrated load at the midspan is used as a model. The deformed configuration (i.e. after buckling) is governed by a system of differential equations with non-constant coefficients which is solved by an approximate analytical technique. A formula is established for determining the critical moment of lateral buckling. Moreover, the boundary between existence and non-existence of adjacent equilibrium is established. The results obtained are compared with those of a conservative load (i.e. remaining vertical after buckling).*

1. INTRODUCTION

In the present work, the lateral buckling response of a simply supported I-beam, subjected to a midspan concentrated follower load is thoroughly discussed. Assuming that the loss of stability occurs through divergence, we consider the equilibrium in a slightly bent configuration in which vertical and lateral deflections as well as angles of twist are developed. This state of equilibrium is described by a system of three differential equations [1,2] with non-constant coefficients. Clearly, a closed form solution of the above system cannot be, in general, obtained. Therefore, one has to resort to approximate analytical solutions. Hence, an analytic approximate technique for solving the above system of differential equations is successfully employed.

The determination of the critical (elastic) level of loading is not only of theoretical, but also of practical importance, since in many current design codes, the design resistance of a beam against lateral buckling, even in the case of inelastic buckling, is based on the corresponding value of elastic buckling.

In many recent publications, concerning problems related to lateral buckling, results are obtained applying approximate procedures [3,4,5,6] as well as finite element [7] or numerical [8] methods. Approximate shape functions are also used, for establishing the

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postbuckling behaviour in cases of lateral [9] (bending without axial force) or lateral-torsional [10,11] (bending and axial compression) buckling.

Reviewing the present state of the art a pertinent work which is worth mentioning is that by Kaschke [12]· This work deals with the lateral buckling of beams on which a concentrated load was applied through a bar subjected to a compressive force of constant direction. Kaschke discussed the cases of a cantilever with a tip force and of a simply supported beam with a force at the midspan using a cumbersome series solution technique. For the same cases of loading Ings and Trahair [13] studied the problem of lateral buckling of beams under "directed" loading with the aid of finite element computer programs.

Non-conservative problems are of paramount importance in modern structural design. Examples range from Takoma Narrows bridge wind-induced collapse to shell-type ovalling oscillations of thin-walled metal chimneys under high wind, to fluid-structure interactions to offshore structures, etc.

The present paper deals with the lateral buckling response of a non-conservative system due to the type of the follower load (associated with a non-conservative parameter n), with the following objectives: (a) to define the limit value of the non-conservatives parameter n (boundary between divergence and flutter instability) for which the beam loses its lateral stability through divergence (static instability) and (b) to present a comprehensive and easily applied technique for solving the above problem.

2. Basic equations

Consider the simply supported beam, shown in Fig la, of length l, having an I section with two planes of symmetry and subjected to a concentrated load P acting in the middle of the span at the center of a section. We assume that this load form, after buckling, an angle $\eta\varphi_0$ ($0 \leq \eta \leq 1$) with the principal axis yy of the cross section (φ_0 is the angle of twist in the midsection of the beam) as it is shown in Fig lb. The case η=1 corresponds to a load which maintains, after buckling, its initial direction, the case η=0 corresponds to a load which, after buckling, remains perpendicular to the flanges of the cross section. We assume also, that the ends of the beam are laterally supported, cannot rotate about z-axis but they are free to warp. The critical value of loading can be obtained from the differential equations of equilibrium in the deformed configuration.

We consider a fixed system of axes x, y, z and a local system ξ, η, ζ (Fig 1) at the center of an arbitrary section αα [1]. The axes ξ and η are the principal axes of the section and ζ is the direction of the tangent to the deflected axis of the beam. The deformation of the beam is defined by the components w and u of the displacement of the center in the x and y directions and by the angle of rotation φ. Components w and u are taken positive in the positive directions of the corresponding axes and the angle φ is taken positive about the z axis according to the right-hand rule of signs.

$$\left.\begin{aligned} M_x &= \frac{1}{2}Pz \\ M_y &= \frac{1}{2}Pz\varphi_0(1-\eta) \\ M_z &= \frac{1}{2}P(u-u_0)+\frac{1}{2}P\varphi_0(1-\eta)(w-w_0) \end{aligned}\right\} \qquad (1)$$

where u_0 ,w_0 the lateral and vertical deflections at the middle-span of the beam.

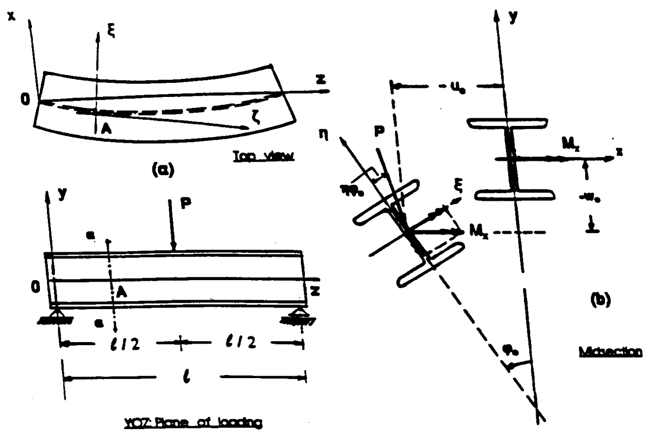

Fig. 1. Lateral buckling of a beam subjected to a concentrated follower load.

Projecting the above moments according to the directions of axes ξ, η, ζ, and assuming the quantities w, u and φ as very small we could have:

$$\left.\begin{aligned} M_\xi &= \frac{1}{2}Pz \\ M_\eta &= \frac{1}{2}Pz\varphi - \frac{1}{2}Pz\varphi_0(1-\eta) \\ M_\varsigma &= \frac{1}{2}P(u-u_0) - \frac{1}{2}Pz\frac{du}{dz} \end{aligned}\right\} \tag{2}$$

The differential equations of equilibrium in the deformed state, taking into account that the curvatures of the deflected axis of the beam in the xz and yz planes can be taken as d^2u/dz^2 and d^2w/dz^2 respectively, as the deflections are assumed small, are

$$\left.\begin{aligned} EI_\xi \frac{d^2w}{dz^2} &= M_\xi \\ EI_\eta \frac{d^2u}{dz^2} &= M_\eta \\ GJ\frac{d\varphi}{dz} - EC_w\frac{d^3\varphi}{dz^3} &= M\varsigma \end{aligned}\right\} \tag{3}$$

(E is the Young's modulus, GJ is the torsional rigidity and EC_w is the warping rigidity of the beam) and in combination with eqs. (2) ($I_\xi = I_x$, $I_\eta = I_y$)

$$\left.\begin{aligned} EI_x\frac{d^2w}{dz^2} &= \frac{1}{2}Pz \\ EI_y\frac{d^2u}{dz^2} &= \frac{1}{2}Pz\varphi - \frac{1}{2}Pz\varphi_0(1-\eta) \\ GJ\frac{d\varphi}{dz} - EC_w\frac{d^3\varphi}{dz^3} &= \frac{1}{2}P(u-u_0) - \frac{1}{2}Pz\frac{du}{dz} \end{aligned}\right\} \tag{4}$$

Eliminating u from the second and the third of eqs (4) we obtain the following differential equation of equilibrium of the beam in the deformed state

$$GJ\varphi'' - EC_w\varphi'''' = \frac{1}{4}\frac{P^2z^2}{EI_y}\varphi + \frac{1}{4}\frac{P^2z^2}{EI_y}\varphi_0(1-\eta) \tag{5}$$

(where the prime denotes differentiation with respect to z). This equation can be also written in non dimensional form as

$$\varphi'''' - \beta^2\varphi'' = 4\overline{M}_0{}^2\xi^2\varphi - 4\overline{M}_0{}^2\xi^2\varphi_0(1-\eta) \tag{6}$$

where

$$\left.\begin{array}{ll} \xi = z/l, & \beta^2 = GJl^2/EC_w \\ M_0 = Pl/4, & \overline{M}_0{}^2 = M_0{}^2l^4/EI_yEC_w \end{array}\right\} \tag{7}$$

Eq (6) is related with the following boundary conditions

$$\left.\begin{array}{l} \varphi(0) = \varphi''(0) = 0 \\ \varphi'(1/2) = 0 \\ u'(1/2) = 0 \end{array}\right\} \tag{8}$$

The last of the above conditions can be replaced by the condition

$$GJ\varphi'(1/2) - EC_w\varphi'''(1/2) = 0 \tag{9}$$

as one can conclude from the last of eqs (4).

3. Application of the approximate method

It is assumed that the variation of the angle of twist along the z-axis of the beam is given by a function of the form

$$\varphi(\xi) = \alpha_4\xi^4 + \alpha_3\xi^3 + \alpha_2\xi^2 + \alpha_1\xi + \alpha_0 \tag{10}$$

where α_0, α_1, α_2, α_3, α_4 are constant coefficients. According to the boundary conditions (8), (9), eq (10) can be written as

$$\varphi(\xi) = \frac{16}{5}\varphi_0(\xi^4 - 2\xi^3 + \xi) \tag{11}$$

(φ_0 is the angle of twist at the midsection).

Introducing eq (11) into the second term of differential equation (6) we can take after two integration

$$\begin{aligned} \varphi'' = {} & \beta^2\varphi + \frac{64}{5}\overline{M}_0{}^2\varphi_0\left(\frac{\xi^8}{56} - \frac{\xi^7}{21} + \frac{\xi^5}{20}\right) - \frac{1}{3}\overline{M}_0{}^2\varphi_0(1-\eta)\xi^4 + \\ & + \frac{1}{6}\overline{M}_0{}^2\varphi_0(1-\eta)\xi - \frac{31}{210}\overline{M}_0\varphi_0\xi \end{aligned} \tag{12}$$

The constants of integration are determined with the aid of boundary conditions (8).

Using, in the second term of eq (12) the expression (11) for the angle of twist, we could have, after two further integration, the more precise function

$$\varphi = \frac{8}{5}\varphi_0\beta^2\left(\frac{\xi^6}{15} - \frac{\xi^5}{5} + \frac{\xi^3}{3} - \frac{\xi}{5}\right) + \frac{8}{105}\left(\frac{\xi^{10}}{30} - \frac{\xi^9}{9} + \frac{\xi^7}{5}\right) - \frac{31}{1260}\overline{M}_0{}^2\varphi_0\xi^3 + \\ + \frac{1717}{100800}\overline{M}_0{}^2\varphi_0\xi - \frac{1}{2}\overline{M}_0{}^2\varphi_0(1-\eta) \\ \left(\frac{\xi^6}{45} - \frac{\xi^3}{18} + \frac{3\xi}{80}\right) = 0 \tag{13}$$

Applying eq (13) for ξ = 1/2, in this case $\varphi = \varphi_0$, we obtain, for the critical value of the dimensionless bending moment $\overline{M}_0$ the expression

$$\overline{M}_{0,cr} = \frac{12.83}{\sqrt{\eta - 0.087}}\sqrt{1 + \frac{61}{600}\beta^2} \tag{14}$$

or, setting $61/600 = 1/\pi^2$,

$$\overline{M}_{0,cr} = \frac{12.83}{\sqrt{\eta - 0.087}}\sqrt{1 + \frac{\beta^2}{\pi^2}} \tag{15}$$

The critical moment of lateral bucking of equation (15) can be compared to the critical moment corresponding to the usual case of a concentrated load, which remains vertical after bucking. This critical moment can be determined by the formula [14].

$$M_{cr} = \frac{4.24}{l}\sqrt{EI_y}GJ\sqrt{1 + \frac{\pi^2 EC_w}{GJl^2}} \tag{16}$$

which, using relations (7), can be written

$$M_{cr} = 13.32\sqrt{1 + \frac{\beta^2}{\pi^2}} \tag{17}$$

It is possible to obtain the same expression, applying equation (15) for η=1. (In this case, the coefficient 13.43 instead of 13.32 is obtained).

As it was expected, according to relation (15), greater values for the critical moment of lateral buckling compared with values obtained for eq (17) are determined. For instance, for η=0.50 the value, corresponding to the follower load, is 1.50 times the value related with the vertical load. For η=0.10 the relation is 8.50 times.

Combining eqs (15) and (17) one can establish the following relation (18) between the dimensionless critical moments of lateral buckling corresponding to the follower ($\overline{M}_{cr}{}^{foll}$) and the remaining vertical, after buckling, ($\overline{M}_{cr}{}^{vert}$) load

$$\overline{M}_{cr}{}^{foll} = \frac{0.968}{\sqrt{\eta / 0.087}}\overline{M}_{cr}{}^{vert} \tag{18}$$

In Fig. 2 the variation of the ratio $\overline{M}_{cr}{}^{foll} / \overline{M}_{cr}{}^{vert}$ for different values of the parameter η is represented.

It is clear that if the approximate technique is repeated, using as initial function for the angle of twist eq (13) instead of eq (11), one could obtain for this angle a more precise formula. It is, however, ascertained above, as well as for many usual loading cases [12] that the first cycle gives results with sufficient accuracy.

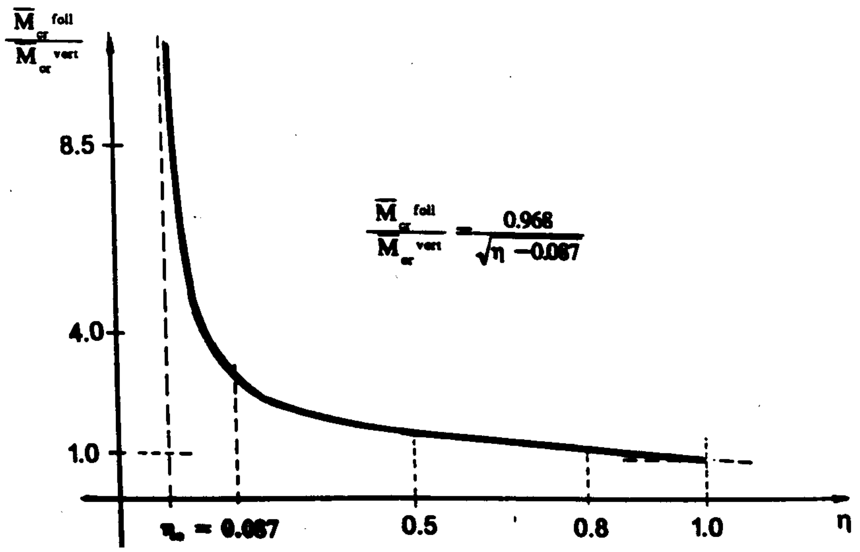

Fig. 2. Variation of the ratio $\overline{M}_{cr}^{foll} / \overline{M}_{cr}^{vert}$ vs η

For $\eta < 0.087$ the above analysis shows that the only possible form of equilibrium of the I-beam is the undeformed one. This is so, because for $\eta < 0.087$ the beam is governed by dynamic (flutter) instability which can be established by using only the dynamic stability analysis. Hence, the boundary between static (lateral) and dynamic (flutter) instability corresponds to $\eta_0 = 0.087$. Clearly, a lateral static buckling analysis is possible for $\eta > \eta_0 = 0.087$.

4. Conclusions

The most important findings based on the model chosen are the following:

(a) A simple and efficient analytic approximate technique is applied for determining the critical moment for lateral buckling in the case of a beam under a concentrated follower load.
(b) Formulas for the critical moments of lateral buckling corresponding to the cases of a follower load and a conservative concentrated load are properly established.
(c) The boundary between divergence and flutter instability related to a certain value of the nonconservativness parameter η is explicitly determined.
(d) In the area of static instability the critical moment of lateral buckling in the case of the follower load is up to 8.50 times greater than the corresponding value of the conservative load.

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BOČNO IZVIJANJE I PROFILA IZLOŽENIH PRATEĆEM OPTEREĆENJU

G. I. Ioannidis

U predloženom radu ispitano je bočno izvijanje I profila pod pratećim opterećenjem. Kao model je korišćena prosta greda sa osloncima izložena koncentrisanom opterećenju na sredini raspona. Deformisana konfiguracija (tj. nakon izvijanja) predstavljena je sistemom diferencijalnih jednačina sa promenljivim koeficijentima koji je rešen približnom analitičkom metodom. Postavljena je formula za odredjivanje kritičnog momenta bočnog izvijanja. Sem toga ustanovljena je granica izmedju postojanja i nepostojanja granične ravnoteže. Dobijeni rezultati su uporedjeni sa rezultatima konzervativnog opterećenja (tj. zadržavanje vertikalnog položaja nakon izvijanja).