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PLANE THERMOELASTIC WAVES IN INFINITE HALF-SPACE CAUSED BY INSTANTANEOUS PRESSURE FORCES ON ITS SURFACE

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Abstract. *The propagation of longitudinal thermoelastic waves in a half-space caused by mechanical influences is considered. One-dimensional dynamic problem of coupled thermoelasticity is solved by using integral transforms. Exact and approximate expressions for calculation of thermomechanical values at each point of a half-space and at arbitrary moment of time are given. Qualitative and quantitative analyses of the effects of damping and dispersion of thermoelastic waves is performed.*

1. INTRODUCTION

In this paper we consider the propagation of thermoelastic waves in an infinite elastic half-space caused by instantaneous (impulse) effect of pressure forces applied to its boundary surface. Interaction between the field of deformation and the field of temperature is taken into consideration. Thermoelastic waves caused by mechanical influences have been rarely studied compared with waves caused by thermal influences. Besides, the solutions in closed mathematical form can be rarely found because of the complexity of the system of differential equations of the problem. The thermal disturbance of the elastic layer caused by instantaneous pressure forces on its surface is considered in ref. [1]. The limit values of the temperature, displacement, and stress are determined for the case when the time increases to infinity. The case of propagation of harmonic waves in infinite half-space is also considered, and it is shown the appearance of damping and dispersion, but without discussion as to what changes the waves of arbitrary form are exposed.

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2. THE BASIC EQUATIONS OF THE PROBLEM. THE BOUNDARY AND INITIAL CONDITIONS

The system of partial differential equations which describes the propagation of plane thermoelastic waves for the coupled case in absence of internal heat sources is given in the form [2], [3]:

$$\frac{\partial^2 u}{\partial x^2}-\frac{1}{c^2}\cdot\frac{\partial^2 u}{\partial t^2}-m\frac{\partial\theta}{\partial x}=0 \tag{1}$$

$$\frac{\partial^2 \theta}{\partial x^2}-\frac{1}{\kappa}\cdot\frac{\partial\theta}{\partial t}-\eta\frac{\partial^2 u}{\partial x\partial t}=0 \tag{2}$$

where $u=u(x,t)$ is the displacement in the direction of x axis, $\theta(x,t)$ represents temperature above the temperature T_o of a body in natural state, $c=\sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$ is the speed of the only elastic longitudinal waves, E, ν, ρ are Young's modulus, Poison's ratio and material density, respectively, $m=\frac{1+\nu}{1-\nu}\alpha_t$ stands for modified coefficient of thermal expansion, α_t is the coefficient of linear dilatation , $\kappa=\lambda/(c_\varepsilon\rho)$ means the coefficient of thermal intensity, λ is the heat conduction coefficient, c_ε is the specific heat at a constant strain, $\eta=\frac{E\alpha_t T_o}{(1-2\nu)\lambda}$ represents the coefficient which takes into consideration the influence of speed of deformation on the change of temperature.

For the case of the action of pressure forces on the boundary x=0 of the half-space $x\geq 0$, at the initial moment of time t=0, the boundary condition is

$$\sigma(x,t)\big|_{x=0}=-S_o\cdot\delta(t) \tag{3}$$

where: $\sigma(x,t)$ is the component of stress tensor in x direction, $\delta(t)$ represents the Dirac function, and S_o means the total impulse of instantaneous forces per unit of surface.

With the help of Duhamel-Neumann relations for stress [3], the boundary condition (3) is reduced to displacement condition, that is

$$\left.\frac{\partial u}{\partial x}\right|_{x=0}=-\frac{S_o}{E_1}\delta(t),\quad t\geq 0\quad,\ E_1=\frac{(1-\nu)E}{(1-2\nu)(1+\nu)} \tag{4}$$

The temperature θ, on the boundary x=0, is kept constant during thermokinetic process, that is

$$\theta(x,t)\big|_{x=0}=0 \tag{5}$$

The assumed initial conditions are:

$$u(x,t)\big|_{t=0}=0;\quad \theta(x,t)\big|_{t=0}=0\ ,\quad \text{for}\quad x\geq 0 \tag{6}$$

3. SOLVING OF DIFFERENTIAL EQUATIONS OF THE PROBLEM

The infinite Fourier cosine and sine transforms of functions $u=u(x,t)$ and $\theta=\theta(x,t)$ are given in the form [4], [5]:

$$
\begin{aligned}
u_c(\xi,t) &= \sqrt{\frac{2}{\pi}}\cdot\int_0^\infty u(x,t)\cos(\xi x)dx, \quad u(x,t) = \sqrt{\frac{2}{\pi}}\cdot\int_0^\infty u_c(\xi,t)\cos(\xi x)d\xi \\
\theta_c(\xi,t) &= \sqrt{\frac{2}{\pi}}\cdot\int_0^\infty \theta(x,t)\cos(\xi x)dx, \quad \theta(x,t) = \sqrt{\frac{2}{\pi}}\cdot\int_0^\infty \theta_c(\xi,t)\cos(\xi x)d\xi
\end{aligned}
\tag{7}
$$

The Laplace transform is given by [6]

$$
f^*(p) = \int_0^\infty f(t)\,\mathrm{e}^{-pt}\,dt; \quad f(t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} f^*(p)\,\mathrm{e}^{pt}\,dp \tag{8}
$$

Let us apply the transforms (7) and (8) to the system of equations (1) and (2), and to the both boundary and initial conditions (4) and (5). By introducing the transforms of the boundary and initial conditions into the transformed equations (1) and (2), a system of algebraic equations can be obtained of which the solutions are:

$$
u_c^*(\xi,p) = \sqrt{\frac{2}{\pi}}\cdot\frac{S_o c^2}{E_1}\cdot\frac{p+\kappa\xi^2}{[p^2+(c\xi)^2](p+\kappa\xi^2)+\varepsilon(c\xi)^2 p} \tag{9}
$$

$$
\theta_s^*(\xi,p) = \sqrt{\frac{2}{\pi}}\cdot\frac{S_o c^2\kappa\eta}{E_1}\cdot\frac{p\xi}{[p^2+(c\xi)^2](p+\kappa\xi^2)+\varepsilon(c\xi)^2 p} \tag{10}
$$

where $\varepsilon = m\kappa\eta = \dfrac{(1+\nu)E\alpha_t^2 T_o}{(1-\nu)(1-2\nu)\rho c_\varepsilon}$ is the coupling coefficient between the field of temperature and the field of strain, for the plane state of strain. If we look at the transforms u_c^* and θ_s^* we can observe that the denominators in both terms are polynomials of the third order, so the application of inverse transforms is complicated and impractical. On the other hand, by neglecting the term $\varepsilon(c\xi)^2 p$, inverse transforms can be found from the tables of Laplace transforms. In that case, however, the sense of the solution will be lost, because the effect of coupling of the two physical fields will be neglected. We can proceed to a solution by linearizing the denominator $[p^2+(c\xi)^2](p+\kappa\xi^2)+\varepsilon(c\xi)^2 p$. Under the assumption that the value of the term $\varepsilon(c\xi)^2 p$ is smaller compared to other terms, the expression can be reduced, with omitting some terms, to the form [7]

$$
[(p+\chi)^2+(c\xi+\varphi)^2]\cdot(p+k\xi^2+\psi) \tag{11}
$$

where χ, φ, ψ are small quantities which ought to be determined. Therefore, both the denominator in the equation (10) and the expression (11) are expanded into polynomials with regard to the variable p, and they are equalized.

$$
\begin{aligned}
& p^3+(\kappa\xi^2)p^2+(c\xi)^2(1+\varepsilon)p+(c\xi)^2\kappa\xi^2 \equiv \\
& \equiv p^3+(\kappa\xi^2+\psi+2\chi)p^2+[2\kappa\xi^2\chi+2\chi\psi+\chi^2+(c\xi+\varphi)^2]p+(\kappa\xi^2+\psi)[\chi^2+(c\xi+\varphi)^2]
\end{aligned}
\tag{12}
$$

By comparing the coefficients at p^3, p^2, p and the free term in the equation (12), and neglecting all the mutual products of values χ, φ, ψ as small quantities of higher order, we can find these small quantities

$$\chi=\frac{\varepsilon}{2}\frac{\kappa\xi^2}{1+\left(\frac{\kappa\xi}{c}\right)^2}\ ,\qquad \varphi=\frac{\varepsilon}{2}\frac{c\xi}{1+\left(\frac{\kappa\xi}{c}\right)^2}\ ,\qquad \psi=-\varepsilon\frac{\kappa\xi^2}{1+\left(\frac{\kappa\xi}{c}\right)^2} \tag{13}$$

The approximation of the denominator in eq. (9), (10) by the expression (11) is justified because the following relations are satisfied:

$$\frac{\varepsilon}{2}c\xi\le\varphi<<c\xi,\quad \left|-\varepsilon\kappa\xi^2\right|\le|\psi|<<\kappa\xi^2,\quad \varepsilon<<1 \tag{14}$$

that is the terms $c\xi+\varphi$ and $\kappa\xi^2+\psi$ in eq. (11) do not differ much from the terms $c\xi$ and $\kappa\xi^2$ in eq. (9), (10). Thus, we obtain more suitable expression for transforms u_c^* and θ_s^*.

$$\begin{aligned} u_c^*(\xi,p)&=\sqrt{\frac{2}{\pi}}\frac{S_oc^2}{E_1}\frac{p+\omega}{[(p+\alpha)^2+\beta^2](p+\gamma)}\ ,\\ \theta_s^*(\xi,p)&=\sqrt{\frac{2}{\pi}}\frac{S_0c^2k\eta}{E_1}\frac{\xi p}{[(p+\alpha)^2+\beta^2](p+\gamma)} \end{aligned} \tag{15}$$

where

$$\begin{aligned} &\alpha=\alpha(\xi)=\frac{\varepsilon}{2}\frac{\omega(\xi)}{1+(\kappa\xi/c)^2}\ ,\qquad \beta=\beta(\xi)=c\xi\left[1+\frac{1}{2}\frac{1}{1+(\kappa\xi/c)^2}\right],\\ &\gamma=\gamma(\xi)=\omega(\xi)\left[1-\beta\frac{1}{1+(\kappa\xi/c)^2}\right],\quad \omega=\omega(\xi)=k\xi^2 \end{aligned} \tag{16}$$

By applying the inverse Laplace transform (8) to the simplified expressions (15), obtained from (9) and (10), we come to the transforms $u_c=u_c(\xi,t)$ and $\theta_s=\theta_s(\xi,t)$ in the domain of time [6]:

$$u_c(\xi,t)=\sqrt{\frac{2}{\pi}}\frac{S_oc^2}{E_1}\left\{\frac{\omega-\gamma}{(\gamma-\alpha)^2+\beta^2}\left(\mathrm{e}^{-\gamma t}-\mathrm{e}^{-\alpha t}\cos\beta t\right)+\frac{1}{\beta}\left[1+\frac{(\gamma-\alpha)(\omega-\gamma)}{(\gamma-\alpha)^2+\beta^2}\right]\mathrm{e}^{-\alpha t}\sin\beta t\right\} \tag{17}$$

$$\theta_s(\xi,t)=\sqrt{\frac{2}{\pi}}\frac{S_oc^2\kappa\eta}{E_1}\xi\left\{\frac{\gamma}{(\gamma-\alpha)^2+\beta^2}\left(\mathrm{e}^{-\alpha t}\cos\beta t-\mathrm{e}^{-\gamma t}\right)+\frac{1}{\beta}\left[1-\frac{(\gamma-\alpha)\gamma}{(\gamma-\alpha)^2+\beta^2}\right]\mathrm{e}^{-\alpha t}\sin\beta t\right\} \tag{18}$$

Let us introduce nondimensional quantities:

$$\bar{u}=\frac{E_1u}{cS_o},\qquad \bar{\theta}=\frac{E_1\theta}{c^2S_o\eta},\qquad \bar{x}=\frac{cx}{t},\bar{t}=\frac{c^2t}{\kappa} \tag{19}$$

The functions α, β, γ and ω can be also reduced to nondimensional form:

$$\begin{aligned} &\bar{\xi}=\frac{\kappa\xi}{c},\quad \bar{\alpha}(\bar{\xi})=\frac{\kappa\alpha}{c^2}=\frac{\varepsilon}{2}\frac{\bar{\xi}^2}{1+\bar{\xi}^2}\ ,\qquad \bar{\beta}(\bar{\xi})=\frac{\kappa\beta}{c^2}=\bar{\xi}\left(1+\frac{\varepsilon}{2}\frac{1}{1+\bar{\xi}^2}\right),\\ &\bar{\gamma}(\bar{\xi})=\frac{\kappa\gamma}{c^2}=\bar{\xi}^2\left(1-\varepsilon\frac{1}{1+\bar{\xi}^2}\right)\ ,\qquad \bar{\omega}(\bar{\xi})=\frac{\kappa\omega}{c^2}=\bar{\xi}^2 \end{aligned} \tag{20}$$

By applying the inverse transforms (7) to equations (17) and (18), with regards to eq. (19) and (20), the final solutions for the fields of displacement and temperature for the half-space $x \geq 0$ can be obtained in nondimensional form

$$\bar{u}(\bar{x},\bar{t})=\frac{2}{\pi}\int_0^\infty\left\{\frac{\bar{\omega}-\bar{\gamma}}{(\bar{\gamma}-\bar{\alpha})^2+\bar{\beta}^2}(\mathrm{e}^{-\bar{\gamma}\bar{t}}-\mathrm{e}^{-\bar{\alpha}\bar{t}}\cos\bar{\beta}\bar{t})+\frac{1}{\bar{\beta}}\left[1+\frac{(\bar{\gamma}-\bar{\alpha})(\bar{\omega}-\bar{\gamma})}{(\bar{\gamma}-\bar{\alpha})^2+\bar{\beta}^2}\right]\mathrm{e}^{-\bar{\alpha}\bar{t}}\sin\bar{\beta}\bar{t}\right\}\cos(\bar{x}\bar{t}\bar{\xi})d\bar{\xi} \tag{21}$$

$$\bar{\theta}(\bar{x},\bar{t})=\frac{2}{\pi}\int_0^\infty\left\{\frac{\bar{\gamma}}{(\bar{\gamma}-\bar{\alpha})^2+\bar{\beta}^2}(\mathrm{e}^{-\bar{\alpha}\bar{t}}\cos\bar{\beta}\bar{t}-\mathrm{e}^{-\bar{\gamma}\bar{t}})+\frac{1}{\bar{\beta}}\left[1-\frac{(\bar{\gamma}-\bar{\alpha})\bar{\gamma}}{(\bar{\gamma}-\bar{\alpha})^2+\bar{\beta}^2}\right]\mathrm{e}^{-\bar{\alpha}\bar{t}}\sin\bar{\beta}\bar{t}\right\}\sin(\bar{x}\bar{t}\bar{\xi})\bar{\xi}d\bar{\xi} \tag{22}$$

The equations for $\bar{u}$ and $\bar{\theta}$ defined by eq. (21) and (22) at any point of the half-space $x \geq 0$ and in any moment of time, could be determined numerically, by applying Filon's procedure for calculation of integral of fast-oscillating functions [7].

4. Approximate solution.

In addition to applying the procedure of numerical integration, it is possible to obtain, with some limitation, the approximate expressions for displacement and temperature which are more suitable for practical use. All the integrands contain the multipliers of the form:

$$\mathrm{e}^{-\bar{\alpha}\bar{t}}=\exp\left(-\frac{\varepsilon}{2}\frac{\bar{\xi}^2}{1+\bar{\xi}^2}\right)\bar{t},\quad \mathrm{e}^{-\bar{\gamma}\bar{t}}=\exp\left[-\left(1-\varepsilon\frac{1}{1+\bar{\xi}^2}\right)\bar{\xi}^2\right]\bar{t}$$

For sufficiently high values of nondimensional time $\bar{t}$, upper terms tend very fast to zero, beginning from some value $\bar{\xi}=\bar{\xi}_o$. Taking for $\bar{\xi}_o$ a sufficiently small value ($\bar{\xi}_o<1$) it can be considered that the following relations are satisfied:

$$\bar{\xi}^2 \ll 1,\quad 1+\bar{\xi}^2\cong 1,\quad 0\leq\xi\leq\bar{\xi}_o<1$$

The functions $\bar{\alpha}(\bar{\xi})$, $\bar{\beta}(\bar{\xi})$ and $\bar{\gamma}(\bar{\xi})$ become simpler:

$$\bar{\alpha}(\bar{\xi})\cong\frac{\varepsilon}{2}\bar{\xi}^2,\quad \bar{\beta}(\bar{\xi})\cong\bar{\xi}(1+\varepsilon/2),\quad \bar{\gamma}(\bar{\xi})\cong\bar{\xi}^2(1-\varepsilon),\quad \bar{\omega}(\bar{\xi})=\bar{\xi}^2 \tag{23}$$

Accepting for $\bar{\alpha}\bar{t}$ a sufficiently high value, taking for example

$$\bar{\alpha}\bar{t}=\frac{\varepsilon}{2}\bar{\xi}_o\bar{t}=50 \tag{24}$$

the exponential function $\exp(-\bar{\alpha}\bar{t})$ for $\bar{\xi}>\bar{\xi}_o$ tends very fast to zero. From eq. (24) the minimum value of the nondimensional time can be defined and it is valid for the performed approximation, that is

$$\bar{t}_{\min}=\frac{100}{\varepsilon\bar{\xi}_o^2} \tag{25}$$

Taking, for example, $\bar{\xi}_o=0.1$ and $\varepsilon=0.01$ (for steel), one obtains $\bar{t}_{\min}=10^6$. This value is not too large because during that time the thermoelastic wave crosses the way [6] $x=vt\cong(1+\varepsilon/2)\cdot 10^6\frac{\kappa}{c}\cong 10^{-3}\mathrm{m}$ (where $v=(1+\varepsilon/2)c$ is the wave velocity), which shows that the condition (25) is not too limiting for the practical use. Since the coupling coefficient ε for steel has the smallest value, this interval is smaller for other materials.

As the term $e^{-\bar{\gamma}\bar{t}}$ decreases faster than the term $e^{-\bar{\alpha}\bar{t}}$, it can be seen that the condition (25) remains appropriate for the definition of the minimum time. After replacing the functions (23) into expressions (21) and (22) one obtains:

$$\bar{u}(\bar{x},\bar{t}) \cong \frac{2}{\pi}\int_0^\infty \left\{ \frac{\varepsilon}{1+\varepsilon}\left[e^{-(1-\varepsilon)\bar{t}\bar{\xi}^2} - e^{-\frac{\varepsilon}{2}\bar{t}\bar{\xi}^2}\cos(\bar{t}(1+\varepsilon/2)\bar{\xi}) \right] + \frac{1}{\bar{\xi}(1+\varepsilon/2)} e^{-\frac{\varepsilon}{2}\bar{t}\bar{\xi}^2}\sin(\bar{t}(1+\varepsilon/2)\bar{\xi}) \right\}\cos(\bar{x}\bar{\xi})d\bar{\xi} \quad (26)$$

$$\bar{\theta}(\bar{x},\bar{t}) \cong \frac{2}{\pi}\int_0^\infty \left\{ \frac{1-\varepsilon}{1+\varepsilon}\bar{\xi}\left[e^{-\frac{\varepsilon}{2}\bar{t}\bar{\xi}^2}\cos(\bar{t}(1+\varepsilon/2)\bar{\xi}) - e^{-(1-\varepsilon)\bar{t}\bar{\xi}^2} \right] + \frac{1}{1+\varepsilon/2} e^{-\frac{\varepsilon}{2}\bar{t}\bar{\xi}^2}\sin(\bar{t}(1+\varepsilon/2)\bar{\xi}) \right\}\sin(\bar{x}\bar{\xi})d\bar{\xi} \quad (27)$$

The upper limit of integration in the above integrals is formally taken infinite. This will not lead to wrong results because of the change of the integrand. That is, the terms $e^{-\bar{\alpha}\bar{t}}$ and $e^{-\bar{\gamma}\bar{t}}$ tend very fast to zero, so the errors of approximation (23) for $\bar{\xi} > \bar{\xi}_o$ will not appear.

If the integration of eqs. (26) and (27) is done with neglecting of terms the order of the magnitude for $\sqrt{\bar{t}}$ being less than the order of the magnitude of retained terms, we can finally obtain [8], [9]:

$$\bar{u}(\bar{x},\bar{t}) \cong \frac{1}{1+\varepsilon/2}\cdot\frac{1}{2}\left\{ 1-\operatorname{erf}\left[\frac{\bar{x}-\bar{t}(1+\varepsilon/2)}{\sqrt{2\varepsilon\bar{t}}} \right] \right\} \quad (28)$$

$$\bar{\theta}(\bar{x},\bar{t}) \cong \frac{1}{1+\varepsilon/2}\cdot\frac{1}{\sqrt{2\pi\varepsilon\bar{t}}}\exp\left\{ -\frac{[\bar{x}-\bar{t}(1+\varepsilon/2)]^2}{2\varepsilon\bar{t}} \right\} \quad (29)$$

where erf(...) is the error function.

The functions in eqs. (28) and (29) show that the thermoelastic wave changes its form during propagation, which is the consequence of dispersion. A clearer form of equations can be obtained by introducing nondimensional distance at the front of the wave in the direction of expanding, that is

$$\bar{x}' = \frac{c}{\kappa}x' = \bar{x} - \bar{t}(1+\varepsilon/2) \quad (30)$$

where $x' = x - vt = x - c(1+\varepsilon/2)t$ is the distance of the observed plane at the front of the wave. In that case the expression for the nondimensional displacement and the temperature become:

$$\bar{u}(\bar{x}',\bar{t}) \cong \frac{1}{1+\varepsilon/2}\cdot\frac{1-\operatorname{erf}\left(\frac{\bar{x}'}{\sqrt{2\varepsilon\bar{t}}}\right)}{2} \quad (31)$$

$$\bar{\theta}(\bar{x}',\bar{t}) \cong \frac{1}{1+\varepsilon/2}\cdot\frac{1}{\sqrt{2\pi\varepsilon\bar{t}}}\exp\left[-\frac{(\bar{x}')^2}{2\varepsilon\bar{t}} \right] \quad (32)$$

From the above equations, for the plain state of strain, the nondimensional strain and stress in the direction of the *x* axis can be found:

$$\bar{\varepsilon}_x(\bar{x}',\bar{t}) = \frac{1}{\bar{t}}\frac{\partial \bar{u}}{\partial \bar{x}} \cong -\frac{1}{1+\varepsilon/2}\cdot\frac{1}{\sqrt{2\pi\varepsilon\,\bar{t}}}\exp\left[-\frac{(\bar{x}')^2}{2\varepsilon\,\bar{t}}\right] = -\bar{\theta}(\bar{x}',\bar{t}) \tag{33}$$

$$\bar{\sigma}_x(\bar{x}',\bar{t}) = \bar{\varepsilon}_x(\bar{x}',\bar{t}) - \varepsilon\cdot\bar{\theta}(\bar{x}',\bar{t}) \cong -\frac{1+\varepsilon}{1+\varepsilon/2}\cdot\frac{1}{\sqrt{2\pi\varepsilon\,\bar{t}}}\exp\left[-\frac{(\bar{x}')^2}{2\varepsilon\,\bar{t}}\right] = -(1+\varepsilon)\cdot\bar{\theta}(\bar{x}',\bar{t}) \tag{34}$$

where $\bar{\varepsilon}_x = \frac{\varepsilon_x E_1 \kappa}{S_o c^2}$ is nondimensional strain and $\bar{\sigma}_x = \frac{\sigma_x \kappa}{S_o c^2}$ is nondimensional stress in direction of x axis. The following relations for determination the strain and stress are used:

$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad \sigma_x = E_1(\varepsilon_x - m\cdot\theta) \tag{35}$$

Nondimensional displacement and temperature as functions of nondimensional distance $\bar{x}'$ and time $\bar{t}$ for the value of the coupling coefficient ε=0.547 (for polyethylene h.p.) are shown in Fig. 1. All calculations are performed according to eqs. (21) and (22). In Fig. 2. the nondimensional displacement, temperature and stress, calculated from eqs. (31), (32) and (34), are shown.

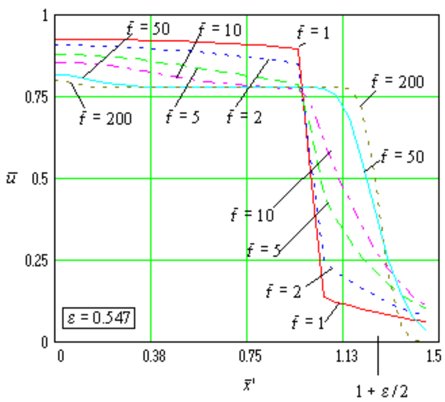

Fig. 1a

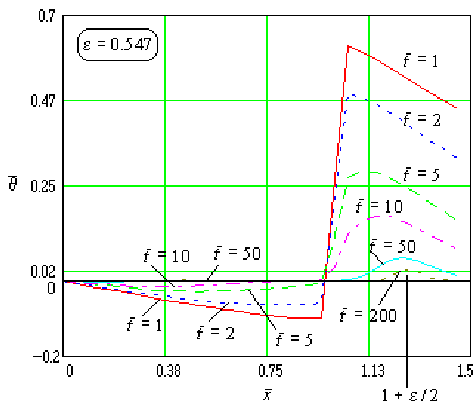

Fig. 1b.

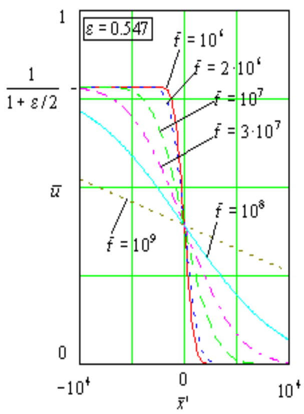

Fig. 2a

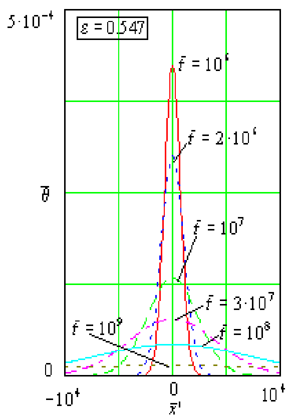

Fig. 2b

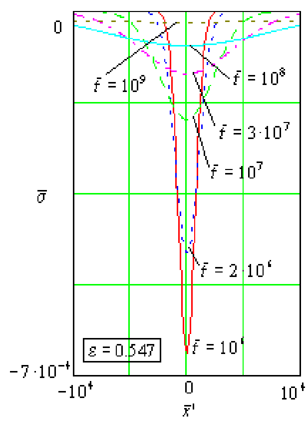

Fig. 2c.

The change of the form of thermoelastic wave which is noticeable in Fig. 2 is the consequence of the effect of damping and dispersion [3], [10]. The thermal wave spreads with the increase of time $\bar{t}$. As a measure of spreading of the waves we take the distance between the planes in which the temperature decreases to 37 percents of the value (1/*e*-part) of the maximum temperature at the front of the wave, that is $\exp[-(\bar{x}')^2/(2\varepsilon\,\bar{t})] = \exp(-1)$. This gives:

$$2x' = 8\varepsilon\, c\, t \tag{36}$$

The coupling coefficient ε is a physical characteristic of the material, so the effect of damping and dispersion of thermoelastic waves depends exclusively on the value of this coefficient. This can be shown best in Fig. 3., where the change of the temperature during the time at the front of the wave $(\bar{x} = (1+\varepsilon/2)\bar{t})$ for some different values of the coefficient ε is presented.

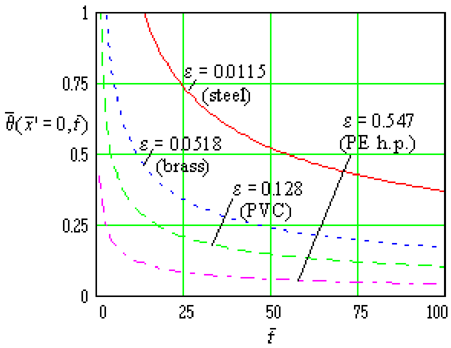

Fig. 3

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RAVNI TERMOELASTIČNI TALASI U POLUPROSTORU IZAZVANI PRITISNIM SILAMA U NJEGOVOJ RAVNI

Rastko Čukić, Dejan Trajkovski

Razmatran je problem prostiranja longitudinalnih talasa u poluprostoru izazvanih mehaničkim uticajima. Jednodimenzioni dinamički problem spregnute termoelastičnosti rešavan je korišćenjem integralnih transformacija. Dati su tačni i približni izrazi za izračunavanje termomehaničkih veličina za svaku tačku poluprostora u proizvoljnom trenutku vremena. Izvršena je kvalitativna i kvantitativna analiza efekta prigušenja i rasipanja termoelastičnih talasa.