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REFERENCE TRACKING VERSUS PATH-FOLLOWING FOR A ONE-LINK FLEXIBLE ROBOT MANIPULATOR

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Abstract. *This paper details two control approaches for a flexible manipulator system, where the non-minimum phase problem is treated. In the first approach, we use the motion planning technique. It searches for proper output trajectories with polynomial form, in order to cancel the effects of the unstable zeros. The second approach is called Path-Following with internal model control. Its primary objective is to steer a physical object to converge to a geometric path, and its secondary objective is to ensure that an object's motion along the path satisfies a given dynamic specification.*

Key words: *Flexible manipulator, Motion planning, Path following, Vibration control*

1. INTRODUCTION

For a long time there has been a standing interest into the use of light and flexible manipulator robots. Amongst the many improved characteristics when compared to rigid robots, perhaps the most appealing one is the higher operation speeds achievable. Unfortunately, the dynamic system of this kind of robots is of high dimension due to the links flexibility, and the control problem that is posed is a non-collocated one, since we apply torque at one end and measure position at the other end of a flexible element. This characteristic makes the control of flexible manipulators one of the areas under great investigation in robotics. The three main objectives in the control of flexible robot arms are:

O1 - Point to point motion of the end-effector,
O2 - Trajectory tracking in the joint space (tracking of a desired angular trajectory),
O3 - Trajectory tracking in the operational space (tracking of a desired tip trajectory).

When comparing flexible manipulators with rigid manipulators, the control methodology differs. Using rigid manipulator robots, we can achieve excellent control using simple linear joints controllers, such as Proportional-Integral-Derivative (PID) controllers. On the other hand, when using a flexible manipulator, we have to consider the oscilla-

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tions that appear at the tip during motion; therefore, we need to design a controller that contemplates feedback of the end-effector position. There are many references in the literature dealing with Non-Minimum phase systems controllers. Isidori [1] presented stabilization via output feedback and Dačić [2] presented three distinct approaches for tracking in the presence of unstable zeros dynamics. He referred to them as: the Internal Model approach, the Flatness approach and the Inversion approach. Also Aguiar et al. [3-7] presented the path-following methodology with an internal model control in order to control a vehicle with unstable load at a constant speed. There are less publications referring to robotic manipulator path-following. Skjetne [8] presents the problem of a robotic cutting tool where the control objective is for the tip of the tool to trace a desired repeatable path at a constant nominal speed. Benosman et al. [9-12] made a profound analysis of the stable inversion approach in order to cancel the unstable zero dynamics.

In this paper we focus on the Internal Model Approach and the Inversion approach. The paper is organized as follows: Section 2 recalls the dynamic equations of a flexible link. In section 3 the trajectory planning method is presented and Section 4 presents the path-following controller. The simulation results are shown in Section 5 and finally the conclusions are presented in Section 6.

2. DYNAMIC EQUATIONS

The research presented is based on the one joint planar flexible robot represented in Fig. 1. It is considered that the flexible link is clamped to a rigid hub with a moment of inertia I_H, radius r, and an input torque τ. The ordinary differential equations of this system may be obtained in the form [13-15]

$$M\ddot{q} + Kq = T \tag{1}$$

Where M is the system inertia matrix, K is the system stiffness matrix, T is the vector of external forces and q is the vector of generalized coordinates,

$$q = [\theta \quad \eta_1 \quad \eta_2]^T \tag{2}$$

$$M\ddot{q} + Kq = T \tag{3}$$

Here, η_i is the modal amplitude of the ith clamped-free vibration mode in the assumed modes spatial discretization procedure. In this work, only the first two clamped-free modes are taken into account.

Considering linear displacements, the link displacement at a distance x from the frame origin along the OX direction can be described as

$$y(x,t) = x\theta(t) + \upsilon(x,t) \tag{4}$$

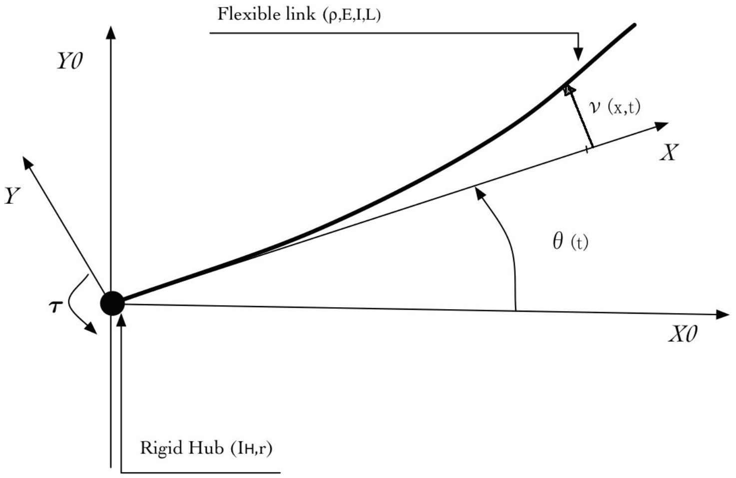

Fig. 1 Mechanical model of a single flexible link

In Eq. (4), the total displacement is given as a function of the rigid body motion $\theta(t)$ and the elastic deflection $\upsilon(x, t)$ where

$$\upsilon(x,t) = \sum_{i=1}^{2} \phi_i(x)\eta_i(t) \tag{5}$$

$\phi_i(x)$ and $\eta_i(t)$ represent the modal functions and modal amplitudes of the i^{th} clamped-free mode respectively. The modal functions considered are cubic Hermite polynomials with the form

$$\phi_1(x) = 3\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right)^3 \tag{6}$$

$$\phi_2(x) = \left(\frac{x}{L}\right)^2 - \left(\frac{x}{L}\right)^3 \tag{7}$$

where L is the link length.

3. MOTION PLANNING – STABLE INVERSION METHOD

We consider a non-minimum phase linear system represented in the following form,

$$P\left(\frac{d}{dt}\right)u(t) = Q\left(\frac{d}{dt}\right)y(t) \tag{8}$$

where P and Q are polynomials in a differential operator d/dt, with degrees m and n respectively ($m < n$). In a first analysis, the time response of the inverse problem in (8) is composed by two terms, the transient and the steady-state, where the transient contains divergent terms due to the unstable zeros. The solution proposed by Benosman et. al. [9-12]

is to plan the output trajectory of (8), $y_d(t)$ (the input of the inverse system), such that the divergent terms of $u(t)$ are cancelled. To this end, a polynomial in time form is considered for the output trajectory

$$y_d(t) = \sum_{i=1}^{p} a_i t^{i-1} \tag{9}$$

where the degree of the polynomial form p depends on the number of output initial and final constraints, as well as the number of unstable zeros in (8). Solving the inverse problem in (8) with the input in (9) we obtain $u(t) = u_t(t) + u_p(t)$ where $u_t(t)$ is the transient solution and $u_p(t)$ is the particular solution.

The transient solution can be written as

$$u_t(t) = \sum_{i=1}^{m} A_i(a_i, t_0, u_0^{(1)}, \cdots, u_0^{(n-1)}) \exp^{r_i, t} \tag{10}$$

where the r_i are all the roots of the characteristic equation. The A_i are linear functions of the coefficients a_i and of all initial conditions. It can be shown that they are given by

$$A_i = u_0 + \sum_{j=1}^{p} \frac{a_j}{zero_i^j} \tag{11}$$

On the other hand, the particular solution can be represented as

$$u_p(t) = \sum_{i=1}^{p} B_i(a_i) t^{i-1} \tag{12}$$

To cancel the effect of the unstable zeros on the transient solution (10), i.e. all the zeros in the right complex plane and pure imaginary zeros, the A_i associated with the unstable zeros must be null

$$A_i(a_i, t_0, u_0^{(1)}, \cdots, u_0^{(n-1)}) = 0 \tag{13}$$

From this constraint and from the initial and final conditions, the output coefficients a_i may be found by solving the system of equations

$$\begin{cases} A_i(a_i, t_0, u_0^{(1)}, \cdots, u_0^{(n-1)}) = 0 \\ y_d^{(i)}(t_0) = initial\ conditions \\ y_d^{(i)}(t_f) = final\ conditions \end{cases} \tag{14}$$

The particular solution for the input (12) may now be calculated since the coefficients B_i are given as functions of the coefficients a_i. An open loop control solution may be readily obtained as

$$u_{ol} = \sum_{i=1}^{p} B_i(a_i) t^{i-1} + \sum_{i=1}^{m} A_{i\,st}(a_i, t_0, u_0^{(1)}, \cdots, u_0^{(n-1)}) \exp^{r_{i\,st}, t} \tag{15}$$

where r_{ist} and A_{ist} are the stable zeros and the corresponding A_i coefficients. In order to increase the robustness of the approach, a loop may be closed around the joint angle yielding

$$u_{cl} = u_{ol} + K[e_\theta \quad e_\theta^{(1)} \quad \cdots \quad e_\theta^{(n-1)}]^T \tag{16}$$

where the joint error is given by $e_\theta(t) = \theta_d(t) - \theta(t)$, and K is the gain matrix.

3.1. Implementation

For the system in Fig. 1, the hub has a radius $r = 0.075$m and the link has length $L = 0.5$m. The system transfer function is given by

$$\frac{y}{\tau} = \frac{92.5s^4 - 4.924\times10^6 s^2 + 1.163\times10^{10}}{s^2(s^4 + 56780s^2 + 1.307\times10^8)} \tag{17}$$

For simplicity, let us define vector N with the coefficients of the numerator, and vector D with the coefficients of the denominator. The zeros of this transfer function are

$$s_{1,2} = \pm225.29 \qquad s_{3,4} = \pm49.77 \tag{18}$$

where we have two unstable zeros. The differential equation in (8) is for this case

$$\sum_{i=1}^{5} N(i)\tau^{(5-i)}(t) = \sum_{j=1}^{7} D(j)y^{(7-j)}(t) \tag{19}$$

with the following initial conditions

$$\begin{cases} \tau^{(i)}(0) = 0, & i = 0,1,2,3 \\ y^{(j)}(0) = 0, & j = 0,1,2,3,4,5 \end{cases} \tag{20}$$

One thus has two unstable zeros, four initial conditions for τ and six initial conditions for y. The degree of the polynomial in (9) is therefore $p = 12$. The output that cancels the unstable zeros may now be found:

$$\begin{cases} A_k = \tau_0 + \sum_{j=1}^{p} \dfrac{a_j}{Zo_k^j} = 0 \\ y^{(i)}(0) = 0 \quad i = 0,1,2,3 \\ y(t_f) = y_f \\ y^{(i)}(t_f) = 0 \quad i = 1,2,3,4,5 \end{cases} \tag{21}$$

Zo_k^j represents the k^{th} unstable zero powered to j. Notice that these conditions are chosen to force the desired torque to be symmetric. The desired a_i coefficients are then directly obtained, solving the system above. In this way it is possible to calculate the transient solution of the system (10). For the particular solution (12), the coefficients B_i are obtained as linear functions of the output coefficients a_i substituting equation (12) and equation (9) into equation (19). As introduced in Section 3, equation (16), in order to bring some robustness to the controller, a close-loop form should be used. Two types may be chosen from:

1) A partial state feedback, based on the joint position and velocity variables:

$$\begin{gathered} T_{cl} = T_{ol} + K_p(\theta_d(t) - \theta(t)) + K_v(\dot{\theta}_d(t) - \dot{\theta}(t)) \\ K_p > 0, \qquad K_v > 0 \end{gathered} \tag{22}$$

2) An open loop form where the input of the system is the angle of the joint.

4. Path-Following for a One Link Non-Minimum Phase Robot Manipulator

The objective of the Path-Following method is to force the non-minimum phase system output to follow a geometric path without a timing law assigned to it. Systems with unstable zero dynamics have limited tracking capabilities, and the only way to change this performance limitation is to change the input-output structure of the system. This can be performed by reformulating the problem as path-following, rather than reference tracking. With this reformulation, a new timing law $\gamma(t)$ is introduced, which is used as an additional control input.

In this section we present the Path-Following problem as demonstrated in [6]. Path-following based on internal model control is used to achieve asymptotic tracking of reference signals. The controller that incorporates an internal model of the exosystem is capable of ensuring an asymptotic convergence of the tracking error to zero for every possible reference signal generated by the exosystem.

The following linear time-invariant system is assumed

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \quad x(t_0) = x_0 \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{23}$$

where $x(t)$ is the state, $u(t)$ the input and $y(t)$ the output. The main objective of this method is to reach and follow a desired geometric path $y_d(\gamma)$. The geometric path $y_d(\gamma)$ can be generated by an exosystem of the form

$$\begin{aligned} &\frac{d}{d\gamma}\omega(\gamma) = S \times \omega(\gamma), \quad \omega(\gamma_0) = \omega_0 \\ &y_d(\gamma) = Q \times \omega(\gamma) \end{aligned} \tag{24}$$

where $\omega \in R^{2n}$ is the exosystem state and $S + S' = 0$. For any timing law $\gamma(t)$, the path-following error can be defined as $e(t) = y(t) - y_d(\gamma(t))$. The following problems are associated with the Path-Following methodology:

<u>Geometric Path Following</u>: For the desired path $\gamma_d(\gamma)$, it is necessary to design a controller that achieves:

- Boundedness: the state $x(t)$ is uniformly bounded for all $t \geq t_0$, and every initial condition $(x(t_0), \omega(\gamma_0))$, where $\gamma_0 = \gamma(t_0)$.
- Error convergence: the path-following error $e(t)$ converges to zero as $t \to \infty$.
- Forward motion: $\dot{\gamma}(t) > c$ for all $t \geq t_0$, where c is a positive constant.

Speed-assigned path-following: Given a desired speed $v_d > 0$, it is required that $\dot{\gamma} \to v_d$ as $t \to \infty$.

As demonstrated in [3,6], one can always assume a small L_2–norm of the path-following error,

$$J = \int_0^\infty \left\| y(t) - y_d(t) \right\|^2 dt = \int_0^\infty \left\| e(t) \right\|^2 dt < \delta \tag{25}$$

that verifies a δ arbitrarily small in order to consider a perfect tracking problem.

4.1. Controller design – Internal Model Control

In [6], a solution is presented to achieve a path controller for (23), such that the closed loop state is bounded. If (A, B, C, D) is a non-minimum phase system, the pair (A, B) is stabilizable, the pair (A, C) is detectable, the number of inputs is as large as the number of outputs and the zeros of (A, B, C, D) do not coincide with the eigenvalues of S (24), then, for the geometric path-following problem, there are constant matrices K and L, and a timing law $\gamma(t)$ such that the feedback law is

$$u(t) = Kx(t) + L(\dot{\gamma}_d)\omega(\gamma(t)) \tag{26}$$

To calculate the matrices K and L, we first solve the following Sylvester equations, which are solvable under the stated conditions

$$\begin{aligned} v_d \Pi S &= A\Pi + B\Gamma \\ 0 &= C\Pi + D\Gamma - Q \end{aligned} \tag{27}$$

Next we solve for K as a minimum quadratic regulator problem that minimizes the cost function

$$J(u) = \int_0^\infty (x^T Q x + u^T R u + 2x^T N u)\, dt \tag{28}$$

and finally calculate L as $L = \Gamma - K \times \Pi$. Once the path controller is designed, an evolution rule to γ has to be created, in a way that $\lim_{t\to\infty} \gamma = \gamma_d$ and $\lim_{t\to\infty} \dot{\gamma} = v_d$.

5. Simulation Results

In this section, simulation results are reported for the IST flexible arm described in Fig. 1.

5.1. Motion Planing – Control where the input is the torque

The method was solved for $t_f = 2.7$s and $y_f = -0.35m$. The closed loop control was obtained using $K_p = 4$ and $K_v = 0.33$. In Figs. 2 and 3 the tracking error and the corresponding elastic displacement of the tip are shown. The tracking error is very small, and this error is due to the fact that the simulated model is quadratic in the deformation and not linear as assumed for the controller [13]. The tip displacement plot shows a nearly antisymmetric curve as is expected for point to point motion without residual vibration. The same behavior is obtained for the input torque as shown in Fig. 4.

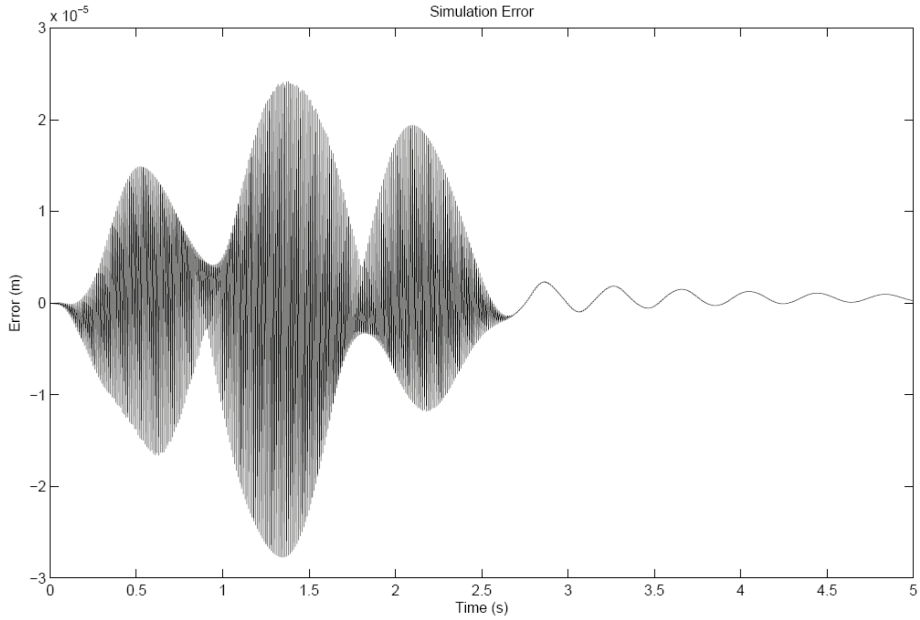

Fig. 2 Tracking error

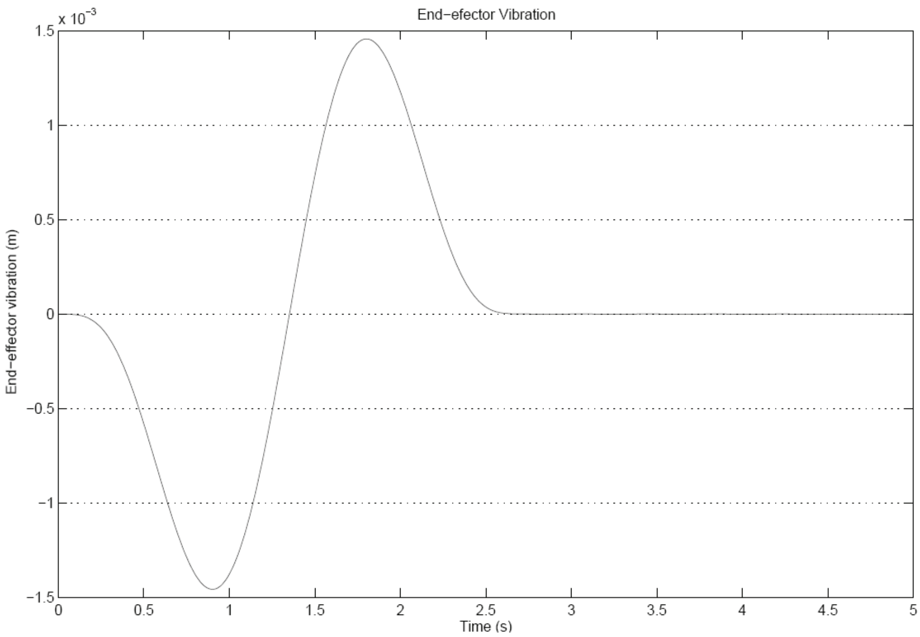

Fig. 3 Tip displacement

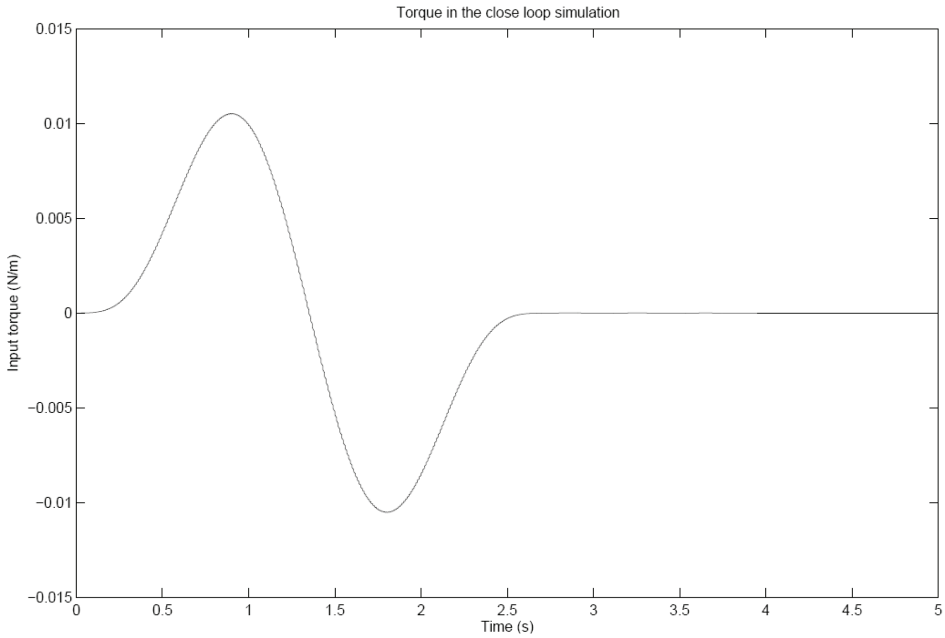

Fig. 4 Input Torque

5.2. Path-following

The path used in this methodology is:

$$\begin{aligned} y_d &= (1 - e^{(-\omega_d \times \gamma)} \times (1 + \omega_d \times \gamma)) y_f \\ \frac{dy_d}{d\gamma} &= (\omega_d e^{(-\omega_d \times \gamma)} \times (1 + \omega_d \times \gamma) - \omega_d e^{(-\omega_d \times \gamma)}) y_f \\ \frac{d^2 y_d}{d\gamma^2} &= (-\omega_d^2 e^{(-\omega_d \times \gamma)} \times (1 + \omega_d \times \gamma) + 2\omega_d^2 e^{(-\omega_d \times \gamma)}) y_f \end{aligned} \tag{29}$$

where the variable ω_d sets the convergence velocity of the system to the final value y_f. For the controller calculus, the Matlab LQR function was used yielding the following gains

$$K = [-10.25 \quad -16.85 \quad -2.32 \quad -181.83 \quad -126.24 \quad 8.12] \tag{30}$$

The value of L is calculated for different values of speed assignments between 0m/s and 5m/s. The five second simulation has $y_f = \pi / 8$ and $\omega_d = 20$.

In Fig. 5 the desired path versus the path variable is plotted. In Figs. 6 and 7 the simulation tracking error and the corresponding deformation of the end-effector are presented. As expected, the tracking error and the end-effector deformation converge to zero.

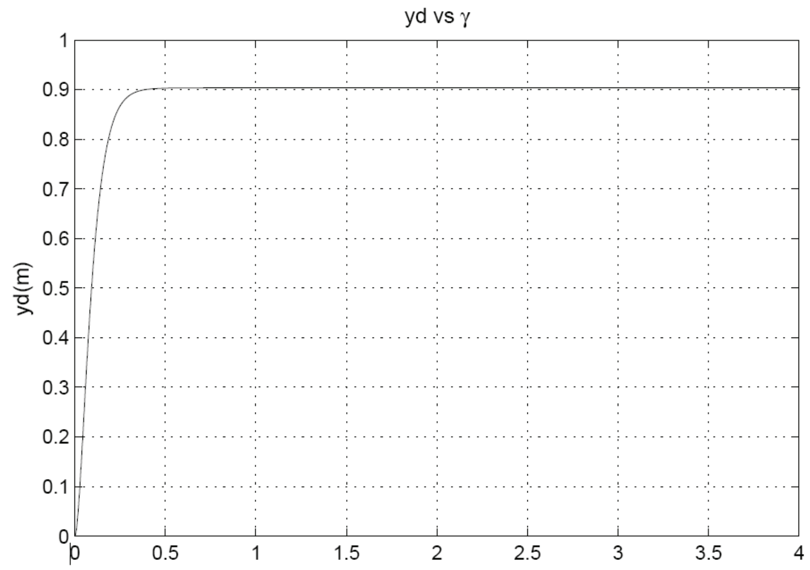

Fig. 5 Desired output versus path variable

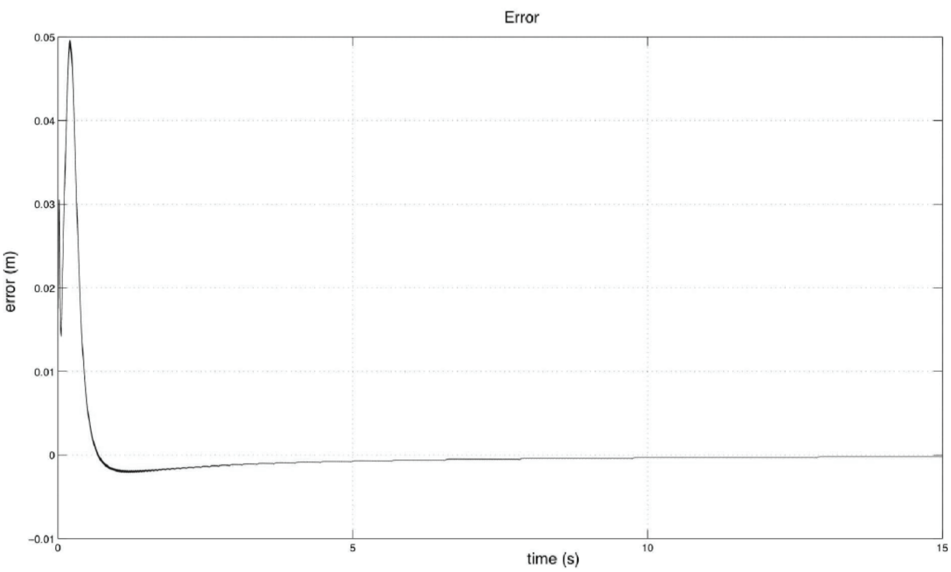

Fig. 6 Tracking error

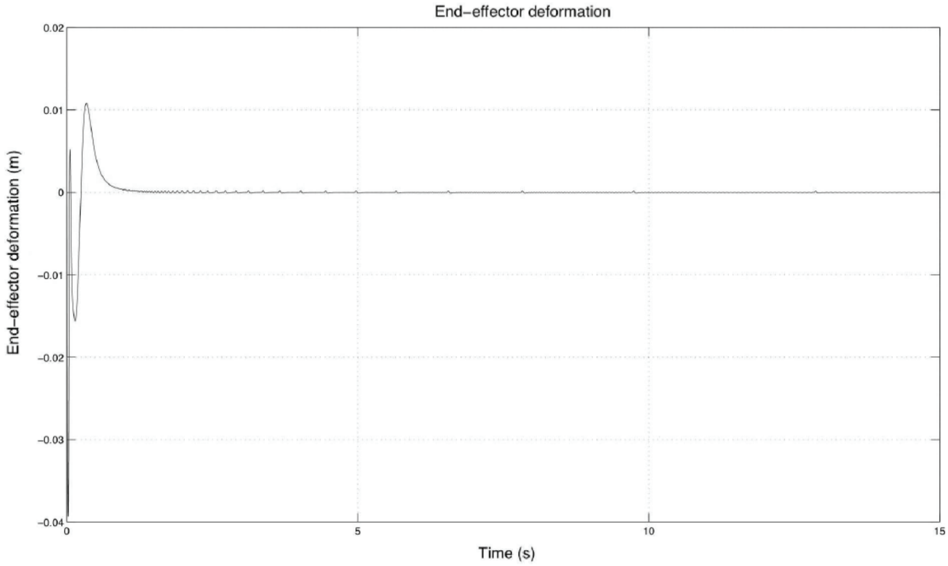

Fig. 7 Deformation of the end-effector

6. CONCLUSIONS

The stable inversion method is a simple and efficient control methodology, where all the computational effort is made offline. Very good results were obtained in the presence of model mismatch due to the higher order terms of the deformation. Path following is still under intensive study, but the present results show its applicability to flexible manipulators. Since the reference-tracking controller is an open-loop controller with respect to the tip, when the reference achieves the final value, the controller becomes passive and it does not observe the end-effector deflection, resulting on a permanent vibration. The path-following ability to separate the dynamic follower controller from the states boundedness controller, improves the control actuation. Even when the system reaches the final value, it still removes the external perturbation. It becomes clear that the path-following controller is a much more developed and robust controller than the reference tracking controller.

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PROSLEĐIVANJE REFERENCE NASPRAM PRAĆENJU PUTANJE ZA FLEKSIBILNI JEDNO-SEGEMENTNI ROBOT MANIPULATOR

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Rad proučava dva pristupa upravljačkog sistema za fleksibilni manipulator gde se tretira dinamički problem neminimalne faze. U prvom pristupu, koristimo tehniku planiranja pokretljivosti. Traži se odgovarajući izlaz za putanju formom polinoma da bi se kompenzovali efekti nestabilnih nula. Drugi pristup nazivamo praćenje putanje sa upravljanjem pomoću internog modela. Njegov osnovni cilj je upravljanje fizičkim objektom kako bi se konvergiralo ka geometrijskoj putanji, a njegov drugi cilj je da se obezbedi da pokretanje objekata putanjom zadovoljava zadatu dinamičku specifikaciju..

Ključne reči: *Fleksibilni manipulator, planiranje kretanja, praćenje putanje, kontrola vibracije*