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**LEONHARD EULER
(1707-1783)
300-TH ANNIVERSARY**

Projekat ON144002 -Seminar

**NONLINEAR DYNAMICS
MILUTIN MILANKOVIC**




IN HONOR OF

LEONHARD EULER
(1707-1783)
300-th Anniversary

At Centre for Nonlinear Dynamics and Active Structures
Mechanical Engineering University of Niš

Leonhard Euler



Born	<u>April 15, 1707</u> <u>Basel, Switzerland</u>
Died	<u>September 18 [O.S. September 7] 1783</u> <u>St Petersburg, Russia</u>
Residence	<u>Prussia</u> <u>Russia</u> <u>Switzerland</u>
Nationality	<u>Swiss</u>
Field	<u>Mathematics and physics</u>
Institutions	<u>Imperial Russian Academy of Sciences</u> <u>Berlin Academy</u>
Alma mater	<u>University of Basel</u>
Religion	<u>Calvinist¹⁾</u>

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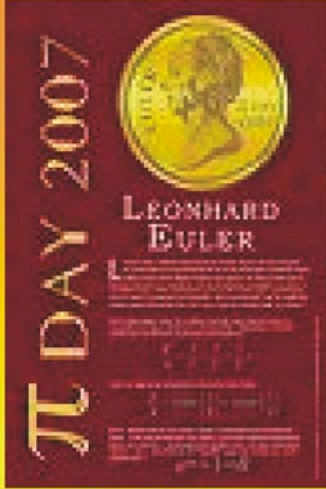
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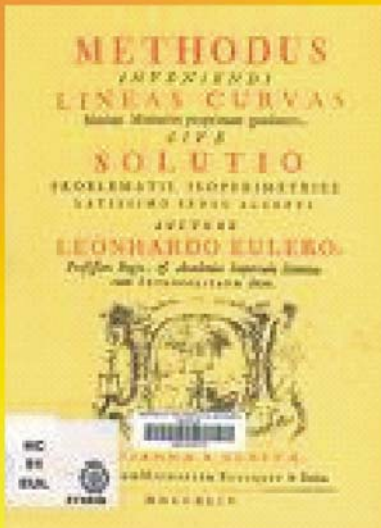
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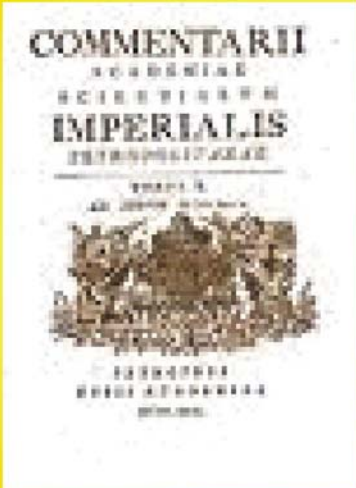
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LEONHARD EULER
(1707-1783)
300-th Anniversary



Selected bibliography

The cover page of Euler's *Methodus inveniendi lineas curvas*.

Euler has an extensive bibliography but his best known books include:

Elements of Algebra. This elementary algebra text starts with a discussion of the nature of numbers and gives a comprehensive introduction to algebra, including formulae for solutions of polynomial equations.

Introductio in analysin infinitorum (1748). English translation *Introduction to Analysis of the Infinite* by John Blanton (Book I, ISBN 0-387-96824-5, Springer-Verlag 1988; Book II, ISBN 0-387-97132-7, Springer-Verlag 1989).

Two influential textbooks on calculus: *Institutiones calculi differentialis* (1755) and *Institutiones calculi integralis* (1768–1770).

Lettres à une Princesse d'Allemagne (Letters to a German Princess) (1768–1772). Available online (in French). English translation, with notes, and a life of Euler, available online from Google Books: Volume 1, Volume 2

Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes, sive solutio problematis isoperimetrici latissimo sensu accepti (1744). The Latin title translates as *a method for finding curved lines enjoying properties of maximum or minimum, or solution of isoperimetric problems in the broadest accepted sense*. [35]

A definitive collection of Euler's works, entitled *Opera Omnia*, has been published since 1911 by the Euler Commission of the Swiss Academy of Sciences.

LEONHARD EULER

Born: 15 April 1707 In Basel, Switzerland
Died: 18 Sept 1783 In St Petersburg, Russia
<http://www.leonhard-euler.ch/>

The Opera Omnia

Published by Birkhauser and the Euler Commission of Switzerland, the Opera Omnia is the definitive printed source for Euler's works. Publication began in 1911, and to date 76 volumes have been published, comprising almost all of Euler's works.

Put simply, the Opera Omnia is the authoritative source of Euler's works. Not only do his writings appear in neatly typeset, edited form, but each volume also includes commentaries, some of them very lengthy and very scholarly, on those of Euler's works in the volume.

In addition to publishing Euler's published works, the Opera Omnia has published three volumes of Euler's correspondence (see link), and has several more volumes in preparation.

Any serious Euler scholarship must take into account this invaluable resource put together by the Euler Commission. The Opera Omnia can be found in many large research libraries, and is well worth the time spent with it.

The Euler Committee of the Swiss Academy of Sciences was founded in 1907 with the task to publish all scientific books, papers and the correspondence of Leonhard Euler (1707-1783) in a scientific edition.

Euler's books and papers are edited in the Series I-III, the correspondence in the Series IV. In the last 90 years 71 volumes of the Series I-III have been published. The last three volumes are in preparation and should appear early in the next century. The Series IV with Euler's scientific correspondence will contain 10 volumes. 4 volumes have been published and three volumes are in active preparation.

This huge endeavour can only succeed with the aid of internationally acclaimed scientists as coworkers and with the financial support of the Swiss National Science Foundation, the Swiss Academy of Sciences and many long-term substantial contributions from Swiss industrial corporations.

Formation and Training

1707 Born 15 April in Basel, the son of the Protestant minister Paul Euler and Margaretha Brucker.

1720 Leonhard entered Basel University, which was founded in 1460. Initially he studied theology, Oriental languages and history, but soon switched to mathematics under Johann Bernoulli (1667-1748), who became the world's most noted mathematician following the death of Isaac Newton (1643-1727). Quick to recognize Euler's mathematical genius, Bernoulli challenged him by having him read the works of the masters, and especially by instructing him personally in contemporary mathematical research.

First Petersburg period

1727 Euler's thesis titled *De Sono* (On Acoustics) formed the basis for his application for a post as professor of physics in Basel, but he was passed over on account of his youth. Through the help of the Bernoullis, he was offered a position in St. Petersburg at the Academy of Science, founded by Peter the Great in 1725. There he worked first as assistant professor, then from 1730 as professor and member of the academy (he had no teaching commitments, though he did write a textbook on elementary mathematics). The principal contributions of this early Petersburg period include a two-volume work on mechanics, a book on music theory and *Scientia navalis* (about hydrodynamics, shipbuilding and navigation), which was eventually published in 1749.

1734 At the beginning of January, Euler married Katharina Gsell, a daughter of the Swiss painter George Gsell, who was working in St. Petersburg. Euler's son Johann Albrecht was born at the end of November, the only one of his offspring to follow in his footsteps as a mathematician and member of the Academy. Only three of Euler's thirteen children would survive him. He had twenty-one grandchildren. 1738 As a result of a severe abscess, Euler lost the sight of his right eye.

Berlin years

1741 Conscious of the political turmoil in the Russian empire, Euler accepted Frederick II's offer of a professorship at the newly established Prussian Academy ("Berlin Academy") and settled with his family in Berlin. There he held a position as director of the mathematics department. Maupertuis, who in 1736 made a name for himself in a famous expedition to Lapland (the purpose of which was to determine whether the Earth was indeed an oblate spheroid) became president of the Academy, though as a scientist, he ranked far below Euler.

In addition to hundreds of treatises written during the Berlin period, Euler produced major works on the calculus of variations, the theory of special functions, differential equations, astronomy as well as a second masterpiece on mechanics and a popular work on physics and philosophy titled *Lettres à une princesse d'Allemagne*. The basic outline of his celebrated work on algebra also dates from the Berlin period. During this time, Euler maintained active connections with the Petersburg Academy, and he helped to promote interactions between the two internationally renowned academies. Euler was a member of all the important academies of his time and received many awards.

Second Petersburg period

1766 Frederick II's bumbling was influential in Euler's accepting an offer from Catherine the Great to return to St. Petersburg, where he remained until his death.

1771 In the aftermath of a failed cataract operation, Euler lost the sight of his remaining good eye and soon became nearly totally blind. During the great St. Petersburg fire, he was barely saved from his burning house by the Basler artisan Peter Grimm. Yet, amazingly, his productivity increased: approximately half of his prodigious output occurred during this second Petersburg period, including three-volume works on integral calculus and optics (*Institutiones calculi integralis* and *Dioptrica*) as well as the authoritative version of his work on algebra.

1773 Following the death of his wife Katherina, in 1776 Euler married her half-sister Abigail Gsell.

1783 On 18 September Euler suffered a stroke and died quickly and painlessly.

57 results "**Opera Omnia**" in the category Books and CD-ROMs

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The "Euler phenomenon"

Three factors go a long way to explaining the "Euler phenomenon": First of all, his - perhaps uniquely - gifted memory. Whatever Euler heard, saw, thought, or wrote he seems to have remembered his whole life long, as countless of his contemporaries would attest. So it is, for example, that at an advanced age he was able to delight his relatives, friends, and acquaintances with a literal (Latin) recitation of any song from Virgil's Aeneis, and that he could recall by heart minutes of academy meetings decades later, to say nothing of his memory for mathematical things. Second, Euler's prodigious memory went hand in hand with a rare ability to concentrate. Noise and bustle in his immediate environment hardly disturbed his thinking: "A child on his knees, a cat on his back - this is how he wrote his immortal works," reported Thiébauld, his colleague from the Berlin academy. The third factor in the "Euler mystery" is, quite simply, constant, calm work.

Reputation

Leonhard Euler's influence and reputation were already impressive during his lifetime. He was (according to Andreas Speiser) for roughly two decades the intellectual leader of the Protestant part of Germany, and (according to Eduard Winter) he performed inestimable services as the "golden bridge between two academies". The 10 volumes of his correspondence testify to this role, as does the fact that, during his Berlin years, Euler published 109 papers in the Petersburger Kommentare as opposed to the 119 he published in the Mémoires of the Berlin academy. And although Euler's energy was sufficient for him to keep up his activities at both institutions, the institutions themselves could not easily cope with the almost inexhaustible tide of Euler's productivity. To judge simply from the extent of his work, Euler is in the company of the most prolific members of the human race, for instance, Voltaire, Leibniz, Telemann or Goethe. The directory of Euler's writings published by G. Eneström (1910-1913) takes up an entire volume and lists almost 900 titles, among them some 40 books.

Productivity

The following table summarizes the extent of Euler's writings specified by him as ready for publication, arranged according to decades (not included are a few dozen works that have not yet been dated):

Year	Works	%
1725–1734	35	5
1735–1744	50	10
1745–1754	150	19
1755–1764	110	14
1765–1774	145	18
1775–1783	270	34

Technical range

With respect to technical discipline, the writings break down approximately as follows:

Algebra, number theory, analysis.....	40%
Mechanics and other physics.....	28%
Geometry, including trigonometry.....	18%
Astronomy.....	11%
Ship theory, artillery, architecture.....	2%
Philosophy, music theory, theology, and anything else not included above.....	1%

The distribution of Euler's purely mathematical works is approximately as follows:

Algebra, combinatorics, and probability theory.....	10%
Number theory.....	13%
Fundamental analysis and differential calculus.....	7%
Infinite series.....	13%
Integral calculus.....	20%
Differential equations.....	13%
Calculus of variations.....	7%
Geometry, including differential geometry.....	17%

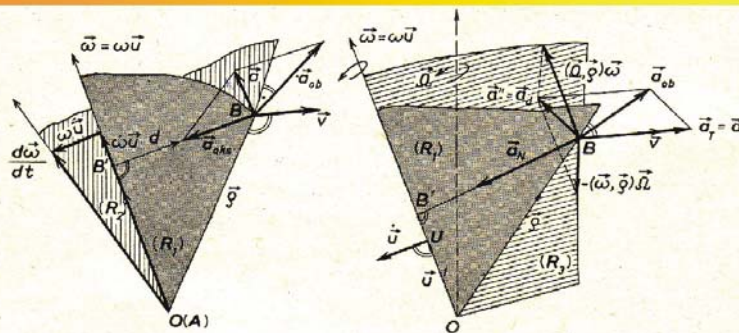
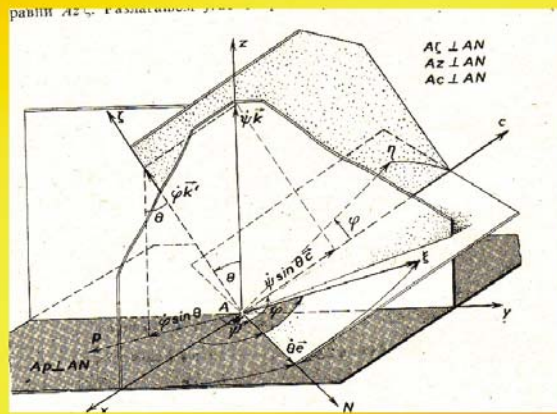
Prizes

Altogether Leonhard Euler won 12 international academy prizes, not counting the prizes of his sons Johann Albrecht (7) and Karl (1), which can essentially also be credited to Euler's account. The Frenchman King Louis XVI awarded Euler 1000 rubles for his "second ship theory", and the Russian empress Katharina II gave him double that amount so that the blind doyen of Petersburg could collect a supplementary salary in 1773.

Influence

On the subject of Euler the judgment of the most important mathematicians is unanimous. Laplace used to say to his students: "Read Euler, read Euler! He is the master of us all!" and Gauss explained emphatically: "The study of Euler's works remains the best instruction in the various areas of mathematics and can be replaced by no other." Indeed, through his books, which are consistently characterized by the highest striving for clarity and simplicity and which represent the first actual textbooks in the modern sense, Euler became the premier teacher of Europe not only of his time but well into the 19th century.

$$\begin{aligned} \omega_x &= \dot{\theta} \cos \psi + \dot{\varphi} \sin \theta \sin \psi; & \omega_z &= \dot{\theta} \cos \varphi + \dot{\psi} \sin \theta \sin \varphi; \\ \omega_y &= \dot{\theta} \sin \psi - \dot{\varphi} \sin \theta \cos \psi; & \omega_\eta &= -\dot{\theta} \sin \varphi + \dot{\psi} \sin \theta \cos \varphi; \\ \omega_z &= \dot{\psi} + \dot{\varphi} \cos \theta; & \omega_\zeta &= \dot{\varphi} + \dot{\psi} \cos \theta, \end{aligned}$$



Диференцирајући израз (225) по времену, угаоно убрзање може се изразити помоћу Ојлерових углова и њихових извода по времену, те је

$$\frac{d\vec{\omega}}{dt} = (\dot{\theta} + \dot{\varphi}\dot{\psi} \sin \theta)\vec{e} + (\dot{\psi} - \dot{\varphi}\dot{\theta} \sin \theta)\vec{k} + \dot{\theta}(\dot{\varphi} \cos \theta - \dot{\psi})[\vec{c}, \vec{k}] + \dot{\varphi}\vec{k}'. \quad (244)$$

Стога су пројекције овог вектора:

a) на осе неизокрећеног триебра $Oxyz$:

$$\begin{aligned} \dot{\omega}_x &= \dot{\theta} \cos \psi - \dot{\theta} \dot{\psi} \sin \psi + \dot{\varphi} \sin \theta \sin \psi + \dot{\theta} \dot{\varphi} \cos \theta \sin \psi + \dot{\psi} \dot{\varphi} \sin \theta \cos \psi; \\ \dot{\omega}_y &= \dot{\theta} \sin \psi - \dot{\theta} \dot{\psi} \cos \psi - \dot{\varphi} \sin \theta \cos \psi - \dot{\theta} \dot{\varphi} \cos \theta \cos \psi + \dot{\varphi} \dot{\psi} \sin \theta \sin \psi; \\ \dot{\omega}_z &= \dot{\psi} + \dot{\varphi} \cos \theta - \dot{\theta} \dot{\varphi} \sin \theta; \end{aligned} \quad (244a)$$

Диференцирајући израз (225) по времену, угаоно убрзање може се изразити помоћу Ојлерових углова и њихових извода по времену, те је

$$\frac{d\vec{\omega}}{dt} = (\ddot{\theta} + \dot{\varphi}\dot{\psi}\sin\theta)\vec{e} + (\ddot{\psi} - \dot{\varphi}\dot{\theta}\sin\theta)\vec{k} + \dot{\theta}(\dot{\varphi}\cos\theta - \dot{\psi})[\vec{e}, \vec{k}] + \ddot{\varphi}\vec{k}. \quad (244)$$

Стога су пројекције овог вектора:

a) на осе *нейокрејној триједра Охуз*:

$$\begin{aligned} \dot{\omega}_x &= \ddot{\theta}\cos\psi - \dot{\theta}\dot{\psi}\sin\psi + \ddot{\varphi}\sin\theta\sin\psi + \dot{\theta}\dot{\varphi}\cos\theta\sin\psi + \dot{\psi}\dot{\varphi}\sin\theta\cos\psi; \\ \dot{\omega}_y &= \ddot{\theta}\sin\psi - \dot{\theta}\dot{\varphi}\cos\psi - \ddot{\psi}\sin\theta\cos\psi - \dot{\theta}\dot{\varphi}\cos\theta\cos\psi + \dot{\varphi}\dot{\psi}\sin\theta\sin\psi; \\ \dot{\omega}_z &= \ddot{\psi} + \ddot{\varphi}\cos\theta - \dot{\theta}\dot{\psi}\sin\theta; \end{aligned} \quad (244a)$$

10. Обртање тела око непомичне тачке

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b) на осе *покрејној триједра Оξηζ*:

$$\begin{aligned} \dot{\omega}_\xi &= \ddot{\theta}\cos\varphi - \dot{\theta}\dot{\varphi}\sin\varphi + \ddot{\psi}\sin\theta\sin\varphi + \dot{\theta}\dot{\psi}\cos\theta\sin\varphi + \dot{\varphi}\dot{\psi}\sin\theta\cos\varphi; \\ \dot{\omega}_\eta &= -\ddot{\theta}\sin\varphi - \dot{\theta}\dot{\varphi}\cos\varphi + \ddot{\psi}\sin\theta\cos\varphi + \dot{\theta}\dot{\psi}\cos\theta\cos\varphi - \dot{\varphi}\dot{\psi}\sin\theta\sin\varphi; \\ \dot{\omega}_\zeta &= \ddot{\varphi} + \ddot{\psi}\cos\theta - \dot{\theta}\dot{\psi}\sin\theta. \end{aligned} \quad (244b)$$

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Динамика

Ова једначина претставља *Ојлерову динамичку једначину у векторском облику* Њој одговарају *три* скаларне једначине за координатне осе покретног триједра *Оξηζ*:

$$\begin{aligned} \dot{L}_\xi + \omega_\eta L_\zeta - \omega_\zeta L_\eta &= \mathfrak{M}_\xi \\ \dot{L}_\eta + \omega_\zeta L_\xi - \omega_\xi L_\zeta &= \mathfrak{M}_\eta \\ \dot{L}_\zeta + \omega_\xi L_\eta - \omega_\eta L_\xi &= \mathfrak{M}_\zeta \end{aligned} \quad (374')$$

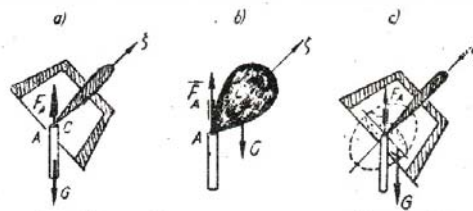
Пројекције замаха на координатне осе одређене су једначинама (370').

Када су координатне осе *главне осе инерције*, Ојлерове динамичке једначине биће простје и, према (370'), гласе:

$$\begin{aligned} J_1 \dot{\omega}_1 - (J_2 - J_3) \omega_2 \omega_3 &= \mathfrak{M}_1 \\ J_2 \dot{\omega}_2 - (J_3 - J_1) \omega_3 \omega_1 &= \mathfrak{M}_2 \\ J_3 \dot{\omega}_3 - (J_1 - J_2) \omega_1 \omega_2 &= \mathfrak{M}_3 \end{aligned} \quad (375)$$

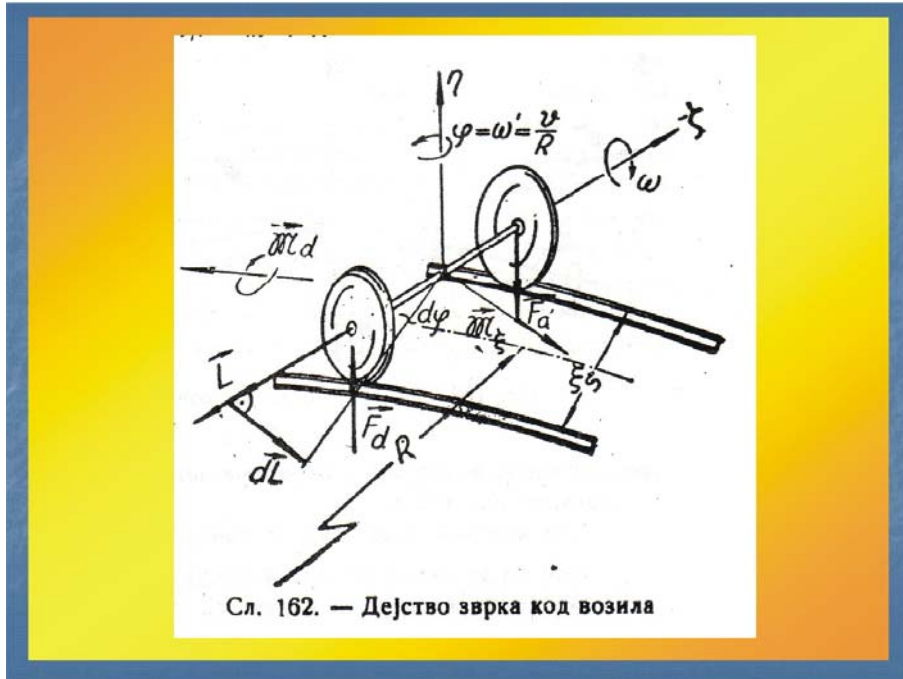
Овде су J_1, J_2, J_3 главни моменти инерције, $\omega_1, \omega_2, \omega_3$ пројекције вектора угаоне брзине на главне осе а моменти $\mathfrak{M}_1, \mathfrak{M}_2, \mathfrak{M}_3$ главни моменти свих спољашњих сила за те осе.

2° **Лагранжево решење.** — Основна је претпоставка код овог решења да је *елипсоид инерције* за *непомичну тачку обрћења*. Тело је *тешко* а тежиште тела налази се на геометриској оси обртног елипсоида — *фигурној оси* (сл. 151 *b*). Ако је оса Oz та оса, онда је $J_1 = J_2$ а тежиште тела налази се на тој оси на удаљењу s од *непомичне тачке* O .



Сл. 151. — Класична решења проблема обртања крутог тела око *непомичне тачке*





$$\vec{p}(t) = \iiint_V \vec{v} dm = \iiint_V [\vec{\omega}, \vec{r}] dm = \omega \iiint_V [\vec{n}, \vec{r}] dm = \omega \vec{S}_O^{(\vec{n})}$$

$$\vec{L}_O = \iiint_V d\vec{L}_O \stackrel{def}{=} \iiint_V [\vec{r}, d\vec{p}] = \iiint_V [\vec{r}, \vec{v}] dm$$

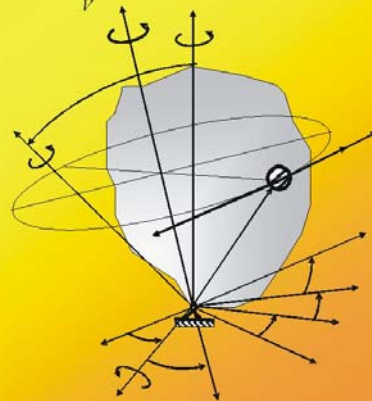
$$\vec{L}_O = \iiint_V [\vec{r}, [\vec{\omega}, \vec{r}]] dm = \omega \iiint_V [\vec{r}, [\vec{n}, \vec{r}]] dm = \omega \iiint_V d\vec{J}_O^{(\vec{n})} = \omega \vec{J}_O^{(\vec{n})}$$

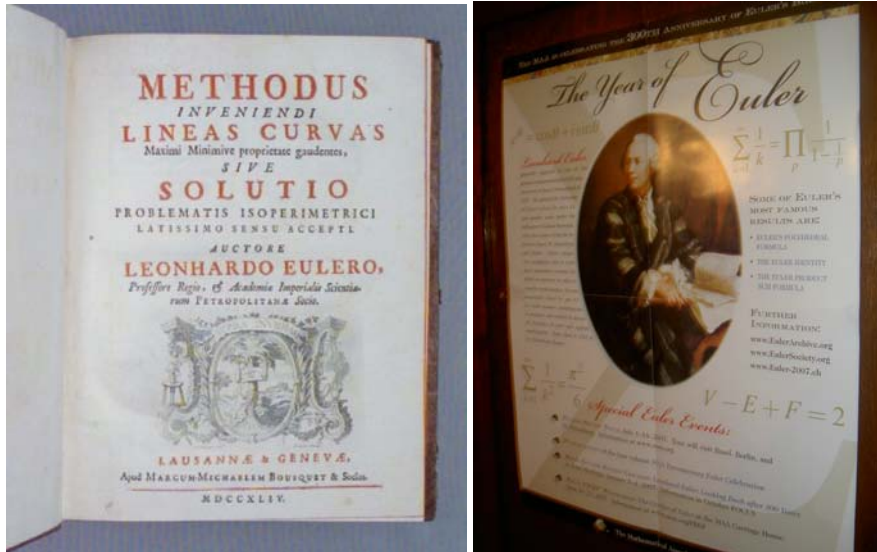
$$\vec{\omega} = \dot{\vartheta} \vec{e} + \dot{\psi} \vec{k} + \dot{\phi} \vec{k}'$$

$$\omega_x = \dot{\vartheta} \cos \psi + \dot{\phi} \sin \vartheta \sin \psi$$

$$\omega_y = \dot{\vartheta} \cos \psi - \dot{\phi} \sin \vartheta \cos \psi$$

$$\omega_z = \dot{\psi} + \dot{\phi} \cos \vartheta$$





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