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# STRING AND BEAM-LIKE MODELS AND THE REDUCTION PROBLEM

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**Abstract**. *In this short communication two types of belt vibrations are discussed and boundaries of their application are established.*

**Key words**: *string, reduction, vibration, boundaries.*

The investigation of moving objects approximated by one-dimensional equations (belts, tapes, and cables) is very important from the view of applications (see [1-6] and the references therein). One may expect that the equations governing the dynamics of the given objects are properly derived for both linear and non-linear cases. However, even for the linear case, the problem is reduced to a consideration of the infinite systems of ordinary differential or algebraic equations (see examples given in references [1-4]).

In this report only a linear case is considered, although the obtained results can be easily generalized into a non-linear case.

As it has been mentioned in reference [1], the belt vibrations can be classified into two types, i.e. that of a string-like type or of a beam-like type, depending on the bending stiffness of a belt.

We are going to establish boundaries of applications of two mentioned models. For a linear case elementary transformations are needed to carry out the study. We show that the obtained linear estimations hold also for a non-linear case.

In the computational scheme, a conveyor belt is modelled by a stretched beam of length $L$ (Figure 1).

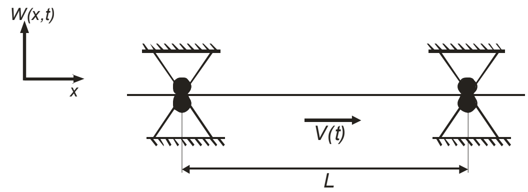


Fig. 1. Schematic model of a conveyor belt [1].

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The governing equations can be reduced to the following form [1, 2]

$$\rho F\frac{\partial^2 W}{\partial t^2}+2V\frac{\partial^2 W}{\partial x\partial t}+V^2\frac{\partial^2 W}{\partial x^2}-T\frac{\partial^2 W}{\partial x^2}+V_1\frac{\partial W}{\partial x}+EI\frac{\partial^4 W}{\partial x^4}=0, \tag{1}$$

where: $W(x,t)$ is the displacement of the belt in the vertical direction; $V$ is the time-varying belt speed, $V_1 = \partial V / \partial t$; $\rho$ is the mass density of the belt; $F$ and $I$ are the area and first moment of the beam cross-section; $t$ is time; $x$ is spatial coordinate; $T$ is constant tension.

The following boundary conditions are applied:

$$W=0 \quad \text{for} \quad x=0,L\,; \tag{2}$$

$$\frac{\partial^2 W}{\partial x^2}=0 \quad \text{for} \quad x=0,L\,. \tag{3}$$

In computations both equation (1) with boundary conditions (2), (3) (beam like approximation), and the so called string-like approximation governed by equation

$$\rho F\frac{\partial^2 W}{\partial t^2}+2V\frac{\partial^2 W}{\partial x\partial t}+V^2\frac{\partial^2 W}{\partial x\partial x}-T\frac{\partial^2 W}{\partial x\partial x}+V_1\frac{\partial W}{\partial x}=0 \tag{4}$$

with boundary condition (2) are applied. It should be emphasized that while solving equation (4), infinite systems appear which cannot be reduced to finite ones [1].

In what follows we show that the occurred difficulties are only of mathematical character and they do not possess any physical insight. We put $V = 0$ and transform equation (1) to non-dimensional form

$$\frac{\partial^2 w}{\partial \tau^2}-\frac{\partial^2 w}{\partial \xi^2}+\varepsilon^2\frac{\partial^4 w}{\partial \xi^4}=0 \tag{5}$$

where: $w=W/h$; $\xi=x/L$; $\tau=tL\sqrt{T/\rho F}$; $\varepsilon^2=EI/(TL^2)$.

Recall that for physically motivated considerations the parameter $\varepsilon$ is small, i.e. $\varepsilon << 1$.

The associated boundary conditions (2), (3) are also transformed into the equivalent non-dimensional form

$$w=\frac{\partial^2 w}{\partial \xi^2}=0 \quad \text{for} \quad \xi=1\,. \tag{6}$$

Eigenfrequencies and associated modes of systems (6), (7) vibrations read:

$$\omega^2=(\pi n)^2+\varepsilon^2(\pi n)^4\,, \tag{7}$$

$$w=Ce^{i\omega\tau}\sin(\pi n\xi)\,. \tag{8}$$

Owing to formula (7) one may convince himself that the string-model (4) can be only applied either for

$$\varepsilon^2(\pi n)^4 << 1 \quad \text{or} \quad n>\frac{1}{\pi\varepsilon^{1/2}}\,. \tag{9}$$

This observation yields a conclusion that the problem of occurrence of infinite systems associated with analysis of string-like model does not appear at all.

## REFERENCES


1. Suweken G., Van Horssen W.T., (2003), On the transversal vibrations of a conveyor belt with a low and time-varying velocity. Part I: the string-like case, *Journal of Sound and Vibration*, 264, 117-133, 2003.
2. Suweken G., Van Horssen W.T., (2003), On the transversal vibrations of a conveyor belt with a low and time-varying velocity. Part II: the beam-like case, *Journal of Sound and Vibration*, 267, 1007-1027, 2003.
3. Suweken G., Van Horssen W.T., (2003), On the weakly nonlinear transversal vibrations of a conveyor belt with a low and time-varying velocity, *Nonlinear Dynamics*, 31, 197-203, 2003.
4. Pelikano F., Vestroni F., (2000), Nonlinear dynamics and bifurcations of an axially moving beam, *Journal of Vibration and Acoustics*, 122, 21-30, 2000.
5. Wickert J.A., (1992), Non-linear vibration of a travelling tensioned beam, *International Journal of Nonlinear Mechanics*, 27(3), 503-517, 1992.
6. Sack R.A., (1954), Transverse oscillations in travelling strings, *British Applied Physics*, 5, 224-226, 1954.
7. Andrianov I.V., Kholod E.G., Olevsky V.I., (1990), Approximate nonlinear boundary value problems of reinforced shell dynamics, *Journal of Sound and Vibration*, 194(3), 369-387, 1990.
8. Awrejcewicz J., Andrianov I.V., Manevitch L.I., (1998), *Asymptotic Approaches in Nonlinear Dynamics: New Trends and Applications*, Springer-Verlag, Heidelberg, Berlin, New York, 1998.
9. Andrianov I.V., Manevitch L.I., (2002), *Asymptotology: Ideas, Methods and Applications*, Kluwer, Dordrecht, 2002.


# MODELI TIPA STRUNE I GREDE U ZADACIMA REDUKCIJE

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*U ovom kratkom radu dva tipa oscilacija trake su prikazana, kao i oblasti njihove primene.*

Ključne reči: *struna, redukcija, vibracije, granice.*