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3D SPUR GEAR FEM MODEL FOR THE NUMERICAL CALCULATION OF FACE LOAD FACTOR

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Ivana Atanasovska[1], Vera Nikolić-Stanojević[2]
[1]Institut IMS a.d., Bulevar vojvode Mišića 43, 11000 Belgrade
e-mail: iviatanasov@yahoo.com
[2] Faculty of Mechanical Engineering, The University of Kragujevac, Sestre Janjić 6, 34000 Kragujevac
e-mail: vnikolic@knez.uis.kg.ac.yu

Abstract. *The aim of this paper is to give a new viewpoint in face of load factor calculation as a part of spur gears load capacity calculation. The described method utilize numerical Finite Element Method models for calculation of stress and deformation values and one completely new access in face of load factor determination.*
With the emphasis on this, a very complex problem of load distribution in gear pair mesh is first described by authors. In accordance with the analysis of mathematical solution the authors developed 3D finite element (FEM) model for simultaneously monitoring strain and stress state of teeth's flanks, teeth's roots and parts of gears. The special types of contact finite elements that define contact of two deformable bodies are used for teeth's flanks contact simulation. One particular part of paper is devoted to solution of the load distribution over gear facewidth when initial mesh misalignment is taken into consideration.
The analysis of obtained numerical results shows that the developed models are very appropriate for research of the described problem.The conclusions that resulted from the analysis of numerical results are one new step and different aspect in the calculation of face load factor for spur gears.

Key words: *spur gears, load distribution, Finite Element Method, contact strain*

1. INTRODUCTION

The problem of load distribution over a gear facewidth could be solved separately from the problem of load distribution over simultaneously meshed tooth pairs when researched meshed gears are cylindrical involute straight-tooth gears (spur gears). Paper [13] describes the solution of load distribution over simultaneously meshed tooth pairs with numerical Finite Element Method (FEM). This solution is used as the starting point in research described in this paper. The fact that maximal stresses appear during single meshed tooth pair period is one of the basic assumptions for the separate solution of load distribution over a gear facewidth.

In real gear working conditions, the load distribution over gear facewidth is non-uniform and can be described with function $q(z)$, which defines the unite load change

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along the tooth pair contact line when B defines the length of tooth bearing pattern. At the same time, it is important to notice that the length of tooth bearing pattern (B) isn't always equal to gear facewidth. In order to reduce overloading of the same parts of meshed tooth flanks, as well as to gears load capacity increase (which is in accordance with the reduction of the fatigue and tooth fracture risk), this deviation should be as small as possible. It can be achieved with the increase of nominal load value and the reduction of mesh misalignment due to manufacturing deviations.

Therefore, the developing of this original procedure for the detailed research of the problem of load distribution over gear facewidth, which is described in this paper, a very important step up and progress in gear calculation.

2. THE MATEMATICAL SOLUTION FOR LOAD DISTRIBUTION OVER GEAR FACEWIDTH

The system of integral equation, which consists of the contact equation and balance equation, represents the starting point for determination of real load distribution over gear facewidth. This system can be presented in the following form:

$$\int_0^B q(z)\cdot K(z,u)dz = \Delta + F_\beta(z) \quad (1)$$

$$\int_0^B q(z)dz = F_{bn} \quad (2)$$

Where: $q(z)$ – is the function of unite load change along the tooth pair contact line; B – is the length of real tooth pair bearing pattern; $K(z,u)$ – is influence function, which defines the relation between u (elastic deformation at one particular point on the contact pattern) and $q(z)dz$ (concentrated load at the same point); z – is the coordinate of the studied point along contact pattern; Δ – is total tooth pair deformation in the direction normal to tooth pair contact pattern; $F_\beta(z)$ – is mesh initial misalignment (deviation between pinion tooth facewidth direction and wheel tooth facewidth direction when the gear pair is nonloaded); F_{bn} – is total normal load value for gear pair in mesh.

It's very hard or almost impossible to determine real values for many factors and variables that have crucial influence on the accurate form of the function $q(z)$, as well as on the value of real tooth pair bearing pattern length B. Also, there is very small possibility to determine and to take into the calculation the real values for gear body deformation, gear rim deformation and deformations of all other parts of gear transmission. Therefore, the determination of function $q(z)$, with very high level of accuracy, is impossible and a system of integral equations defined with the expressions (1) and (2) can be solved only by numerical method usage with the same simplification and assumptions.

It is important to notice and analyze the discrete method for solving the problem of load distribution over gear facewidth, which was developed by prof. K.I.Zablonski, [3]. During single meshed tooth pair period, the main principle of this method defines tooth pair contact pattern like the final number of equal segments. Generally, a length of these segments is nearly a value of gear pair module m, figure 1 Then, value of load $q_j(z)$ that acts on the j^{th} segment (part) of tooth pair contact pattern substitutes with uniform unite load $\overline{q_j}$, i.e. with concentrated force $F_j = \overline{q_j}\cdot\Delta z_j$. Finally, the discrete method developed by prof. Zablonski gives the system of integral equations that can be translated to the equivalent system of algebraic equations. A lot of numerical methods can be use for this translation.

The matrix form of the mentioned equivalent system of algebraic equations is:

$$\begin{bmatrix} K_{11} & K_{12} & . & . & . & K_{1n} \\ K_{21} & K_{22} & . & . & . & K_{2n} \\ \vdots & \vdots & & & & \vdots \\ K_{n1} & K_{n2} & . & . & . & K_{nn} \\ 1 & 1 & . & . & . & 1 \end{bmatrix} \cdot \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_n \end{bmatrix} = \begin{bmatrix} \Delta_1 + F_{\beta 1} \\ \Delta_2 + F_{\beta 2} \\ \vdots \\ \Delta_n + F_{\beta n} \\ F_{bn} \end{bmatrix} \quad (3)$$

Where each matrix element K_{jk} represents the sum of all influence factors for both gears, i.e. the total coefficient for influences of force that acts on the k^{th} segment of tooth pair contact pattern to the deformation of the j^{th} segment of the tooth pair contact pattern. The defined total influences coefficient can be represented with the following expression:

$$K_{jk} = K_{jk}^{\ 1} + K_{jk}^{\ 2} + \ . \ . \ . \ + K_{jk}^{\ s} \quad (4)$$

K_{jk}^1, K_{jk}^2, . . . , K_{jk}^s – is the influence coefficients from tooth bending, tooth contact strain, as well as from torsion, bending and shear of shafts, deformations of bearings and gear case.

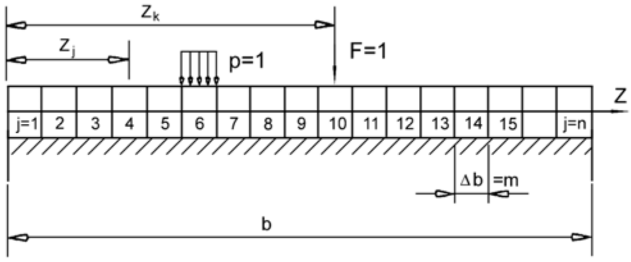

Fig. 1. Partition of tooth pair contact pattern on finale number of equal segments

In equation (3), F_j, j=1,n are unknown forces that act on the center points of segments and their values define the load distribution over gear facewidth, and Δ_j, j=1,n is the displacements of the center points of contact pattern segments which are increased for corresponding mesh misalignments due to manufacturing and assembly ($F_{\beta j}$, j=1,n). Δ_j and $F_{\beta j}$ are corresponding values in the direction which is identical to the direction of normal load. The solution of the system of algebraic equations (3) gives values F_j and this define the real load distribution over facewidth and real length of tooth bearing pattern during transmission of external load defined with force F_{bn}.

This discrete method defined above gives the possibility to derive solution for complex problem of load distribution in tooth mesh. However, it carries some disadvantages. More precisely, it is so generalized that doesn't permit detection of simplifying tasks, which can be easily perceived during researches of real cases. The described method is used in many papers, [7], [10] and [12], which include trials for solution of the complex problem of load distribution over gear facewidth. Those trials are based on the reduction of the number of influence coefficients that are components of the sum of all influence factors K_{jk}, (4). The equivalent system of algebraic equations (3) is especially suitable to solving with numerical methods. Most quoted practical researches of load distribution over gear facewidth in spur gear mesh, [6], [10], [11] and [12], solve the defined problem by numerical Finite Element Method and the research influence of mesh misalignments and various gear body constructions. This research is used in this paper to define the initial principles of FEM models developed in order to solve the problem of load distribution over gear facewidth in real working conditions. These principles are:

- to use the discrete method with the same simplifications that enables simplex calculation of defined problem and easy observation of important values influence on the gear load capacity;
- to disregard deviations and elastic deformations of other gear transmission elements and to take into consideration teeth deformations and gear body deformations, as well as mesh misalignments and inconstancy of tooth stiffness over gear facewidth;
- to use the Finite Element Method (FEM) to obtain numerical solution for load distribution, because FEM method is the best choice for this purpose;
- to improve FEM models of other authors, especially with the accent on complex determination of total tooth deformation along tooth contact pattern;
- to develop such FEM model that enables obtaining of very precise values for tooth deformation u_H – an effect of strains and stresses in tooth contact zone;
- to assume linear tract of total initial mesh misalignment and take maximum value of standard mesh misalignment deviation (ISO 1328-1, [14]) as total initial mesh misalignment.

3. 3D SPUR GEAR FEM MODEL

The physical model chosen for the verification and analysis is a particular gear pair for large transport machines. Its main characteristics are: number of teeth $z_1 = 20$, $z_2 = 96$; addendum modification coefficients $x_1 = 0$, $x_2 = 0.328$; facewidth $b = 350mm$; module $m = m_n = 24$; pressure angle $\alpha = \alpha_n = 20°$; helix angle $\beta = 0°$; rotational wheel speed $n_2 = 4.1596 min^{-1}$; pinion torque $T_1 = 526.41667 KN{\cdot}m$; wheel torque $T_2 = 2526.8 KN{\cdot}m$; material designation (according to JUS) Č 4520; $E = 206\ 000 N/mm^2$; $\nu = 0{,}3$.

In accordance with investigations given in paper [5] and [9], expected maximum contact stress point on path of contact is point B – point of passing from period with two tooth pairs in contact to single meshed tooth pair period. Because of that, 3D FEM tooth contact model is developed for contact in point B and for single meshed tooth pair period.

The finite element types chosen for 3D FEM gear pair model developing are:

- 3D isoparametric structural solid element defined by eight points – for 3D gear modeling; and
- 3D point to surface contact element – for tooth contact modeling.

3D FEM gear pair models are derived like sweeped (copied) 2D model in normal direction along the length equal to gears facewidth, [13]. The facewidth value for studied gear pair is equal to facewidth value used in correspond 2D FEM model, [13], i.e. $b=30\ mm$ (from real $b = 350mm$). Normal forces used in the analysis as an external load $F_{bn} = F_{bn1} = F_{bn2} = 50KN$ (the quarter of real external load). Fifteen sections (segments) along the gear facewidth, which exist in developed 3D FEM models, give a possibility for very precise determination of stress state and load distribution along gear facewidth. Two symmetrical FEM models have been developed for the studied gear pair, one for determining pinion deformation and stress state and other for determining wheel deformation and stress state. The contact nonlinearity in tooth contact zones has been modeled by symmetric contact elements groups. Figure 2 represents discrete models (finite element meshes) for symmetrical 3D FEM contact models developed for the tooth pair contact in point *B*.

Figure 3 represents 3D FEM models displacement constraints and external load application. The boundary conditions on any 3D FEM model are defined by displacement constraints at the direction normal to the surfaces which separate the modeled gear segment from the rest of gear body. Also, displacement constraints at the direction normal to the transverse plane (z-axis of Global Cartesian Coordinate System) are set at the nodes placed on gear rim border. The tooth elements of meshed gear (elements for external load transmission) also have displacement constraints at the direction normal to the load direction (x-axis of Global Cartesian Coordinate System) set at all nodes on tooth borders. In order to achieve statically stable models, the elements-teeth have displacement constraints at the direction normal to teeth transverse plane (z-axis of Global Cartesian Coordinate System) set at all nodes placed on teeth transverse planes. The external load is defined on the elements-teeth by few concentrated forces at the path of contact direction (y-axis of Global Cartesian Coordinate System) set at nodes placed on the middle part of gear facewidth, like fig.3 represents.

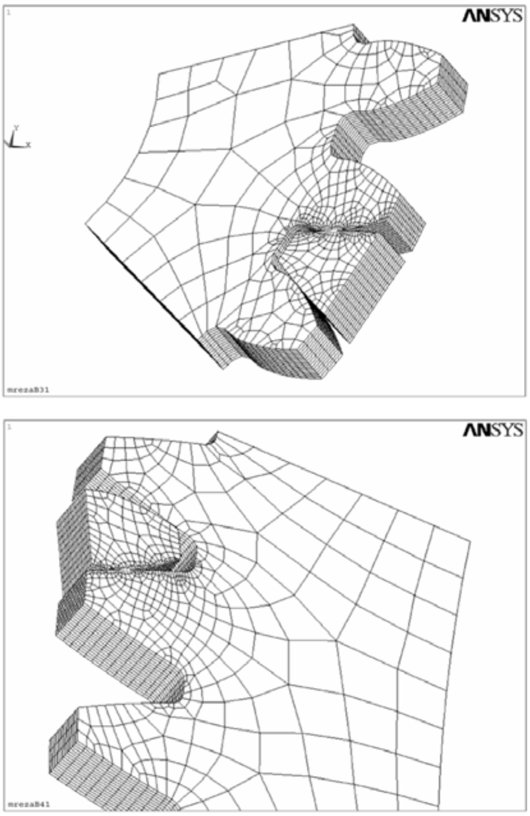

Fig. 2. 3D FEM model for gear pair in mesh

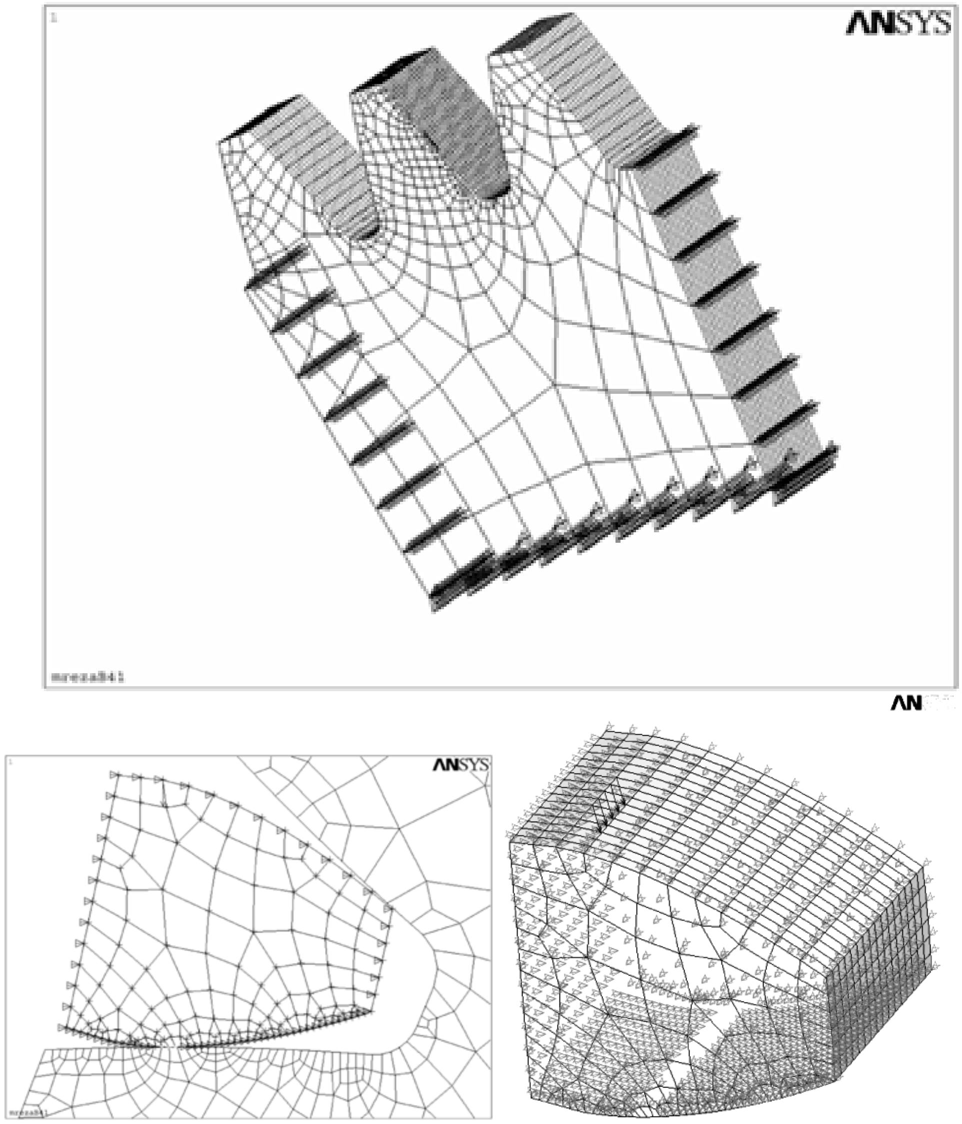

Fig. 3. Displacement constraints and external load application for 3D FEM wheel model

4. THE NUMERICAL RESULTS FOR FACE LOAD FACTOR

FEM calculations for developed FEM gear models described above, have given the numerical results for deformation and stress states of meshed gears. Fig.4 and fig.5 represent equivalent stresses and direct stresses in the path of contact direction obtained for pinion tooth and wheel tooth respectively. On these figures the defined stresses in teeth contact zone can be readily recognized, as well as in teeth root and through teeth body. The obtained results correspond with expected results, based on the results of other authors, [8]. The values of equivalent stress σ_{INT} and direct stress at *y*-direction σ_y which defined the stress state of contact zone can be compared with corresponding results for stresses in contact zone of two pressed cylinders in contact, analyzed in detail in paper [1]. This procedure confirmed that the developed FEM gear models give very precise results for stresses values, especially because of the fact that points with maximum direct stress values lie on contact surfaces, while points with maximum equivalent stress values lie under contact surfaces on small deep.

Fig. 6 represents values of equivalent stress σ_{INT} ($\sigma_{INT} = MAX(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|)$, σ_1, σ_2 and σ_3 – direct stresses) for a pinion tooth in different cross sections along gear facewidth, i.e. in the transverse plane (fig.6a) and on the half of gear facewidth where the maximum values of contact stress and tooth root stress are expected (fig.6b). These results in relation with results obtained through the similar research of other authors, [7], [10], and [12], show that the developed models are very appropriate for gear stress state solving.

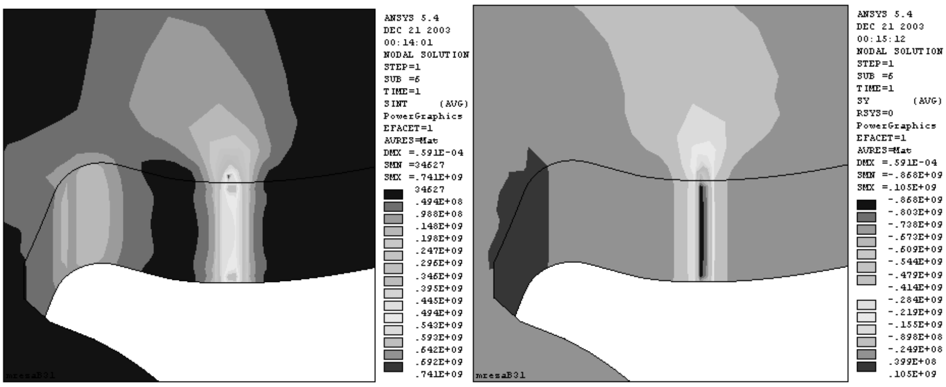

Fig. 4. Values of equivalent stress σ_{INT} and direct stress in y-direction σ_y for a pinion tooth

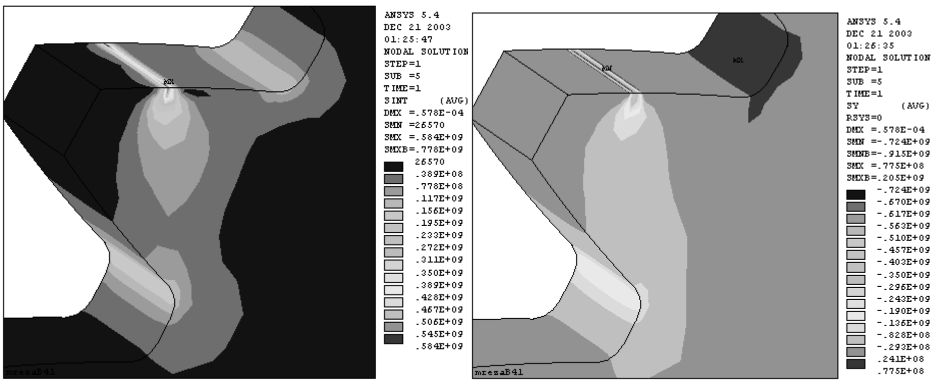

Fig. 5. Values of equivalent stress σ_{INT} and direct stress in y-direction σ_y for a wheel tooth

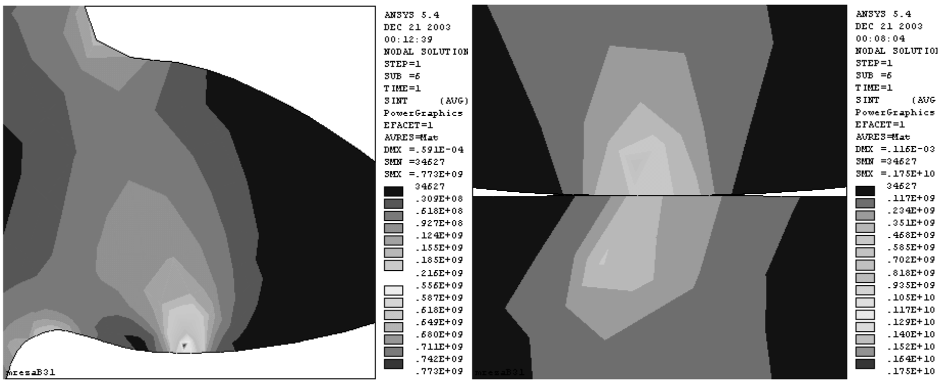

Fig. 6. Values of equivalent stress σ_{INT} for a pinion tooth in different cross sections along gear facewidth

Diagrams shown on fig. 7 represent the obtained results for some important variables: single tooth stiffness c' and unit nominal load value q. These diagrams are very suitable for monitoring and analyzing the variation of these variables over gear facewidth.

During the determination of real stress state competent for meshed gears load capacity, the standard procedure uses corresponding factors in order to take into account the effect of no-uniformly load distribution over the facewidth: $K_{H\beta}$ – for contact stress and $K_{F\beta}$ – for tooth root stress.

Face load factor $K_{H\beta}$ takes into account the influence of load distribution over gear facewidth on contact stress and is defined as a ration between the maximum load per unit facewidth w_{max} and average load per unit facewidth w_m:

$$K_{H\beta} = \frac{w_{max}}{w_m} = \frac{F_{max}/b}{F_m/b} \tag{5}$$

Then, in accordance with expression (5), value of face load factor for contact stress $K_{H\beta}$ for studied gear pair takes value of $K_{H\beta} = q_{max} / q_{sr} = 1.1093$. This very low value of face load factor for contact stress is expected and is the consequence of very small gear facewidth and small value of external load. According to the standard analytical calculations for face load factor for contact stress, a detail described in [4], this factor takes similar value.

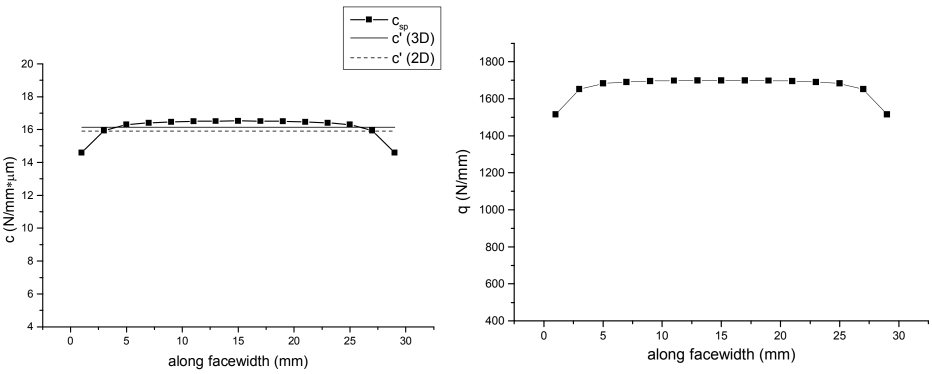

Fig. 7. Diagrams for monitoring of tooth stiffness and unit nominal load variation over facewidth – point B on path of contact (single meshed tooth pair period)

The diagram for monitoring of contact stress variation over facewidth are shown on fig. 8. The change of direct stress in *y*-direction σ_y is studied during this analysis. Also, the diagrams give a possibility to monitor the maximum equivalent stress values σ_{INT} over gear facewidth, which appear in points that lie on small deep under pinion tooth contact surface.

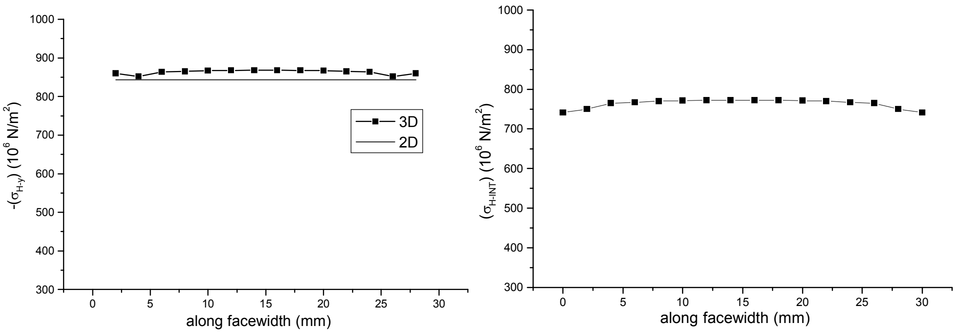

Fig. 8. Diagrams for monitoring of contact stress variation over facewidth – point B on path of contact (single meshed tooth pair period)

On the other hand, in order to analyze stress states in meshed teeth roots, the diagrams for monitoring of total stresses on pinion tooth root and wheel tooth root (σ_{F1} and σ_{F2}) variation over facewidth are created and shown on fig. 9. The values of these stresses are represented comparatively with corresponding values obtained through 2D FEM calculations, [2], while the influence of load distribution over gear facewidth is ignored.

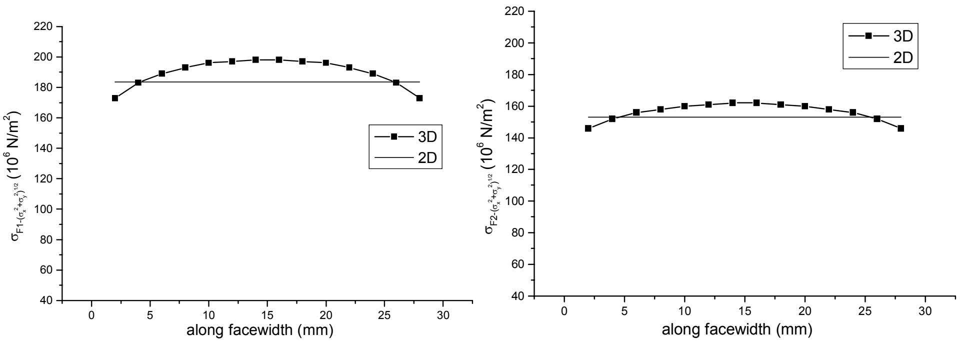

Fig. 9. Diagrams for monitoring of total toot root stresses variation over facewidth – point B on path of contact (single meshed tooth pair period)

According to the definitions based on common gear theory, [4], the face load factor for tooth root stress $K_{F\beta}$ can be calculated as ration between maximum value of equivalent stress when uninformed load distribution over gear facewidth is taking into consideration ($\sigma_{F\text{-}INT}$ from 3D FEM calculation) and maximum value of equivalent stress from calculation that neglects this influence and keep all other calculation requirements unmodified ($\sigma_{F\text{-}INT}$ from 2D FEM calculation). So, the face load factor $K_{F\beta(1)}$ – for pinion tooth and the $K_{F\beta(2)}$ – for wheel tooth for modeled gear pair takes the following values:

$$K_{F\beta(1)} = \left(\frac{\sigma_{F1-3D}}{\sigma_{F1-2D}} \right) = \frac{198.05}{183.604} = 1.07868 \tag{6}$$

$$K_{F\beta(2)} = \left(\frac{\sigma_{F2-3D}}{\sigma_{F2-2D}} \right) = \frac{161.98}{152.95} = 1.05904 \tag{7}$$

According to above analysis, the conclusion is that uneven load distribution over gear facewidth influences more on pinion tooth's root stress than on wheel tooth's root stress, even when the teeth contact line and external load have very small values. This fact raises a question of standard used principle of the same face load factor values for calculation of pinion tooth's root load capacity and wheel tooth's root load capacity, especially in cases of gear pairs with large contact ratio and high values of external load.

5. THE NUMERICAL RESULTS FOR FACE LOAD FACTOR WHEN FACEWIDTH DEVIATIONS EXIST

The real gears always have facewidth deviations, so this chapter represenst the 3D FEM models developed for the same gear pair and also for single meshed tooth pair period contact in point *B* (the point of passing from period with two tooth pairs in contact to single meshed tooth pair period), but with mesh misalignment in accordance with initial principles defined in chapter 2. For the chosen gear pair with the main characteristic variables defined in chapter 3, the maximum value of standard mesh misalignment deviation (ISO 1328-1, [14]) is equal to $F_{\beta y}=11\ \mu m$. Figure 10 represents the variation of equivalent stress σ_{INT} and direct stress in y-direction σ_y along facewidth for pinion tooth.

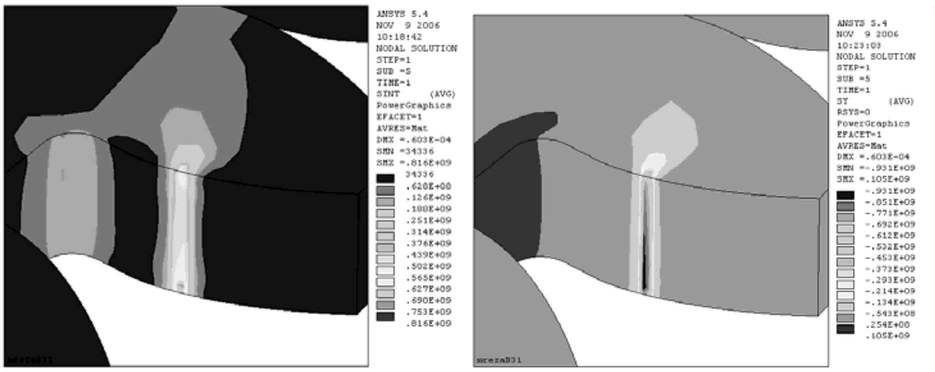

Fig. 10. Influence of mesh misalignemt on equivalent stress σ_{INT} and direct stress in y-direction σ_y – pinion tooth

The diagrams for monitoring of unit nominal load, equivalent contact stress and total root toot stress along gear facewidth are drawn from numerical results of FEM calculations when mesh misalignment exists. Figure 11 ÷ 13 represent these diagrams comparatively with corresponding diagrams derived from FEM calculations when facewidth deviations don't exist.

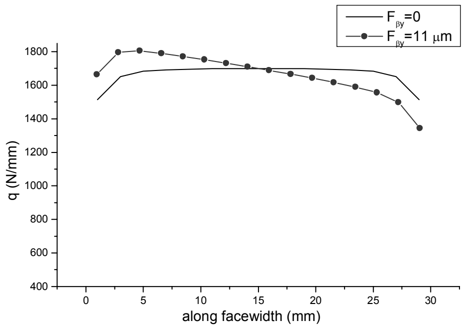

Fig. 11. Influence of mesh misalignemt on unit nominal load variation along facewidth

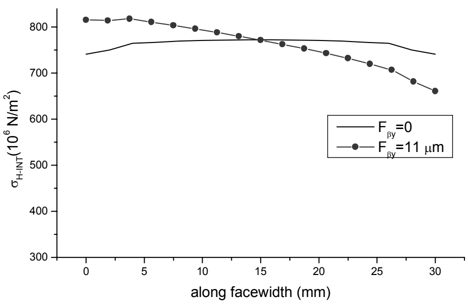

Fig. 12. Influence of mesh misalignemt on equivalent contact stress variation along facewidth

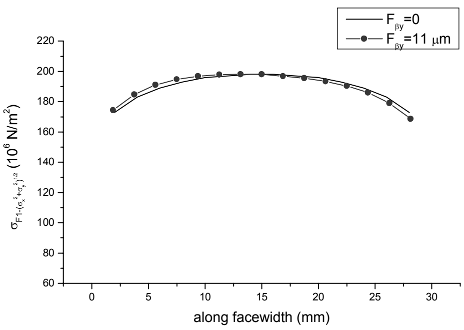

Fig. 13. Influence of mesh misalignemt on total root toot stress variation along facewidth – pinion tooth

From FEM results for this case of gear pair model and from expression (5), the face load factor for contact stress has value of $K_{H\beta} = 1.0831$ (FEM calculation for same gear pair without mesh misalignment gives $K_{H\beta} = 1.0193$ in chapter 4). Also, from the expression (6) face load factor for pinion tooth root stress has the value of $K_{F\beta(1)} = 198.06375 / 183.604 = 1.07875$ when mesh misalignment exists (FEM calculation for same gear pair without mesh misalignment gives $K_{F\beta(1)} = 1.07868$)

These results enable us to examine the effects of load distribution over gear facewidth with mesh misalignment on the stress state of teeth's flanks and teeth's roots. The described models and procedure are the completely new access in investigations of gear load capacity calculation.

6. Conclusions

The analysis of numerical results obtained in this paper by FEM calculations confirms that developed 3D FEM gear models give excellent results. 3D models for meshed gears register influence of load distribution over gear facewidth even when teeth contact lines have very small length and external load has value inherently smaller than real working load value. Otherwise, in such cases of nearly uniform values of deformation, stiffness and stress are expected along the facewidth.

It can be concluded that the developing of 3D FEM model for meshed gear pair in accordance with defined assumptions and goals, represents the successful solution for load distribution over gear facewidth and its influence on the gear load capacity. Also, it's very important to notice that real geometry conditions in teeth contact zone have been modeled with contact finite elements, which permit modeling of two deformable bodies in contact.

The solution for face load factor calculation described in this paper, also with the simultaneous incorporation of this influence in gear load capacity calculations, represents a very important step in comparison to former studies of other authors, which are all used the discrete method of Zablonski as the only principle for solution development. All the existed solutions, except the one described in this paper, are based on the large equation systems with many assumptions and only partial usage of numerical methods.

One special contribution of the described research is the possibility for taking into consideration the initial mesh misalignment (the deviation of meshed teeth's direction), as well as its different influence on stress values in contact zone and in tooth's roots. Also, the precise FEM models and calculations give us the fact that the face load factor for teeth's root stress state is not equal for pinion and wheel gear. This fact is opposite to the assumption used in widely used standard methods for gear load capacity calculation.

The described methodology and results give the basis for future research in order to incorporate the obtained conclusions in standard calculation methods.

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REFERENCES

1. Atanasovska I.: *The analysis of load distribution over simultaneously meshed tooth pairs with accent on the flanks load capacity*, M.A. Thesis, Kragujevac, 1999.
2. Atanasovska (Cvejić) I., *Load Distribution Influence on the Cylindrical Involute Gears' Load Capacity*, Dh Thesis, Kragujevac, 2004.
3. Zablonski К. И.: *Зубчатые передачи*, Техника, Киев, 1977.
4. Nikolić V.: *Machine parts, theory and examples*, Kragujevac, 1995.
5. Atanasovska (Cvejić) I., Nikolić V.: *The analysis of the factor of load distribution over tooth pairs in mesh, in respect of contact stress of tooth flanks*, Journal Technique, Mechanical Engineering 49, Belgrade, june 2000., page M1...M5
6. Linke H., Gajewski G.: *Breitenlastverteilung unter besonderer Berücksichtigung der Breitenballigkeit bei Verzahnungen*, Maschinenbautechnik, *N°*10, 1983., 445-449.
7. Linke H., Mitschke W.: Senf M.: *Einfluß der Radkorpergestaltung auf die Tragfähigkeit von Stirnradverzahnungen*, Maschinenbautechnik, 1983., 32, *N°*10, 450-457.
8. Nikolić, V., Bogdanović, G.: *Determination of stress state of gear drive at static and dynamic loadings*, MVM-International Journal for Vehicle Mechanics, Engines and Transportation Systems, Volume 20, Number 3, September, 1994.
9. Ristivojević, M.: Development of the theoretical model for determining the stress athoritative for chacking the gear tooth volume strength, Facta universitatis, Series: Mechanical Engineering Vol.1, Nº9, 2002., 1135-1145
10. Weck M., Bong B.: *Zur Kraftverteilung in Gleit – und Wälzkontakten*, VDI – Z, Bd.129 (1987) Nr11, 72-75.
11. Weck M., Neupert B.: *Berechnung der Beanspuchungen von Stirnradern mit Zahnflanken – Abweichungen*, Industrie, *N°*36v.5.5 1982/104., 35-39.
12. Moriwaki I., Saito K.: *Calculation of tooth deflection by global local finite element method (sampling points for accurate interpolation)*, International congress – Gear Transmissions'95, 26-28.09.1995., Sofia, 95-98.
13. Nikolić V, Atanasovska I.: *The finite element method solution of the load distribution over simultaneously meshed tooth pairs for cylindrical involute straight-tooth gears*, Conference with international participation IRMES'04, Kragujevac, September 16-17, 2004.
14. Standard ISO 1328-1: Cylindrical gears – ISO system of accyracy, Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth, 1995.

3D MKE MODEL ZUPČANIKA SA PRAVIM ZUPCIMA ZA NUMERIČKI PRORAČUN FAKTORA RASPODELE OPTEREĆENJA DUŽ DODIRNE LINIJE

Ivana Atanasovska, Vera Nikolić-Stanojević

Cilj ovog rada je da prikaže novi pristup u proračunu faktora raspodele opterećenja duž dodirne linije zubaca, koji je sastavni deo proračuna nosivosti evolventnih cilindričnih zupčanika sa pravim zupcima. Opisana metoda koristi numeričku metodu konačnih elemenata (MKE) i jedan potpuno novi pristup u pogledu određivanja faktora raspodele opterećenja duž dodirne linije.

U tom cilju, prvo je opisan kompleksni problem raspodele opterećenja u sprezi zupčastog para. U skladu sa analizom prikazanog matematičkog rešenja, autori su razvili 3D MKE model za istovremeno praćenje stanja napona i stanja deformacije i spregnutih bokova zubaca, podnožja zubaca i delova tela zupčanika. Za simulaciju kontakta bokova zubaca korišćeni su specifični kontaktni konačni elementi koji definišu kontakt dva deformabilna tela. Jedan deo rada posvećen je rešavanju raspodele opterećenja duž dodirne linije zubaca sa uzimanjem u obzir postojanja odstupanja u sprezi.

Analiza dobijenih numeričkih rezultata pokazuje da su razvijeni modeli veoma prikladni za istraživanje opisanog problema. Zaključci koji su dobijeni predstavljaju korak napred i novi aspekt u proračunu faktora raspodele opterećenja duž dodirne linije kod cilindričnih evolventnih zupčanika sa pravim zupcima.

Ključne reči: *zupčanici, raspodela opterećenja, metoda konačnih elemenata, kontaktna naprezanja*