OPTIMAL LQ CONTROLLER WITH ADDITIONAL DYNAMICS
FOR THE ACTIVE VIBRATION SUPPRESSION OF A CAR ROOF

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Abstract. Embedding active materials into mechanical structures, with the aim of the vibration suppression in the presence of critical excitations and disturbances, which can cause resonant states, represents an important field of research, with significant potentials for the application in the automotive industry. This paper presents the controller design and simulation for the active vibration suppression of a car roof with attached piezoelectric patches used as actuators and sensors. The controller is designed in order to suppress vibrations (which can also be the source of noise) in the presence of periodic disturbances/excitations having the same frequencies as the eigenfrequencies of the car roof. Such disturbances/excitations can cause vibrations in the resonance bandwidth and therefore they should be suppressed or avoided. Suggested controller is an optimal LQ controller with additional dynamics, which includes the knowledge about the frequencies of the excitations which can cause resonant states. For the controller design and simulation a state space model was developed using the finite element method (FEM) approach and the modal reduction. Piezoelectric material was modelled using the Semiloof shell elements, which have mechanical and electrical degrees of freedom. The controller performs significant suppression of the vibration magnitudes in comparison with the uncontrolled case.

Key words: optimal LQ controller, additional dynamics, vibration suppression, car roof, piezoelectric structure

1. INTRODUCTION

Development and implementation of actively controlled structures gains an increasing significance in the recent years. Active structures, which involve mechanical
part together with actuators and sensors and appropriate control strategy, are the subject of research and application in many areas of engineering, including automotive applications as well. The aim of this paper is to present the controller design strategy, which can be used for the vibration suppression in the presence of the periodic excitations which can cause resonant states. A potential application in the automotive industry is demonstrated on the simulation example of an actively controlled car roof with attached piezoelectric patches as actuators and sensors, with the aim of the vibration magnitudes suppression in the presence of disturbances. Controller design is based on the state space model obtained using the FEM approach and modal reduction. Suggested controller is an optimal LQ controller with additional dynamics, which includes the knowledge about the frequencies of the excitations, which can cause resonant states.

The concept of the paper can be summarized in the following way. First the theoretical backgrounds of the FEM modelling and controller design are explained. The FEM modelling was performed using the finite element software COSAR [1]. Optimal LQ controller design with additional dynamics, which compensates for the presence of the periodic excitations, is introduced subsequently. Implementation of the suggested control technique is performed through a simulation on the car roof model of a Volkswagen (VW) test car "Bora" [5]. Simulation results of the vibration suppression in the presence of disturbances are presented in the subsequent section. Frequency responses and time-domain simulation results show significant vibration magnitude reduction not only in comparison with the uncontrolled case, but also compared with results of the standard optimal LQ controller. Finally the main conclusions are presented.

2. FEM APPROACH TO MODELING OF PIEZOELECTRIC STRUCTURES

For the controller design purposes the state space model of a structure is obtained using the FEM approach, which takes into account piezoelectric behaviour of active materials. Electro-mechanical behaviour of such structures is described using basic equations of piezoelasticity and the finite element formulation [6], [7]. The coupled electromechanical behavior of a piezoelectric smart material can be described with adequate accuracy by linearized constitutive equations:

\[
\sigma = \frac{\partial H}{\partial \varepsilon} = C \varepsilon - e E_f, \quad D = -\frac{\partial H}{\partial E} = e' \varepsilon + \kappa E_f
\]

(1)

which can be derived by partial differentiation from the expression of the potential function (2) represented in a quadratic form of the primary field variables: mechanical strain \(\varepsilon\) and electric field \(E_f\):

\[
H = \frac{1}{2} e' C \varepsilon - e' E_f e' - \frac{1}{2} \kappa E_f
\]

(2)

where \(\sigma\) represents the mechanical stress vector, \(C\) symmetric elasticity matrix, \(e\) piezoelectric matrix, \(D\) vector of electrical displacement, \(\kappa\) symmetric dielectric matrix. The system of equations, which describe electromechanical behaviour, consists of the constitutive equations (1) together with the mechanical equilibrium and electric equilibrium (charge equation of electrostatics resulting from the 4th Maxwell equation):
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\[
\text{div} \, \sigma + P - \rho \dot{u} = 0, \quad \text{div} \, D = 0
\]  \hspace{1cm} (3)

where \( P \) represents the body force vector, \( u \) is the vector of mechanical displacements described in Cartesian system of coordinates and \( \rho \) is the mass density.

As a result of the FE analysis, behaviour of a structure is approximated by an arbitrary number of finite elements. Based on the equations of motion of a single element, through the element assembly procedure and adding up all element contributions, the resulting assembled equation of motion of a piezoelectric structure can be expressed in a general form:

\[
M \ddot{q} + D \dot{q} + K q = \bar{F}
\]  \hspace{1cm} (4)

where \( q \) is the vector of generalized displacements (mechanical displacements and electric potentials) and \( M, D, \) and \( K \) are the mass matrix, the damping matrix and the stiffness matrix, respectively. The total load vector \( \bar{F} \) is divided into the vector of the external forces \( F_E \) and the vector of the control forces \( F_C \)

\[
\bar{F} = F_E + F_C = \bar{E} w(t) + \bar{B} u(t)
\]  \hspace{1cm} (5)

where the forces are generalized quantities including also electric charges (or voltage). Vector \( w(t) \) represents the vector of external disturbances and \( u(t) \) is the vector of the controller influence on the structure. Matrices \( \bar{E} \) and \( \bar{B} \) describe the positions of the forces and the control parameters in the finite element structure, respectively.

Assembled equation of motion (4), which contains many degrees of freedom, is transformed to a form convenient for the controller design, i.e. to a state space formulation using the procedure of modal reduction [8]. Modal reduction is performed by taking into account eigenmodes in the frequency range of interest, reducing thus the higher-order finite element models to a reasonable order convenient for the controller design. Together with the modal form of the measurement equation, the state space model of the structure is obtained in the form:

\[
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t) + Ew(t) \\
y &= Cx(t) + Du(t) + Fw(t)
\end{aligned}
\]  \hspace{1cm} (6)

where \( x \) is the modal state vector, \( A \) denotes the state matrix, \( B \) is the input matrix, \( E \) is the disturbance coupling matrix, \( C \) output matrix and \( F \) is disturbance-to-output coupling matrix.

Discrete-time equivalent of the continuous-time state space model (6) is used as a starting point for the controller design:

\[
\begin{aligned}
\begin{array}{c}
x[k + 1] = \Phi x[k] + \Gamma u[k] + e w[k] \\
y[k] = C x[k] + D u[k] + F w[k]
\end{array}
\end{aligned}
\]  \hspace{1cm} (7)

where

\[
\begin{aligned}
\Phi &= e^{A T}, & \Gamma &= \int_0^T e^{A \tau} B d\tau, & e &= \int_0^T e^{A \tau} E d\tau
\end{aligned}
\]  \hspace{1cm} (8)

and \( T \) is the sampling time.
3. DESIGN OF THE OPTIMAL LQ CONTROLLER WITH ADDITIONAL DYNAMICS

Optimal LQ controller shows good performances when used for the vibration suppression. Nevertheless, for specific excitation/disturbance cases, e.g. periodic excitations, an optimal LQ tracking system with additional dynamics [4], [8], [10] has shown even better performances, especially in critical cases, when disturbances can excite resonant states. The controller design includes available a priori knowledge about occurring disturbance type contained in the additional dynamics, which represents an important part of the controller design procedure. Additional dynamics is introduced in order to compensate for the presence of disturbance, providing at the same time tracking of the reference trajectories described by the models with the same poles as those of disturbances. Such controller with additional dynamics features serves controlling purposes if the reference input to be tracked and the disturbance acting upon the structure can be described by a rational discrete function. This condition is fulfilled by a sine function used as a disturbance model. A special interest in investigation of this type of disturbances has arisen from the fact that periodic disturbances with frequencies corresponding to the eigenfrequencies of the structure can cause resonance.

Additional dynamics is formed from the coefficients of the polynomial:
\[
\delta(z) = \prod_j (z - e^{j\omega_j})^n = z^n + \delta_1 z^{n-1} + \ldots + \delta_n
\]

where \(\lambda_j\) are the poles of the reference input and/or excitation/disturbance. A state space realization of the additional dynamics is expressed in the form of matrices:

\[
\Phi_s = \begin{bmatrix}
-\delta_1 & 0 & \cdots & 0 \\
-\delta_2 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
-\delta_{n-1} & 0 & \cdots & 1 \\
-\delta_n & 0 & \cdots & 0
\end{bmatrix}, \quad \Gamma_s = \begin{bmatrix}
-\delta_1 \\
-\delta_2 \\
\vdots \\
-\delta_{n-1} \\
-\delta_n
\end{bmatrix}
\]

(10)

In the case of multiple-input multiple-output (MIMO) systems, additional dynamics must be replicated in \(q\) parallel systems (once per each output), where \(q\) is the number of outputs. Replicated additional dynamics is described by:

\[
\Phi_d = diag(\Phi_{s1}, \ldots, \Phi_{sq}), \quad \Gamma_d = diag(\Gamma_{s1}, \ldots, \Gamma_{sq})
\]

(11)

The discrete-time design model \((\Phi_d, \Gamma_d)\) is formed as a cascade combination of additional dynamics \((\Phi_a, \Gamma_a)\) and \((\Phi, \Gamma)\) and the discrete-time plant model \((\Phi, \Gamma)\):

\[
x_{d}[k+1] = \Phi_{d} x_{d}[k] + \Gamma_{d} u[k]
\]

(12)

\[
\Phi_{d} = \begin{bmatrix} \Phi & 0 \\ \Gamma C & \Phi \end{bmatrix}, \quad \Gamma_{d} = \begin{bmatrix} \Gamma \\ 0 \end{bmatrix}, \quad x_{d} = \begin{bmatrix} x[k] \\ x_{d}[k] \end{bmatrix}
\]

(13)
where $\Phi^i$ denotes $\Phi_1$ or $\Phi_2$, and $\Gamma^i$ represents $\Gamma_1$ or $\Gamma_2$, depending on whether the controlled structure is modelled as a single-input single-output or a multiple-input multiple-output system, respectively. Gain matrix $L$ of the optimal LQ regulator is calculated on the basis of the design model (12) in such a way that the feedback law $u[k] = -Lx_d[k]$ minimizes the performance index (14) subject to the constraint (12), where $Q$ and $R$ are symmetric, positive-definite weighting matrices.

$$J = \frac{1}{2} \sum_{k=0}^{\infty} (x_q[k]^TQx_q[k] + u[k]^TRu[k])$$

State variables of a model obtained using described FEM approach are modal variables, which are not measurable and their estimation is therefore necessary. For the estimation of the state variables the Kalman filter can be used. Equations for the Kalman filter design based on the current estimator [2] assume the state space equation of the controlled structure in the form (7) and the measurements depending on the state variables and influenced by the measurement noise. The Kalman estimator is then defined by the following equations:

$$\dot{x}[k] = A[x[k]] + L_{est}[k](y[k] - Cx[k])$$
$$\hat{x}[k] = \Phi\hat{x}[k-1] + \Gamma u[k-1]$$

where $L_{est}$ represents the Kalman gain matrix, which is obtained by solving a discrete Riccati equation [2], [8]. Optimal LQ tracking control system with additional dynamics and Kalman estimator is represented in Fig. 3.

4. VIBRATION SUPPRESSION OF THE CAR ROOF

Vibration suppression using the suggested control technique was shown through a numerical simulation performed on a test structure – car roof of the Volkswagen (VW) test car "Bora" (Fig. 1a). Piezoelectric patches attached to the surface of the car roof are used as actuators and sensors. Excitation by shakers at prescribed points is intended for the experimental investigations (Fig. 1b). For the experimental modal analysis the car roof was excited by an impulse excitation at predefined points. The FEM model including the piezoelectric effects of the actuator/sensor groups was obtained on the basis of the CAD geometry (Fig. 2b) of the car roof using the FEM software COSAR [1].

Based on the generated FEM mesh, an optimization of the actuator/sensor placement was performed under consideration of the eigenmodes of interest and the controllability index [3], [9] for the selected eigenmodes as a criterion for the optimal placement. The actuator/sensor placement shown in Fig. 3 describes one of the test cases, which was calculated based on the controllability index. Comparison of the calculated an experimentally determined eigenfrequencies showed a good agreement in the considered frequency range.
The state space model used for the simulation of the controlled vibration suppression was obtained on the basis of the finite element model with totally 10453 degrees of freedom and 3566 finite elements, thereof 3320 shell elements and 246 beam elements. For the controller design a modally reduced state space model was used, which takes into account five selected eigenfrequencies: $f_1=48.45\text{Hz}$, $f_2=51.12\text{Hz}$, $f_3=63.23\text{Hz}$, $f_4=64.67\text{Hz}$ and $f_5=68.00\text{Hz}$. Accordingly, the state space model of the form (6) has order 10. Corresponding eigenforms are represented in Fig. 2a.

Fig. 1. a) VW test car "Bora"; b) passenger compartment and inner surface of the car roof with attached piezoelectric actuators/sensors and exciting shakers

Fig. 2. a) Selected eigenforms of the car roof obtained on the basis of the FEM model; b) CAD model and the FEM mesh of the car roof;
Using the control concept with optimal LQ controller, additional dynamics and Kalman estimator (shown in Fig. 3) the simulation of the vibration suppression performances was performed in order to show the potentials of the control strategy.

The influence of the excitations is described by disturbance coupling matrix $E$ in the continuous state space model (6) or by the corresponding matrix $\varepsilon$ in the discrete-time state space model (7), (8). The elements of the vector of excitations $w = [e_1 e_2]^T$ correspond to the excitations in prescribed points 1 and 2, respectively.

In accordance with described controller design procedure, periodic sine excitations were considered, since they can cause resonant states. It was investigated how the controller can suppress the vibration amplitudes, measured by the voltage at the sensor patches in the presence of such sinusoidal disturbances with frequencies which are equal to the selected eigenfrequencies of the car roof. The eigenfrequencies $f_1 = 48.45$Hz, $f_2 = 51.12$Hz, $f_3 = 63.23$Hz were selected as the frequencies of the exciting sinusoidal signals. The signals obtained as the sum of the three sinusoids with these frequencies were also considered as the excitations.

![Fig. 3. Actuator/sensor placement and control concept](image)
Obtained results are represented in the time domain as well as in the frequency domain. As a starting point for the controller design a discrete time state space model obtained with the sampling time $T_s = 0.0001s$ was used. For the optimal controller design with additional dynamics, which takes into account the eigenfrequencies $f_1$, $f_2$, and $f_3$, a design model of the order 22 was formed according to (13). Such a controller can handle different excitation cases: a single sinusoidal excitation with the frequency equal to either of the eigenfrequencies $f_1$, $f_2$, or $f_3$ or an excitation which is obtained as a sum of different sinusoidal signals with the frequencies corresponding to the selected eigenfrequencies. In order to design an optimal LQ controller with additional dynamics, the weighting matrices in (14) were selected as follows: $Q = 1000 I_{22 \times 22}$, $R = 1000 I_{4 \times 4}$. For the Kalman estimator, the Kalman gain $L_{est}$ was determined using the Matlab function `kalman`, where the process noise and the measurement noise covariances are assumed to be $Q_w = 0.0706 I_{2 \times 2}$ and $R_v = 0.01 \text{ones}(2,2)$, respectively. In order to show the performance of the optimal LQ controller with additional dynamics in comparison with the standard LQ controller, the simulations were also performed with the standard LQ controller, where the weighting matrices were selected in a similar manner: $Q = 1000 I_{10 \times 10}$, $R = 1000 I_{4 \times 4}$. Lower order of the weighting matrix $Q$ in this case is due to the order of the state space model of the car roof used for the controller design, which now does not take into account the additional dynamics. Simulation results of the vibration suppression in the time domain are represented in Fig. 4.

Fig. 4. Time responses of the controlled and uncontrolled vibrations
It is assumed that the excitation consisting of the sum of the three sinusoidal signals acts as the excitation $e_1$. Outputs $y_1$ and $y_2$ correspond to the voltage signals at sensors S1 and S2 (Fig. 3), respectively. Some portions of the output diagram for $y_2$ are zoomed in order to show the difference between the results obtained with the standard LQ controller and the one with additional dynamics.

In order to show how the controller influences the frequency responses of the car roof, a selection of results for the frequency response functions of the uncontrolled and controlled system is represented in the following figures. Frequency response functions are obtained as a ratio between the Fast Fourier Transforms (FFT) of the output signal to input signal. Since the model of the car roof represents a multiple-input multiple-output system with four control inputs (actuator signals), two outputs (sensor signals) and two excitation inputs, the frequency responses are obtained considering specified excitation inputs and outputs in the presence or absence of control.

Frequency response can be obtained as a FFT of the impulse response. Comparison between the impulse response of a car roof model without control and the frequency responses in the presence of the sine and impulse excitation in the presence of a standard optimal LQ controller is represented in Fig. 5. Sinusoidal excitation is composed as a sum
of the three sinusoids with frequencies equal to the eigenfrequencies of the car roof $f_1, f_2, f_3$. Solid line represents the frequency response obtained as a FFT of the impulse response at the output $y$ (output measured at appropriate sensor patch) when the appropriate excitation point is excited by an impulse. It was assumed that the control input equals zero in order to show the pure frequency response excitation–sensor. Considering the same appropriate excitation–sensor pairs, the control input was introduced in order to see the effect of the controlled magnitudes reduction. As a result the dash-dotted line shows the influence of the standard optimal LQ controller. The same controller shows the frequency domain performance represented by the dashed line in the presence of the sin excitation composed as a sum of three sinusoidal signals. As a result of the superposition, the peak amplitude is present in the response.

The influence of different excitations to frequency responses can be seen from the Fig. 6. If an excitation represents a sum of several sinusoids with different frequencies equal to the eigenfrequencies of the car roof, the peak magnitudes corresponding to these eigenfrequencies become larger. It can be seen from the upper three lines (dotted, solid and dash-dotted) of the frequency responses in Fig. 6. Implementation of the optimal LQ controller without additional dynamics results in the magnitude reduction even at the critical frequencies, but strongly emphasized peaks at certain frequencies as a result of the sinusoidal superposition are still present (dashed line).

Fig. 6. Frequency responses to different sinusoidal excitations without control and with standard optimal LQ controller without additional dynamics
Additional dynamics introduced to reduce the vibration amplitudes in the presence of the periodic excitations affects the frequency response in the sense of moving the frequency response diagram to lower magnitude values and reducing obviously the peaks at the eigenfrequencies. Such an influence is shown in the Fig. 7, where the frequency responses for the excitation $e_2$ composed as a sum of three sinusoidal signals is shown with respect to the outputs $y_1$ and $y_2$. Since the frequency responses in the presence of the same excitation signal exerted at the other excitation point (corresponding to the excitation $e_1$) are represented by the similar diagrams, they are not shown in this paper. In comparison with the optimal LQ controller without additional dynamics (dashed), much greater magnitude reduction can be observed (solid).

![Frequency responses in the presence of the sinusoidal excitation](image)

Fig. 7. Frequency responses in the presence of the sinusoidal excitation which is a sum of the three sinusoidal signals with frequencies equal to the eigenfrequencies: without controller, with the standard LQ controller and with the optimal LQ controller with additional dynamics

5. CONCLUSION

In this paper a controller design procedure is suggested based on the state space model obtained through the FEM approach, which can be used for the successful vibration suppression of the car roof with attached piezoelectric actuators and sensors. The comparison of the uncontrolled and controlled cases shows significant reduction of the vibration magnitudes in the presence of the controller. Results are presented in the frequency and time domains. The controller was also compared with the standard optimal LQ controller without additional dynamics which compensates for the presence of the periodic sinusoidal excitations with critical frequencies. The comparison shows much better vibration suppression in the presence of the controller with additional dynamics.

Presented work represents simulation investigations of the controller behaviour. It is also meaningful in the case when the optimal placement of the actuators and sensors should be confirmed, which was here the case. In the early design phases, besides the overall controllability index [9] as a criterion for the optimal actuator/sensor placement, the simulation results of the controller performance can also be an indication of the quality of the placement. The advantage is the possibility of the iterative changing of the actuator/sensor placement without influencing the real structure. In a similar manner the
contribution to the controller design and its performance testing can be viewed in the frame of the possibility to change the controller parameters and to test different controller design approaches in order to achieve optimal performances.

As a next investigation step an experimental verification would be necessary in order to confirm the performance of the controller in the presence of different excitation types under real operating conditions and model uncertainties. The robust performance of the controller should compensate for the deviations of the modelled case from the real conditions, as well as for the model inaccuracies originating from the variation of properties in a series of products.

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OPTIMALNI LQ KONTROLER SA DODATNOM DINAMIKOM ZA AKTIVNU REDUKCIJU OSCILACIJA KROVA AUTOMOBILA

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Integrisanje aktivnih materijala u mehaničke strukture u cilju prigušenja vibracija u prisustvu kritičnih pobuda i poremećaja koji mogu izazvati rezonantna stanja predstavljaju značajan oblast
optimal LQ controller with additional dynamics for the active vibration suppression of a car roof

istraživanja sa velikim potencijalom za primenu u automobilskoj industriji. U radu je prikazano projektovanje upravljanja i simulacija aktivnog redukovanja oscilacija krova automobila sa aktivnim piezoelektričnim davačima u ulozi aktuatora i senzora. Upravljanje je projektovano sa ciljem redukovanja oscilacija (koje se mogu smatrati i izvorom buke) u prisustvu periodičnih poremećaja/pobuda sa frekvencijama koje se poklapaju sa sopstvenim frekvencijama krova automobila. Takvi poremećaji/pobude mogu prouzrokovati oscilacije u rezonantnom frekventnom opsegu, zbog čega je neophodno njihovo redukovanje. Projektovani upravljački sistem bazira se na optimalnom LQ kontroleru sa dodatnom dinamikom, koja uključuje informacije o pobudama koje mogu izazvati rezonantna stanja. U cilju projektovanja upravljačkog sistema razvijen je model u prostoru stanja na osnovu modeliranja metodom konačnih elemenata i postupkom modalne redukcije reda modela. Piezoelektrični materijal je modelovan primenom Semiloof elementa tipa ljuske, sa mehaničkim i električnim stepenima slobode. Zapažena je znatna redukcija amplituda oscilacija u prisustvu upravljanja u poređenju sa slučajem bez primene upravljačkog sistema.

Ključne reči: optimalni LQ kontroler, dodatna dinamika, redukovanje oscilacija, krov automobila, piezoelektrična struktura.